PRACTICAL EXAMPLES OF REINFORCED CONCRETE DESIGN

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PRACTICAL EXAMPLES OF REINFORCED CONCRETE DESIGN

IN ACCORDANCE WITH THE CODE OF PRACTICE FOR THE USE OF REINFORCED CONCRETE IN BUILDINGS (1934) AND THE LONDON COUNTY COUNCIL BUILDING BY-LAWS (1938)

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INTRODUCTION

THE Building By-laws issued in January 1938 by the London County Council contain a number of features relating to reinforced concrete design that differ from earlier regulations. Most of the variations concern superimposed loadings and permissible stresses, and an apparent difference is the absence of working formulæ. This, however, is largely compensated for by the Memorandum issued by the Council which gives formulæ and rules interpreting the By-laws for the purpose of the practical design of reinforced concrete.

This volume presents a series of convenient aids to design in accordance with the London County Council's By-laws and Memorandum and also in accordance with the "Recommendations for a Code of Practice for the use of Reinforced Concrete in Building" issued in 1934 by the Building Research Board. The subject is dealt with from the point of view of the complete design of a reinforced concrete building planned to combine as many aspects of the By-laws and the Code as possible. Although the hypothetical structure considered shows a simple beam-and-slab construction for the floors, alternative designs embodying slabs spanning in two directions and flat-slab floors are prepared. In the final chapters designs for foundations of various types are considered, together with the design of a small concrete tank. The book is therefore a comprehensive guide to present-day British practice in reinforced concrete in accordance with the London County Council's By-laws and the Recommendations for a Code of Practice.

In many respects the recommendations of the Code comply with the requirements of the By-laws or Memorandum and, unless comments to the contrary are made, the tables, calculations, and designs given in this volume apply equally to the Code and the By-laws. Where differences occur, matter that complies with the Code only is indicated by a black line in the margin.

Although the By-laws and Code are here applied to a limited number of designs, the tables accompanying the calculations are of general application.

In those few places where the wording of either the By-laws or the Code involves slight ambiguity or is lacking in comprehensiveness, the author's interpretation of the spirit of the By-laws or Code is given. No doubt usage will, in time, remove any ambiguity and precedents will be established for such matters as are left to the discretion of the District Surveyor.

The author's thanks are due to the London County Council for kind permission to reproduce the Building By-laws, the explanatory Memorandum, and the regulations concerning welding and the use of high-yield-point steel.

C. E. R.

May 1938

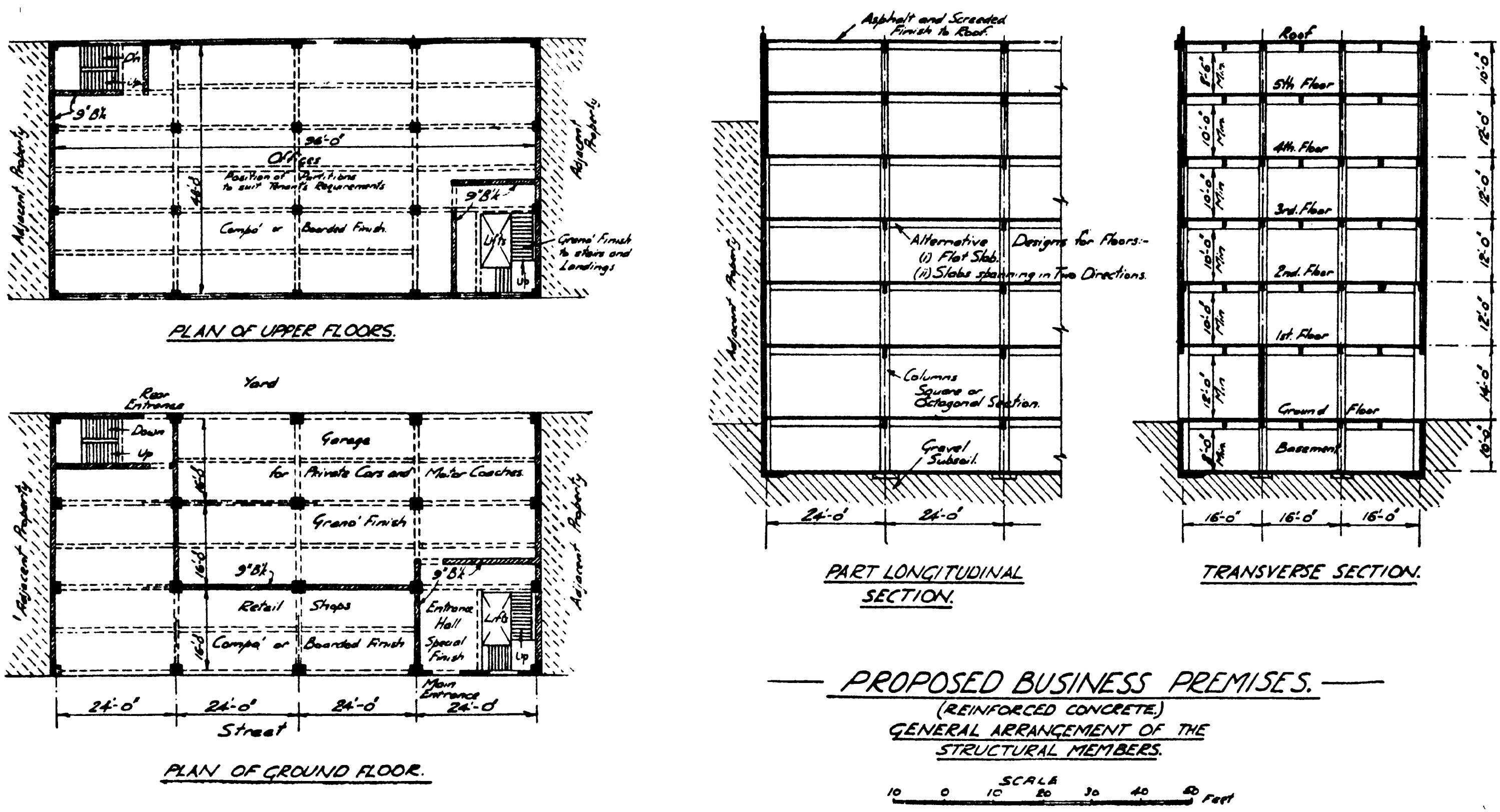


FIG. 1.—PROPOSED BUILDING.

CHAPTER I

LOADS

BEFORE commencing the detail design of the building illustrated in *Fig.* 1, loadings, working stresses, and design factors will be considered as these concern all parts of the subsequent calculations.

Superimposed Loads on Slabs and Stairs.

For the consideration of superimposed loads, By-law 4 * divides floors, stairs, and roofs into eight classes, of which classes Nos. 1 to 6 inclusive and Class No. 8 are given on *Table* 1. Class No. 7, omitted from *Table* 1, includes any purpose for which a floor may be used that is not specified in any of the other classes. For floors of Class No. 7 the loading to be used in design is to be ascertained and approved by the district surveyor.

In the Code of Practice † many more types of floors are named than in the By-laws and those that are omitted from the latter are given on *Table* 1 for guidance in unspecified cases. It should be observed that the class reference numbers in the Code do not conform to those in the By-laws.

With the exception of roofs (Class No. 8) and garage floors under Class No. 5, there is specified for each class a total minimum uniformly distributed superimposed load as well as the normal intensity of superimposed load. The values of both these loadings are summarised on *Table* 1, and *Table* 2 has been prepared to facilitate assessment of the load for which any span of slab in any class should be calculated.

In the case of slabs spanning in two directions the Code recommends that the shorter span should determine the superimposed load to be allowed, but the By-laws give no ruling on this point.

It is important to observe that the By-laws stipulate that the alternative superimposed load shall operate on "an otherwise unloaded floor." Although the meaning is clear when a single span slab is considered, it is not obvious whether in the case of a series of continuous spans it is intended that only one span of the series should be so loaded at any one time, or whether it is necessary to arrange the alternative loads on two or more spans in such sequence as to give the maximum bending moments at various critical sections. Adoption of the latter meaning will give moments substantially in excess of those obtained by the former. The equivalent loadings given on *Table* 2 (and *Table* 5 for beams) will apply to all cases of single or multiple spans, the difference being only in the basis of calculating the maximum moments. Conditions where either interpretation

* The London County Council's By-laws are given in Appendix I, page 217.

† The Recommendations for a Code are given in full in "Explanatory Handbook on the Code of Practice for Reinforced Concrete," by W. L. Scott and W. H. Glanville (see page 259).

TABLE 1.

LOADS ON FLOORS, ROOFS AND STAIRS.

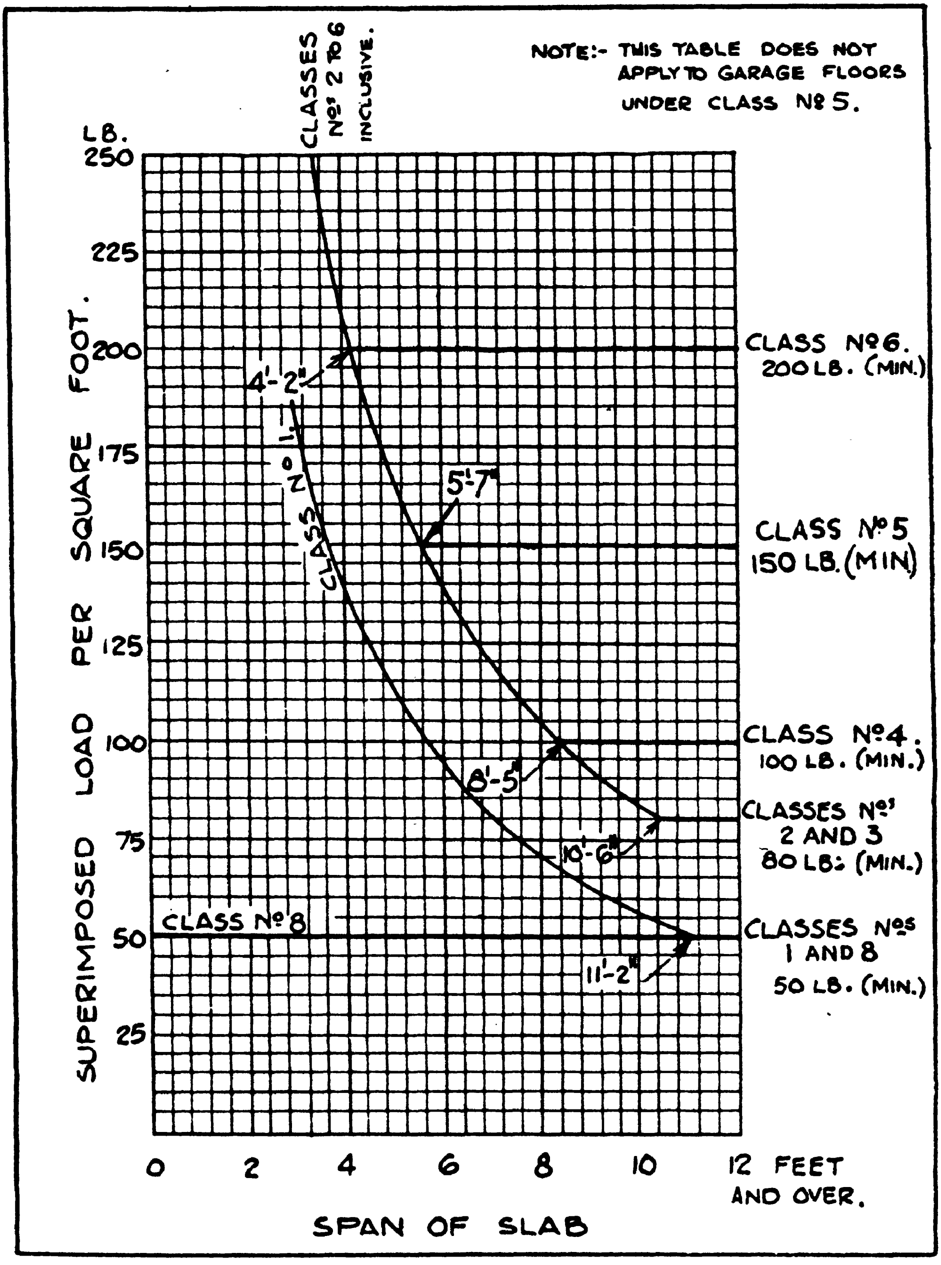
(By-laws and Code.)

FLOORS, ETC. SPECIFIED IN LONDON BUILDING BY-LAWS.						FLOORS SPECIFIED IN CODE OF PRACTICE BUT NOT SPECIFICALLY MENTIONED IN LONDON BY-LAWS.	
		SUPERIMPOSED LOADING.					
		SLAB.		BEAMS.			
CLASS NO.	DESCRIPTION	LB. PER SQ. FOOT OF FLOOR AREA.	MIN. TOTAL LOAD PER FT. WIDTH. (UNIFORMLY DISTRIBUTED).	LB. PER SQ. FOOT OF FLOOR AREA.	MIN TOTAL LOAD. (UNIFORMLY DISTRIBUTED).	DESCRIPTION	SUPERIMPOSED LOADING
1	RESIDENTIAL ROOMS CORRIDORS STAIRS AND LANDINGS WITHIN CURTILAGE OF RESIDENCE.	50	¼ TON.	40	1 TON.	HOTEL BEDROOMS HOSPITAL ROOMS AND WARDS.	AS CLASS 1
2	OFFICE FLOORS ABOVE ENTRANCE FLOOR	80	⅜ TON	50	2 TONS.	–	–
3.	OFFICE FLOORS: ENTRANCE FLOOR AND BELOW ENTRANCE FLOOR. RETAIL SHOPS. GARAGES FOR MOTOR CARS NOT EXCEEDING 2¼ TONS NET WEIGHT.	80	⅜ TON.	80	2 TONS.	CHURCHES. SCHOOLS. READING ROOMS. ART GALLERIES.	AS CLASS 3
4.	CORRIDORS, STAIRS AND LANDINGS. EXCEPT THOSE UNDER CLASS 1.	100 (MIN.)	⅜ TON.	100 (MIN.)	2 TONS.	ASSEMBLY HALLS. DRILL HALLS. DANCE HALLS. GYMNASIA. PUBLIC SPACES IN HOTELS AND HOSPITALS THEATRES. CINEMAS RESTAURANTS. GRANDSTANDS	AS CLASS 4
5.	WORKSHOPS. FACTORIES.	150 (MIN.)	⅜ TON.	120 (MIN.)	2 TONS.	–	–
	GARAGES FOR MOTOR VEHICLES EXCEEDING 2¼ TONS NET WEIGHT.	150	1·5 × MAX. COMBINATION OF WHEEL LOADS	120	1·5 × MAX. COMBINATION OF WHEEL LOADS		
6.	WAREHOUSES. BOOK STORES. STATIONERY STORES.	200 (MIN.)	⅜ TON.	200 (MIN.)	2 TONS.	–	–
8.	FLAT ROOFS. ROOFS INCLINED AT NOT MORE THAN 20° TO HORIZONTAL.	50	–	30	–	–	–
–	ROOFS INCLINED AT MORE THAN 20° TO HORIZONTAL.	15 LB. PER SQ. FT. NORMAL TO SURFACE ACTING INWARDS ON WINDWARD SIDE. 10 LB. PER SQ. FT. NORMAL TO SURFACE ACTING OUTWARDS (NOT SIMULTANEOUSLY) ON LEEWARD SIDE.				–	–

TABLE 2.

MINIMUM SUPERIMPOSED LOADS ON SLABS.

(By-laws.)



may be either unduly sanguine or severe are discussed in Chapter VII where a table presenting additional data for moment calculation is given. The intensity of loadings given for Classes Nos. 4, 5, and 6 are absolute minimum values, and for storage spaces they should be increased to the calculated weights of the contents. For instance, it is usual when designing paper warehouses, printing works, and similar buildings to calculate for a superimposed load of not less than 336 lb. per square foot.

For the floors of garages for vehicles exceeding $2\frac{1}{4}$ tons net weight the alternative superimposed loads are either 150 lb. per square foot or a concentrated load of " $1\frac{1}{2} \times$ maximum wheel load." The minimum value of the wheel load must be 1 ton.

The By-laws do not specify over what area this load can be spread, but the Code recommends that it can be considered as spread over an area 2 ft. 6 in. square. There is no suggestion that this area can be further increased to allow for dispersion through the thickness of the slab, and for the present purpose it is assumed that an area of 2 ft. 6 in. by 2 ft. 6 in. is the maximum allowable for all slab thicknesses. A further difference between the Code and the By-laws is the loading specified for garage floors. The border line between light and heavy motor vehicles is taken as 2 tons in the Code compared with $2\frac{1}{4}$ tons in the By-laws. The minimum superimposed load in accordance with the Code is 200 lb. for beams and slabs compared with 150 lb. and 120 lb. per square foot respectively. The Code recommends that the alternative minimum superimposed load on garage floors should be " 1·5 times maximum wheel load but not less than 1 ton," whereas the By-laws, as mentioned, require the minimum wheel load to be 1 ton, giving a minimum design load of $1\frac{1}{2}$ tons.

To assist the design of garage floors, typical weights of common types of motor vehicles are given on *Table* 4, while the curves on *Table* 3 give the equivalent uniformly distributed load per foot width of slab for bending moment due to a load of 1 ton distributed over the permissible area. The equivalent uniformly distributed loads for concentrated loads exceeding 1 ton will be proportional. These curves are drawn for simply supported spans and for fixed-end spans; intermediate conditions of continuity can be interpolated. The possibility of goods and passenger vehicles using the garage while fully laden should be considered, and for this purpose the maximum loaded axle weights given on *Table* 4 should be of value. It will be observed from this table that most British-made private cars impose loadings falling within Class No. 3.

For roof slabs outside Class No. 8, that is, inclined at an angle of more than 20 deg. to the horizontal, the superimposed load (as indicated on *Table* 1) is taken as 15 lb. per square foot acting inwards on the windward side and 10 lb. per square foot acting outwards on the leeward side; both loads are taken normal to the slope of the roof slab, and are not to be taken as acting simultaneously.

From the slab loading curves we find the following principal superimposed loads for the building shown in *Fig.*-1.

FROM TABLE 2:

Flat roof (Class No. 8), span 8 ft.—50 lb. per square foot.

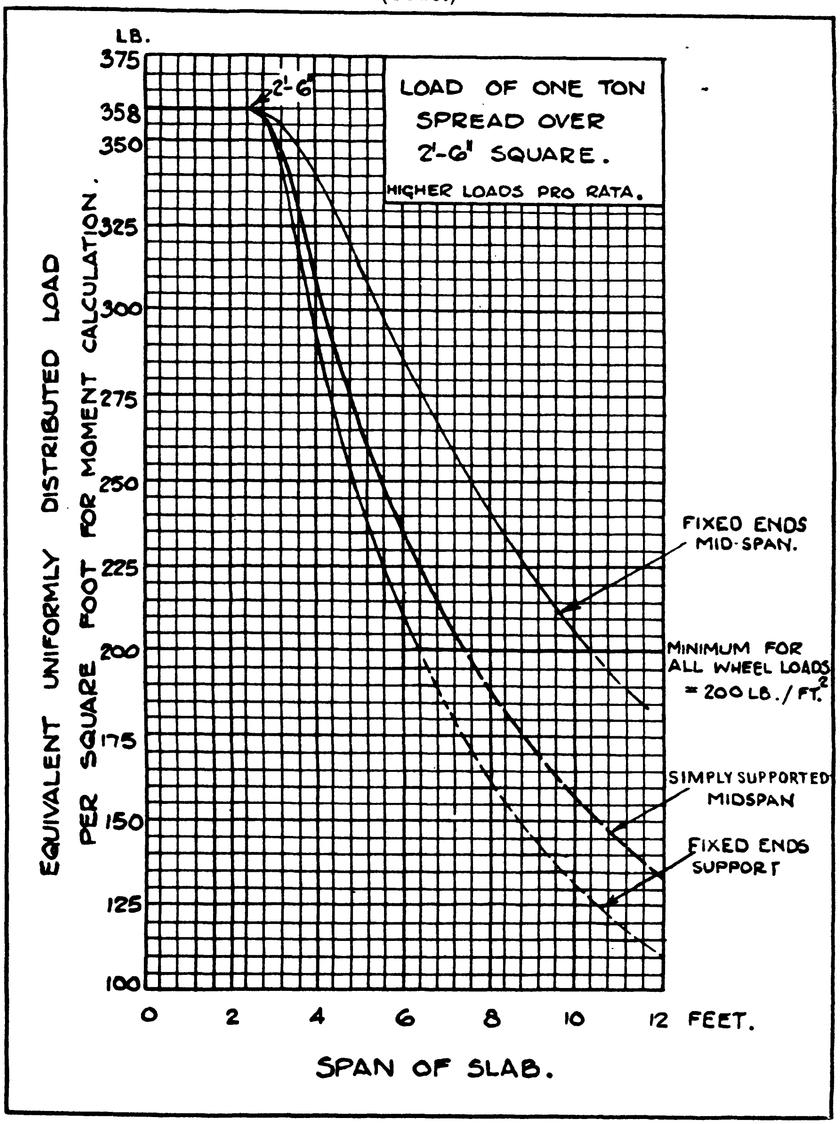
Upper floors: Offices (Class No. 2), span 8 ft.—105 lb. per square foot.

Ground floor: Retail shops: (Class No. 3), span 8 ft.—105 lb. per square foot.

TABLE 3.

SUPERIMPOSED LOADS ON GARAGE FLOOR SLABS.

(Code.)



[*Note:* This table is only applicable when it is permissible to spread the load over an area of 2 ft. 6 in. square as recommended in the Code. Observe that the minimum superimposed load in accordance with the By-laws is 150 lb. per square foot.]

TABLE 4.

TYPICAL WEIGHTS OF MOTOR VEHICLES.

Type of Vehicle	Weights in Tons				
	Total	Front Axle	Centre Axle	Rear Axle	Maximum wheel load plus 50 per cent.
Private cars :					
10 H.P. (Austin)	0·75	0·375	—	0·375	Less than 1·5 ton.
20 H.P. (Austin)	1·875	0·875	—	1·0	
40/50 H.P. (Daimler) . . .	2·75	1·25	—	1·5	
Public vehicles and lorries (loaded weights) :					
Light motor bus	6·0	2·0	—	4·0	3·00
Motor coach	8·0	3·25	—	4·75	3·56
Double-deck bus	9·5	4·0	—	5·5	4·13
4-wheel lorry	12·0	4·0	—	8·0	6·00
6-wheel lorry	19·0	4·0	7·5	7·5	5·63

FROM TABLES 3 AND 4 :

Ground floor : Garage (Class No 5, as motor coaches exceed 2½ tons in weight) —from *Table* 3, continuous spans of 8 ft., minimum equivalent superload per ton load = 162 lb. per square foot, from curve marked " fixed ends support." From *Table* 4 the " maximum wheel load plus 50 per cent. " for motor coach = 3·56 tons ; thus the minimum equivalent superload = 3·56 × 162 = 577 lb. per square foot. As even this minimum load exceeds 150 lb. per square foot, the design of the garage portion of the ground floor slab will be determined by the bending moments due to the wheel loads.

The method of calculating these moments, together with the method of assessing the loads on the stairs, landings, and rectangular panels of slab in the entrance hall, will be dealt with later.

Superimposed Loads on Beams.

As for slabs, total minimum uniformly distributed superimposed loads are given in the By-laws for beams, except for garage floor beams in Class No. 5. For Class No. 1 floors the minimum superimposed load on a beam of any span is 1 ton uniformly distributed, and 2 tons for Classes No. 2 to No. 6 inclusive. The incidence of the minimum superimposed loads on beams, when considering continuous spans, is subject to the same observations as have been made in the case of slabs. Beams or ribs must be designed for slab loading if the ribs are not spaced more than 2 ft. 6 in. apart. In the Code this limitation is recommended as 3 ft.

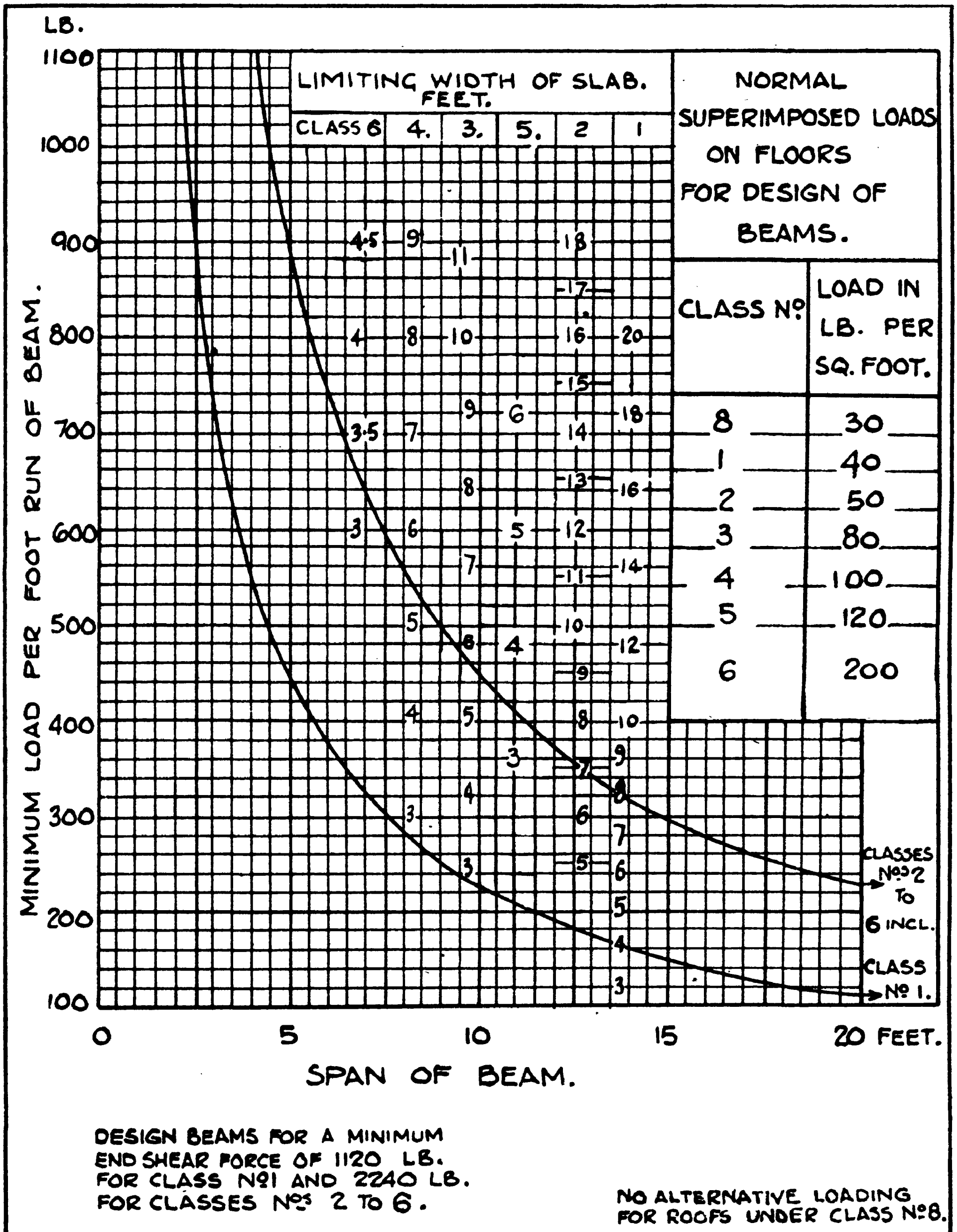
The curves on *Table* 5 combine the alternative loads in such a way that the load for which any beam span should be designed can be easily determined. On this diagram also are tabulated the normal floor and roof loads for beam design ; if the appropriate normal load multiplied by the width of the portion of the floor supported by the beam is less than the load read off the curve, the latter should be used ; otherwise the normal floor loads will apply. The figures tabulated under the heading of " Limiting Width of Slab " determine the critical alternative

TABLE 5.

SUPERIMPOSED LOADS ON BEAMS.

(Except Garage Floors under Class No. 5.)

(By-laws.)



CLASS No	LOAD IN LB. PER SQ. FOOT.
8	30
1	40
2	50
3	80
4	100
5	120
6	200

load. If the width of the slab carried by the beam does not exceed the tabulated value, the load taken from the curve will determine the design of the beam.

For an example, consider a beam spanning 10 ft. with Class No. 2 loading. The 10-ft. span ordinate intersects the Class No. 2 curve at about 450 lb. per foot run; proceeding horizontally from this intersection, the column headed "Class No. 2" is entered at the tabulated figure of "9 ft." Therefore if the width of slab carried by the beam does not exceed 9 ft., the load of 450 lb. read from the curve will control the design; if the width of slab exceeds 9 ft., the normal superimposed load should be used in the design.

In practical cases, only short spans will be affected by the alternative loading; the curves on *Table* 5 have therefore been curtailed at 20 ft. span. Longer spans, for example the internal secondary beams in the upper floors of the building (*Fig.* 1), can be treated as follows. These beams are subject to Class No. 2 loading and span 24 ft. From *Table* 5 the alternative load is seen to be less than 200 lb. per foot run, whereas by calculation from the normal load and area of floor supported the load is 50 lb. per square foot × 8 ft. = 400 lb. per foot run. The normal superimposed load will therefore control the design. The external secondary beams require closer consideration, as the margin between the alternative loads appears to be less. These beams support about 4 ft. width of the slab. From the normal load and the area of the floor, the load on the beam is 50 lb. × 4 ft. = 200 lb. per foot run; from the minimum total load, the load on the beam is $\frac{2 \text{ tons}}{24 \text{ ft.}} = 187$ lb. per foot run. Therefore the normal superimposed load will control the design of these beams also.

The main beams, being subjected to point loads, require special treatment. The magnitude of eacn point load on an internal main beam from the internal secondary beams is 50 lb. × 8 ft. × 24 ft. = 9600 lb. The minimum total superimposed uniform load on the beam is 2 tons; by inspection, this distributed load will give lower moments and shears than the point load of 9600 lb. The normal superimposed load will again control the design.

For garage floors in Class No. 5 the alternative to the uniformly distributed load for beams is identical with that for slabs, and the equivalent distributed load for the purpose of bending moment calculations can be determined from the data given on *Table* 6. The load factors, w, have been calculated on the assumption that the whole wheel load is carried by one beam, but by virtue of the 2-ft. 6-in. square distributive area the load will be partly taken on adjacent beams. A reduction factor (f) has therefore been included, and its value will approach unity for wide beams and for widely spaced beams. Any attempt to derive an accurate value for this factor involves the relation between the clear span of the slab, the distance between the beam centres, and the slab thickness. Therefore the tabulated factors are only approximate, but should be sufficiently accurate for normal loading determinations. If there is any doubt regarding the propriety of making this adjustment, a value of $f = 1$ should be used.

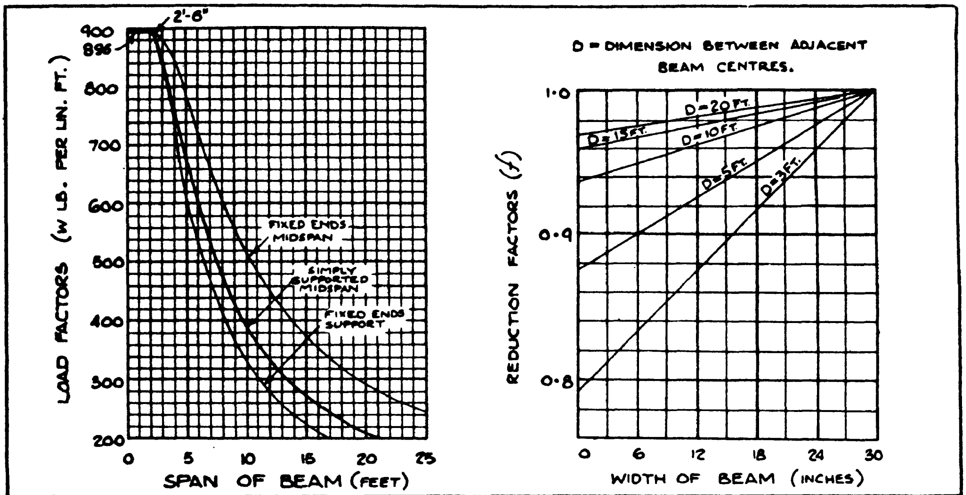
Curves are given for freely supported spans and for spans with fixed ends; intermediate conditions of continuity can be interpolated. No account has been taken of any dispersion of load (for example, through the depth of the beam) beyond the 2 ft. 6 in. recommended in the Code. The minimum shear force for which end sections of a garage floor beam should be designed is 3360 lb.

TABLE 6.

ALTERNATIVE SUPERIMPOSED LOADS ON GARAGE FLOOR BEAMS.

(Code.)

(This table is only applicable when it is permissible to spread the load over an area 2 ft. 6 in. square.)



FOR BENDING MOMENT CALCULATION :—
ALTERNATIVE EQUIVALENT UNIFORMLY DISTRIBUTED LOAD:
$= 1{\cdot}5\ wfP$ LB. PER FOOT RUN OF BEAM.
P = MAX. WHEEL LOAD IN TONS.
$1{\cdot}5P$ NOT TO BE LESS THAN ONE TON.
DESIGN FOR MINIMUM SHEAR FORCE = $3360\ Pf$ LB. (APPROX.)

[*Note :* For By-laws P must not be less than one ton.]

The moments and shears given by the alternative load would be over-ruled if greater moments and shears are given by the normal superimposed load of 120 lb. per square foot* of floor area. In designing garage floors the possibility of beams being subjected at any one time to the loads from two or more wheels should be investigated, and these conditions designed for accordingly. This aspect of the loading will be elaborated when dealing with the detail design of the ground floor beams.

Tie beams, braces, and similar non-loaded beams need not be designed for the minimum total superimposed loads.

Where moving loads are involved By-law 4 requires the effect of vibration, impact, acceleration and deceleration to be taken into account. In the case of garage floors, the 50 per cent. increase on the wheel loads allows for such effects, but in such cases as supports of lifts, crane supports, etc., an allowance on the static load should be made depending on the loads, speeds, etc., stated by the manufacturers of the machinery. The following allowances, taken from practical designs, may serve as a guide:

When designing the supports for overhead cranes the longitudinal thrust due to braking is taken as 20 per cent. of each wheel load. The allowance for impact and vibration is usually taken as 25 per cent. of the total weight of the crane

* 120 lb. per sq. ft. for By-laws; 200 lb. per sq. ft. for the Code.

and load, but for structures erected under the By-laws it would no doubt be preferable to allow an increase of 50 per cent. for these effects. The side-racking thrusts may amount to 10 per cent. of the total load.

With lifts and other moving loads the effect of the acceleration is to increase the static or net load W_D to a value W_M given approximately by the expression

$$W_M = W_D(1 + 0{\cdot}03a_D)$$

where a_D = maximum acceleration in feet per second per second.

The average acceleration of a lift might be in the order of 2 ft. per sec. per sec. for which value $W_M = 1{\cdot}06W_D$. The maximum acceleration may be considerably greater than the average and a value of $1{\cdot}25W_D$ would be required to cover all dynamic effects. It should also be noted that a load applied instantaneously is equivalent to a dead load of twice the applied load.

The load for which lift supports should reasonably be designed however should bear some relation to the load on, and factor of safety in, the suspension ropes. Thus if the "dynamic" load on the ropes is W_M and the ropes have a factor of safety of ten, the live load on the supports ought not to be taken as less than $\dfrac{10W_M}{4} = 2\tfrac{1}{2}W_M$.

Superimposed Loads on Columns, Walls, and Foundations.

For calculating column loads the alternative beam and slab loads that cause the equivalent superimposed load on beams and slabs to vary with the span do not apply. The superimposed load on a column is the product of the area of floor carried by the column and the normal unit load for beams given on *Table* 1. No distinction is made between heavy garage floors and other floors in Class No. 5 for the purpose of column design, except that in the case of garages for very heavy vehicles it may be necessary to compare the possible total weight of vehicles with the minimum of 120 lb. per square foot before adopting the latter.

The superimposed load from roofs inclined at an angle with the horizontal of more than 20 deg. is taken as a vertical load of 10 lb. per square foot of covered area for the purposes of column design.

For columns in buildings under Classes Nos. 1 to 4 inclusive (with beam loads not exceeding 100 lb. per square foot) the superimposed loads can be reduced in accordance with the schedule given in *Table 7*. The load factors tabulated are given to assist in the calculation of the maximum load on a column supporting a number of floors when the superimposed loading (w lb. per square foot) on each floor is the same and the area (A sq. ft.) of floor supported is the same for each story. The total superimposed load on the column is then

$$FwA + R \text{ lb.}$$

where R = total superimposed load from roof (lb.), and
F = reduction factor.

Applying this formula to a typical interior column in the proposed building, we have for the lift between the ground and first floors (static reactions)

$R = 16$ ft. $\times$ 24 ft. $\times$ 30 lb. $= 11{,}520$ lb.
F for five floors $= 4{\cdot}0$.
$w = 50$ lb. per square foot for Class No. 2.
$A = 16$ ft. $\times$ 24 ft. $= 384$ sq. ft.

The total superimposed load on the column below first floor level = (4·0 × 50 × 384) + 11,520 lb. = 88,320 lb., to which the dead load of the floors and roof must be added.

TABLE 7.

REDUCTION OF SUPERIMPOSED LOADS ON COLUMNS.

(By-laws and Code.)

Floor	Proportion of load	Load Factors	
		No. of floors	F
Roof	Full superimposed load	2	1·9
Top floor	do.	3	2·7
1st floor below top	90 per cent. of ditto.	4	3·4
2nd floor below top	80 per cent. of ditto.	5	4·0
3rd floor below top	70 per cent. of ditto.	6	4·5
4th floor below top	60 per cent. of ditto.	7	5·0
5th floor below top	50 per cent. of ditto.	8	5·5
All lower floors	50 per cent. of ditto.	9	6·0
		10	6·5
		11	7·0
Applies only to buildings of more than two stories in Classes Nos. 1 to 4 inclusive		12	7·5
		13	8·0
		14	8·5
		15	9·0

The superimposed loads carried on the foundations or on walls and piers supporting the structure are calculated in the same way as for column loads, and are subject to the same reductions where these apply.

Wind Pressure.

The superimposed loads normal to inclined roofs (*Table* 1) are deemed to include wind load on the roof. A building as a complete structure must be designed, according to By-law 6, to withstand a wind pressure of 15 lb. per square foot acting horizontally on the upper two-thirds of the surface and an additional pressure of 10 lb. per square foot upon all projections above the general roof level. This general wind load can be neglected in any direction where the width of the building exceeds one-half of its height, so long as the structure is such that all wind load is transmitted safely to the ground as is the case when the structure is stiffened by floors or walls or both. According to the By-laws, however, local loading due to wind pressure must be taken into account, but no guidance is given on this point. It seems reasonable to ensure that external wall panels can withstand in bending and shear a uniformly distributed horizontal pressure of 15 lb. per square foot, and that the connections to columns and beams framing the panel are sufficient to transmit the reactions from the panel.

Since the building in *Fig.* 1 is approximately 72 ft. high and its minimum width is 48 ft., the width is more than half the height and the wind pressure on the structure as a whole can be neglected. If conditions are such that allowance must be made for wind pressure, the effect on the building is to induce bending moments and shearing forces in the columns and extra bending moments in the beams. A simple method of assessing these bending moments, etc., at any floor level is to calculate first the total horizontal pressure on one bay of the building

above the floor level considered. This pressure represents the total horizontal shear on a single line of columns at the level considered, and should be divided between the columns in such proportions that external columns take half the shear taken on internal columns. The bending moment on each column is then the product of the shear on the column by half the story-height. The bending moment in the floor beams is equal to the sum of the bending moments in the columns above and below the joint considered.

Dead Loads.

The primary dead load is the weight of the concrete structure, to which should be added all wall, floor, ceiling, and roof finishes, brickwork, masonry, steelwork, partitions, fixed tanks, and machinery and similar permanent construction comprised within the building.

By-law 3 stipulates that for dead loading the weights of materials are to be those given in British Standard Specification No. 648 (1935). The Specification gives 115 lb. per cubic foot for plain brick concrete, 140 lb. per cubic foot for plain ballast concrete, and 150 lb. per cubic foot for reinforced concrete containing about 2 per cent. of steel. This value for reinforced concrete is less convenient than the value of 144 lb. per cubic foot conventionally adopted. The By-laws state that the weights given in B.S.S. No. 648 are to be used unless otherwise agreed with the District Surveyor, and for ordinary construction the Memorandum * permits 144 lb. per cubic foot to be adopted for reinforced concrete. The dead weight of compact reinforced concrete may, however, exceed this figure considerably.

The Code also recommends a weight of 144 lb. per cubic foot for reinforced concrete.

Loads on slabs and beams due to partitions should be included in the dead load, and when the weight of the partitions and the position of the latter are definitely known they should be designed for accordingly. If the positions are not known an addition must be made to the dead load on the floor to allow for the type of partition to be adopted. In the case of office floors (Classes Nos. 2 and 3) the minimum additional dead load for partitions should, in accordance with By-law 5, be taken as 20 lb. per square foot of floor area. In the design of the building in *Fig.* 1, it will be assumed that an allowance of 20 lb. per square foot will be ample for the partitions on the upper floors. The weight of the brick walls around the stair wells will be computed and allowed for accordingly. An allowance of 20 lb. per square foot is generally only sufficient for timber and other light partitions, and if solid $4\frac{1}{2}$-in. brick walls are to be carried a higher allowance should be made.

* London County Council Memorandum on the Computation of Stresses (Appendix IV).

CHAPTER II

MATERIALS AND STRESSES

Concrete Mixes and Working Stresses.

THE requirements of the By-laws relating to the proportions and working stresses of various concrete mixes are summarised on *Table* 8. Two qualities of concrete suitable for reinforced work are recognised in By-law 14: "Ordinary" quality (Mixes I, II, and III) and "Quality A" (Mixes IA, IIA, and IIIA). For plain concrete the mixes designated as IV and V can be used, while mixes VI and VII cannot be used in the construction of any part of a building. The By-laws define the mixes in terms of the quantity of aggregate to 1 cwt. of cement; the equivalent approximate volumetric proportions based on a cubic foot of cement weighing 90 lb. are also given on *Table* 8. The proportions as specified for reinforced concrete are all of the form $1 : n : 2n$, but if circumstances necessitate any change in the proportion of fine to coarse aggregate the proportions may be varied within the limits of $1 : n : 1\frac{1}{2}n$ and $1 : n : 2\frac{1}{2}n$. For such non-specified mixes the properties must be based on the ratio of the sum of the separate volumes of fine and coarse aggregates to the quantity of cement.

For each of the tabulated mixes the By-laws specify a minimum 28-day compressive strength; if the higher working stresses permissible with Quality A mixes are to be used, preliminary tests must give certain minimum strengths. Generally the maximum permissible working stress in compression due to bending is in accordance with By-law 99 one-third of the 28-day strength of cubes made on the site. The maximum direct compressive stress is 80 per cent. of the allowable bending stress, and the permissible shear stress is 10 per cent. of the bending stress. The permissible punching shear stress is twice the tabulated permissible shear stress, and the permissible bond stress is equal to the shear stress plus 25 lb. per square inch. The bond stress caused by variation of tensile stress due to bending may be twice the tabulated bond stress.

The tabulated direct stress for reinforced concrete applies to columns in which the ratio of effective length to least radius of gyration does not exceed 50; to comply with By-law 101, for ratios between 50 and 120 the stress must be reduced according to the expression given in *Table* 8.

Where proportions other than those defined in the By-laws are used (but not leaner than Mix I or richer than Mix III or IIIA), the minimum 28-day crushing strength (c) according to By-law 99, can be obtained by proportion from the two adjacent defined mixes. The following expression can be used for this purpose:

$$c = \frac{(V_2 - V)(c_1 - c_2)}{V_2 - V_1} + c_2$$

TABLE 8.

CONCRETE MIXES, STRESSES, AND DESIGN FACTORS.

(By-laws.)

CONCRETE MIX						MINIMUM CRUSHING STRENGTH AT 28 DAYS.		WORKING STRESSES. TONS PER SQ. FT. BEARING PRESSURE ON PLAIN CONCRETE				WORKING STRESSES. LBS. PER SQ. IN.				DESIGN FACTORS $t = 18{,}000$ LB. PER SQ. IN.			
APPROXIMATE VOLUMETRIC PROPORTIONS	QUANTITIES: CEMENT LB.	FINE AGG. CU. FT.	COARSE AGG. CU. FT.	REFERENCE	QUALITY	PRELIM. TESTS.	WORKS TESTS.	WALLS & PIERS: GENERAL	WALLS & PIERS: LOCAL	FILLING	SUPPORTS FOR WALLS & PIERS	COMPRESSION: BENDING	COMPRESSION: DIRECT	SHEAR	BOND	RATIO OF STRESSES	NEUTRAL AXIS FACTOR	LEVER ARM FACTOR	RESISTANCE MOMENT FACTOR $\left(\frac{BM}{bd^2}\right)$
1:1:2	112	1¼	2½	I	ORDINARY	-	2925	40	48	-	40	975	780	98	123	18·5	0·45	0·85	186
				IA	QUALITY A	5625	3750	-	-	-	-	1250	1000	125	150	15·2	0·51	0·83	265
1:1½:3	112	1⅞	3¾	II	ORDINARY	-	2550	35	42	-	35	850	680	85	110	21·2	0·415	0·86	152
				IIA	QUALITY A	4950	3300	-	-	-	-	1100	880	110	135	16·4	0·48	0·84	221
1:2:4	112	2½	5	III	ORDINARY	-	2250	30	36	-	30	750	600	75	100	24	0·385	0·87	126
				IIIA	QUALITY A	4275	2850	-	-	-	-	950	760	95	120	19	0·44	0·85	179
1:6	112	7½		IV	-	-	1480	20	24	20	20	-	-	-	-	THESE MIXES NOT TO BE USED FOR REINFORCED CONCRETE.			
1:8	112	10		V	-	-	1110	15	18	15	15	-	-	-	-				
1:10	112	12½		VI	-	-	740	10	12	10	-	THESE MIXES NOT TO BE USED IN BUILDING CONSTRUCTION							
1:12	112	15		VII	-	-	370	5	6	5	-								

NOTES:-

$\frac{H}{B} = \frac{\text{HEIGHT OF PIER, ETC.}}{\text{LEAST WIDTH}}$

$\frac{L}{r} = \frac{\text{EFFECTIVE COLUMN LENGTH}}{\text{LEAST RADIUS OF GYRATION}}$

c = MINIMUM CRUSHING STRENGTH AT 28 DAYS (WORKS TEST)

TABULATED PRESSURES APPLY WHEN $\frac{H}{B} \not> 2$ (SUPPORTS FOR WALLS & PIERS)

$\not> 6$ (WALLS & PIERS)

REDUCTION FACTORS FOR OTHER RATIOS

$1{\cdot}2 - \frac{H}{10B}$ (SUPPORTS FOR WALLS & PIERS)

$1{\cdot}6 - \frac{H}{10B}$ (WALLS & PIERS $\frac{H}{B} \not> 12$)

DIRECT STRESS VALUES ONLY APPLY IF $\frac{L}{r} \not> 50$.

REDUCTION FACTORS FOR OTHER RATIOS UP TO $\frac{L}{r} = 120$

$= 1{\cdot}5 - \frac{L}{100r}$

WORKING STRESSES CAN BE INCREASED BY 33⅓% IF INCREASE IS DUE TO WIND ONLY.

MODULAR RAT[IO] 15.

WORKING STRE[SSES] -

COMPN. BENDING $= \frac{c}{3}$

DIRECT $= 0{\cdot}8 \times \frac{c}{3}$

SHEAR: $= 0{\cdot}1c$

BOND: $= 0{\cdot}1c + 25$

PUNCHING SHEAR: $= 0{\cdot}2c$

where V = volume of fine plus coarse aggregate per cwt. of cement in the proposed mix,
V_1 and V_2 = ditto in the two adjacent defined mixes, and
c_1 and c_2 = the minimum 28-day crushing strengths specified for the two adjacent defined mixes.

Consider a mix of 112 lb. of cement, $2\frac{1}{12}$ cu. ft. of fine aggregate and $4\frac{1}{6}$ cu. ft. of coarse aggregate (approximately $1 : 1\frac{7}{8} : 3\frac{1}{3}$) which is commonly used for piles, waterproof concrete, and other purposes where a slightly richer mix than 1 : 2 : 4 is advantageous.

$$V = 2\tfrac{1}{12} + 4\tfrac{1}{6} = 6{\cdot}25 \text{ cu. ft.}$$

For Mix III (ordinary quality) $V_2 = 2\frac{1}{2} + 5 = 7{\cdot}50$ cu. ft.
$c_2 = 2{,}250$ lb. per square inch.
For Mix II (ordinary quality) $V_1 = 1\frac{7}{8} + 3\frac{3}{4} = 5{\cdot}625$ cu. ft.
$c_1 = 2{,}550$ lb. per square inch.

$$\text{Hence } c = \frac{(7{\cdot}50 - 6{\cdot}25)(2550 - 2250)}{7{\cdot}50 - 5{\cdot}625} + 2250 = 2451 \text{ lb. per square inch.}$$

Therefore the maximum working stresses for this mix are:

		(lb. per square inch)
Compression due to bending	$= \frac{2451}{3}$	$= 817$
Direct compression	$= 0{\cdot}8 \times 817$	$= 653$
Shear	$= \frac{817}{10}$	$= 82$
Bond	$= 82 + 25$	$= 107$
Punching shear	$= 2 \times 82$	$= 164$
Bond due to variation in tension due to bending	$= 2 \times 107$	$= 214.$

According to By-law 103, the specified maximum concrete working stresses may be exceeded by one-third provided the excess is due only to wind loading. This increase, however, is not applicable to secondary beams in floors or to roof construction.

The allowable bearing pressures—specified in By-laws 34, 35, and 60—on plain concretes used as filling, as supports for piers and walls, or as piers and walls are also given on *Table* 8, and attention is drawn to the limitations of the height to width ratio and the reduction factors applicable when the limiting ratio is exceeded.

The stipulations in the Code in connection with concrete mixes and the permissible working stresses on the concrete and reinforcement are summarised on *Table* 9 together with the appropriate design factors for use in bending resistance calculations. For the convenience of those who prefer to specify concrete mixes in actual quantities instead of in approximate volumetric proportions, these quantities are included on the table.

The concrete stresses specified are the maximum permitted for each mix and for each grade recognised by the Code. The stress used for Special-grade concrete is determined from the result of preliminary compression tests. The conditions controlling the various grades are fully laid down in the Code, to which the reader

TABLE 9.

Concrete Mixes, Stresses and Design Factors.

(Code.)

			Mix IV			Mix IVA			Mix III			Mix II			Mix I			
Proportions.	Mix Reference		Mix IV			Mix IVA			Mix III			Mix II			Mix I			Units
	Approximate volumetric proportions.		1 : 2 : 4			1 : 1⅔ : 3⅓			1 : 1½ : 3			1 : 1⅕ : 2⅖			1 : 1 : 2			
	Cement.		112			112			112			112			112			Lb.
	Fine aggregate.		2½			2 1/12			1⅞			1½			1¼			Cu. ft.
	Coarse aggregate.		5			4⅙			3¾			3			2½			
	Grade of concrete		Ord.	High.	Spec.	Ord.	High	Spec.	Ord.	High	Spec.	Ord.	High	Spec.	Ord.	High	Spec.	
Working stresses.	Compⁿ	Bending	750	950	1188	817	1051	1314	850	1100	1375	925	1200	1500	975	1250	1563	Lb. per square inch.
		Direct	600	760	950	653	841	1051	680	880	1100	740	960	1200	780	1000	1250	
	Shear		75	95	119	82	105	131	85	110	138	93	120	150	98	125	150	
	Bond		100	120	150	107	130	150	110	135	150	118	145	150	123	150	150	
Design factors (bending).	Modular ratio		17·8	14·0	11·2	16·3	12·6	10·1	15·7	12·1	9·7	14·4	11·1	8·9	13·7	10·7	8·5	
	Resistance moment factors ($\frac{BM}{bd^2}$) — Permissible steel stresses	18,000	137	174	217	149	192	240	155	201	251	169	219	274	178	228	285	
		20,000	130	165	206	142	182	228	147	191	238	160	208	260	169	218	271	
		25,000	115	146	183	125	161	202	131	169	211	142	185	231	150	192	240	
	Economic percentage of steel.	18,000	0·89	1·13	1·40	0·97	1·25	1·56	1·01	1·30	1·63	1·10	1·42	1·77	1·15	1·48	1·85	
		20,000	0·75	0·95	1·19	0·82	1·05	1·31	0·85	1·10	1·38	0·93	1·20	1·50	0·98	1·25	1·56	
		25,000	0·52	0·66	0·83	0·57	0·73	0·91	0·59	0·77	0·96	0·64	0·84	1·04	0·68	0·87	1·09	
	Neutral axis factors	18,000	0·426	Neutral axis and lever arm factors are constant for all grades and all mixes at a given steel stress and maximum concrete stress.														
		20,000	0·400															
		25,000	0·348															
	Lever arm factors.	18,000	0·858															
		20,000	0·867															
		25,000	0·884															

Maximum working stresses :- $m = \frac{40,000}{3X}$

Concrete: compression (bending) = X = ⅓ (28 day strength of site-made cubes).

Direct compression = 0·80 X

Shear = 0·1 X Bond = 0·1 X + 25   [Neither shear nor bond stress may exceed 150 lb. per in.²]

Steel : Rolled mild steel (B.S.S. Nº 15). = 18,000 lb. per sq. in.
High yield point mild steel (B.S.S. Nº 15) = 20,000 lb. per sq. in.
Hard drawn steel wire (B.S.S. Nº 165) = 25,000 lb. per sq. in.

is referred for particulars. The tests for Ordinary-grade concrete are optional, and there are no special design requirements for either the Ordinary or High-grade concretes. Structures designed in accordance with the Special-grade stresses must be calculated throughout as monolithic frames, but it should be realised that the application of the Code to normal buildings, with any grade of concrete, requires provision to be made for the moments in external columns, although the internal columns under normal circumstances may be designed for direct load only.

The tables in the Code define Mixes Nos. I, II, III, and IV, but on *Table* 9 an additional mix, given the reference " IVA " and approximating to $1 : 1\frac{2}{3} : 3\frac{1}{3}$, has been included. The properties of this mix are calculated in accordance with Code requirements in a similar manner to that for the By-laws.

Reinforcement.

Reinforcing steel as specified in By-law 15 is normally mild steel complying with B.S.S. No. 15 (1936) [now incorporated in No. 785 (1938)], the ultimate stress being between 58,000 and 72,000 lb. per square inch.

According to By-law 100, the maximum stress allowed on this quality of steel is 18,000 lb. per square inch in tension other than in helical binding in columns, in which case the maximum tensile stress is limited to 13,500 lb. per square inch. Compression reinforcement in beams can be stressed to 18,000 lb. per square inch if designed on the " steel beam " theory, that is, neglecting the compressive resistance of the concrete. For compression in beams and columns where the compressive resistance of the concrete is taken into account the stress in the reinforcement is taken as the stress in the surrounding concrete multiplied by the modular ratio.

According to By-law 103, the stresses in reinforcement may exceed those given above by one-third provided that such excess is due entirely to stresses produced by wind. This increase does not apply to secondary floor beams or to roof structures.

Under By-law 15 the use of high-tension steel complying with B.S.S. No. 548 (1934) [now incorporated in B.S.S. No. 785 (1938)], is permitted and slab reinforcement may be hard drawn steel wire complying with B.S.S. No. 165 (1929) [now superseded by B.S.S. No. 785 (1938)]. The conditions under which these and other special steels may be used are set out in supplementary regulations (see Appendix No. III). The material must comply with the appropriate specification, although materials not covered by any British Standard Specification will be considered. The permissible working stresses are based on the yield-point stress as determined on the reinforcement bar in the state in which it is incorporated in the concrete. Normally the maximum working tensile stress allowed is 50 per cent. of the yield-point stress, while the maximum compressive stress, when the concrete is neglected, is 40 per cent. of the yield-point stress. When the compressive resistance of the concrete is taken into account, the working compressive stress is taken as the modular ratio multiplied by the stress in the surrounding concrete. Since the elastic modulus of any quality of steel is approximately constant, the liability to cracking of the concrete depends on the working stress in the reinforcement. Hard drawn steel wire, originally B.S.S. No. 165 (now No. 785), may exhibit a yield-point stress as high

as 75,000 lb. per square inch, but it would be unreasonable to stress such steel to 50 per cent. of this value, so to limit the liability of cracking the concrete the London County Council allows 27,000 lb. per square inch in cold-drawn wire mesh, and also in twisted bars. Expanded metal for use as concrete reinforcement as specified in B.S.S. No. 405 is normal mild steel having an ultimate strength between 26 and 32 tons per square inch. As for steel under the original Specifications Nos. 15 and 165 no yield-point stress is specified, but a medium tensile steel is specified in B.S.S. No. 785 (1938) having an ultimate strength between 33 tons and 38 tons per square inch and with minimum yield-point stresses of 19·5 tons per square inch for bars not exceeding 1 in. in diameter, 18·5 tons per square inch for bars over 1 in. but not exceeding 1½ in., and 17·5 tons per square inch for bars over 1½ in. but not exceeding 2 in. At 50 per cent. of the yield-point stress the working stress would be between 19,600 lb. and 21,800 lb. per square inch.

High-tensile steel under original B.S.S. No. 548 (now 785) has an ultimate strength between 37 tons and 43 tons per square inch, that is, between 83,000 and 96,000 lb. per square inch. The minimum yield-point stresses for this steel are specified as 51,520 lb. per square inch for bars not exceeding 1 in. in diameter, 49,280 lb. per square inch for bars greater than 1 in. but not more than 1½ in. in diameter, and 47,040 lb. per square inch for bars greater than 1½ in. and not over 2 in. in diameter. Thus the working stress would vary from, say, 23,500 lb. to 25,000 lb. per square inch if the full 50 per cent. of the yield-point stress is admitted.

The working stress in the reinforcement according to the Code depends on the class of steel used, and the maximum stresses given on *Table* 9 apply to bending only. The value of 25,000 lb. per square inch is only applicable to solid slabs, excluding flat slabs, when reinforced with hard drawn steel wire having a yield-point stress of not less than 55,600 lb. per square inch.

For ordinary rolled mild steel bars the maximum tensile stresses are as given in the By-laws. Similarly for beams, except that the Code values for the modular ratio would apply when the stress depends on the compression stress in the surrounding concrete. In columns subject to axial loading, neither longitudinal reinforcement nor helical binding must be stressed beyond 13,500 lb. per square inch.

For mild steel complying with B.S.S. No. 15 (now No. 785) having a yield-point stress of not less than 44,000 lb. per square inch, the permissible tensile stress for bars in members subject to bending and for the compression reinforcement in beams designed on the "steel beam" theory is 20,000 lb. per square inch, but for shear reinforcement the tensile stress is limited to 18,000 lb. per square inch. For longitudinal bars and helical reinforcement in columns subject to direct load the stress in compression and tension respectively must not exceed 15,000 lb. per square inch.

Generally high-yield-point steel in solid slabs, except flat slabs, can be stressed to 0·45 times the yield-point stress up to a limit of 25,000 lb. per square inch so long as the total area of the tension steel does not exceed 1 per cent. of the effective area of the slab.

Factors for Resistance to Bending:

By-law 99 specifies a modular ratio of 15 for all mixes. From this and the permissible compressive stresses due to bending and a tensile stress of 18,000 lb. per square inch in the reinforcement, the stress ratios and neutral-axis, lever-arm and resistance-moment factors given on *Table* 8 can be calculated. The factors apply when the maximum working stresses in both concrete and steel are obtained simultaneously. For other combinations of stresses, *Tables* 10 and 11 have been prepared. The former gives the neutral-axis and lever-arm factors and the steel percentages in relation to a range of stresses for a modular ratio of 15.

Resistance-moment factors $\left(Q = \frac{R.M.}{bd^2}\right)$ are given on *Table* 11 for a suitable range of stresses based on $m = 15$. Methods of applying *Tables* 10 and 11 are explained in the calculations for the beams and slabs in the building under consideration.

The modular ratios tabulated in the Code are given to the nearest whole number, but permission is given to use the more exact values (as given in *Table* 9) calculated from the general formula. Owing to the general variation of the modular ratio, charts for the ready calculation of design factors based on $m = 15$ are inadequate. Therefore additional curves are given on *Table* 10 to cover the modular ratios specified in the Code. The neutral-axis, lever-arm, resistance-moment factors, and the "economic" steel percentage given in *Table* 9 apply when the maximum working stresses in both concrete and steel are simultaneously obtained. For other combinations of stress, *Tables* 10 and 12 have been prepared to facilitate computation. The former gives the neutral-axis and lever-arm factors and the steel percentages in relation to a range of stresses for the modular ratios of the mixes specified in the Code, but curves for other mixes can be interpolated most conveniently from the appropriate value of the modular ratio. *Table* 12 gives the resistance-moment factors $Q = \frac{B.M.}{bd^2}$ for 1 : 2 : 4 mix of Ordinary and High-grade concrete, as these are the concretes most commonly adopted in beam-and-slab construction.

Design Data.

For the purpose of the design of the building in *Fig.* 1 it is assumed that Quality A concrete, Mix IIIA (equivalent to 1 : 2 : 4), will be used except in the columns. The basic stresses will therefore be 950 lb. per square inch (*Table* 8) in compression due to bending and 18,000 lb. per square inch in tension in the reinforcement. The resistance moment factor (*Table* 11) will be 179 with a modular ratio of 15 and maximum working stresses.

Where calculations and design according to the Code differ from the corresponding details for design in accordance with the By-laws, in the present example it is assumed that the control of site operations is such that the requirements for High-grade concrete are satisfied. Except in certain columns a 1 : 2 : 4 mix will be used with ordinary rolled mild steel bars, the basic stresses being

TABLE 10.

BEAM DESIGN FACTORS.

(By-laws and Code.)

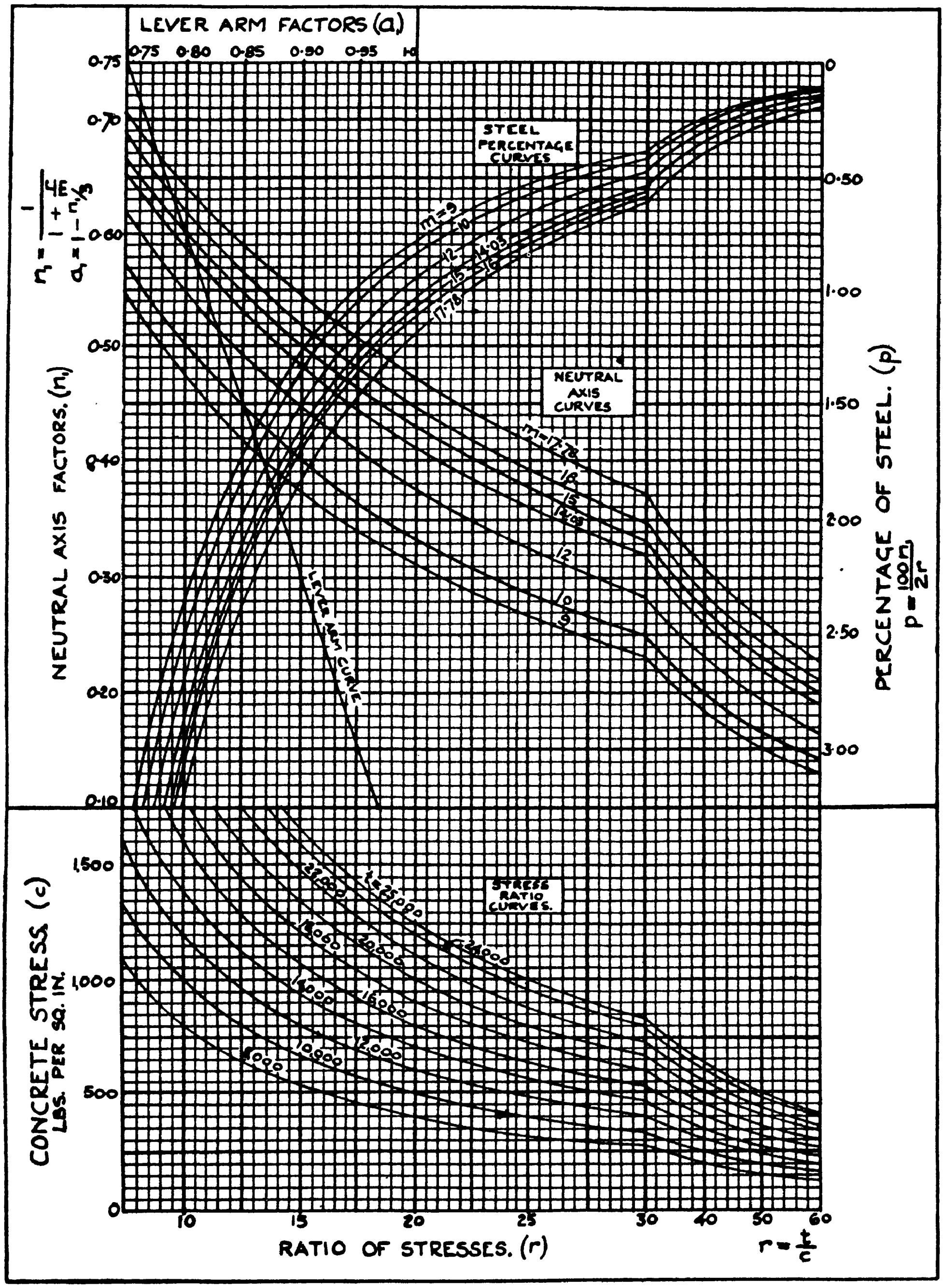


TABLE II.

Resistance Moment Factors.

(By-laws.)

RESISTANCE MOMENT FACTORS $\left(Q = \frac{R.M.}{bd^2}\right)$

	CONCRETE STRESS	STEEL STRESS. 5,000	8,000	10,000	12,000	14,000	15,000	16,000	17,000	18,000
CONCRETE MIXES CORRESPONDING TO MAX. COMPV. STRESS	200	33	25	21	-	-	-	-	-	-
	400	89	74	67	59	54	52	50	48	46
	500	120	102	92	84	77	74	71	69	66
	550	135	116	105	97	89	86	83	80	77
	600	151	130	119	110	102	98	95	92	88
	650	168	146	134	124	115	111	108	104	101
	700	184	160	148	138	129	124	120	117	113
MIX III	750	199	176	163	152	142	137	134	129	126
-	800	215	192	178	167	156	151	147	143	139
MIX II	850	231	210	193	181	171	168	161	157	152
-	900	250	223	207	196	186	179	175	168	166
MIX IIIA	950	265	247	224	212	200	194	189	183	179
MIX I.	975	272	250	232	222	206	202	196	190	186
-	1,000	281	255	240	226	214	209	203	198	193
MIX IIA.	1,100	314	287	271	256	243	237	231	226	221
MIX IA.	1,250	364	334	318	303	290	283	277	271	265

MODULAR RATIO
$m = 15$

ALL STRESSES ARE IN LB. PER SQ. INCH

$R.M = Qbd^2$

$$Q = \frac{n a_1 c}{2}$$

$$n_1 = \frac{1}{1 + \frac{t}{mc}}$$

$$a_1 = 1 - \frac{n_1}{3}$$

c = MAX. COMPV. STRESS IN CONCRETE

t = MAX. TENSILE STRESS IN REINFORCEMENT.

n_1 = NEUTRAL AXIS FACTOR.

a_1 = LEVER ARM FACTOR.

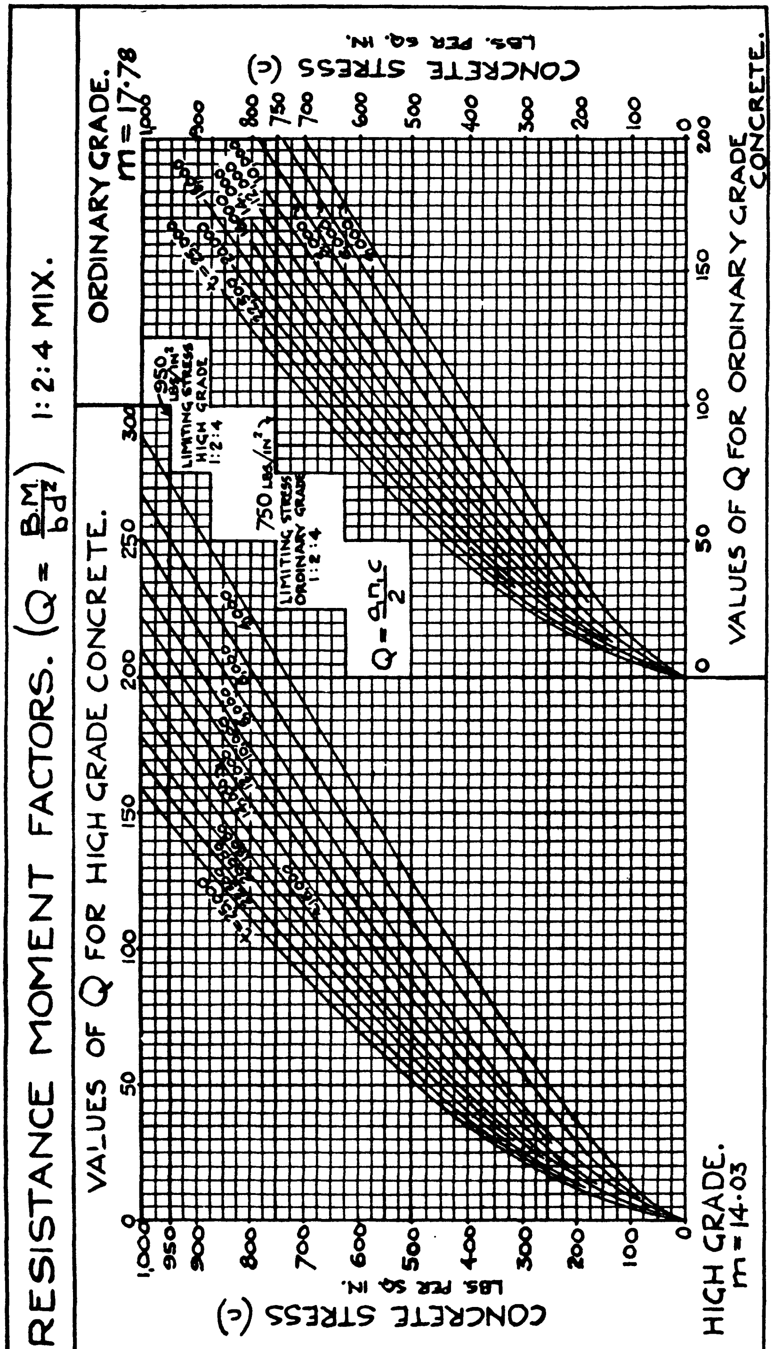
b = BREADTH OF SECTION.

d = EFFECTIVE DEPTH OF SECTION.

TABLE 12.

Resistance Moment Factors.

(Code.)



therefore 950 lb. and 18,000 lb. per square inch. The appropriate value of the modular ratio for the mix adopted is 14·0 from *Table* 9, or by calculation

$$m = \frac{40{,}000}{3x} = \frac{40{,}000}{3 \times 950} = 14{\cdot}03.$$

For this grade and mix of concrete it is necessary to make preliminary tests of the concrete strength, using the materials and consistency intended for the actual work unless other satisfactory evidence of the strength can be produced. During the progress of the work the test cubes must be made weekly, or whenever the materials are changed. The results of tests on the preliminary cubes should show a strength of not less than 4,275 lb. per square inch ($= 4{\cdot}5x$) at 28 days, and the progress cubes at the same age should show at least 2,850 lb. per square inch ($= 3{\cdot}0x$). Daily consistency tests should be made by means of slump apparatus; the maximum slump should not exceed 6 in. and should preferably be less.

The design factors for bending given on *Tables* 10, 11, and 12 are derived from the basic assumptions described in the Memorandum, that is, that all tensile stresses are taken by the reinforcement, that the concrete and reinforcement are elastic, and that plane sections remain plane after bending. The Memorandum suggests that simple and sufficiently approximate methods of calculation are preferable to complex processes, and that shrinkage and expansion of the concrete can be neglected. This point of view is maintained in the succeeding calculations and detail designs which, as required by the Memorandum, conform to commonly-accepted practice, and an endeavour has been made to give due weight to practical considerations.

CHAPTER III

BENDING MOMENTS

Continuous Spans.

ALTHOUGH the By-laws do not deal with the method of calculating bending moments for beams and slabs continuous over a number of spans, the Memorandum gives the following bases of calculation.

One method is to apply (as in common practice) approximate moment coefficients of $\frac{1}{10}$ for the penultimate supports and at the middle of the end spans, and $\frac{1}{12}$ for other interior supports and at the middle of the interior spans. The use of these coefficients is restricted to uniformly distributed loads over approximately equal spans freely supported on end supports and applies to the combined dead and live loads. Consecutive spans are considered approximately equal when the shorter does not differ by more than 15 per cent. of the longer.

An alternative method is to calculate the theoretical moments throughout the beam, which may frequently lead to lower moments at certain sections than are given by the approximate method. It also seems necessary to adopt this more accurate method when the load is not a uniform distribution. In calculating the moments the following assumptions are allowable: (1) The beams are free to rotate about the supports. This does not make any allowance for continuity with internal columns, but where end spans frame into external columns the appropriate negative moment should be designed for in accordance with the requirements of the Memorandum. (2) For maximum moments at midspan and support sections the spans subjected to superimposed load are shown in *Fig.* 2.

The effect of the first assumption is to overrate slightly the estimated moments at intermediate supports, while the arrangement of superimposed loading specified for the support moment does not give the true theoretical maximum moment at the support. To obtain the latter it would be necessary to load the spans indicated by broken lines in addition to those required by the Memorandum. The difference between the maximum negative moments obtained by the two sequences of loading is not materially great; for example, with four continuous spans and uniformly distributed load, the Memorandum loading on two adjacent spans gives support moments which are 95 per cent. of the theoretical maximum moments due to the superimposed load only on three spans.

The two permissible methods of moment assessment are applicable both to beams and to slabs spanning in one direction, but it is also open to the designer to consider beams as forming a monolithic frame in conjunction with the internal and external columns and to compute the bending moments in the beams and all

columns accordingly. The moments in slabs spanning in two directions and the moments in flat slabs are considered later.

An important concession when adopting the more accurate of the alternative beam and slab moment coefficients is that whereby any calculated support

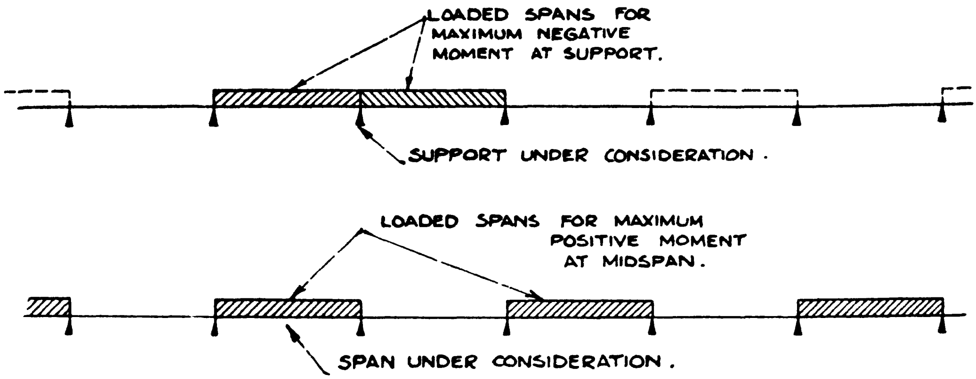


FIG. 2.—LOADING CONDITIONS FOR MAXIMUM MOMENTS.

moment may be reduced by not more than 15 per cent. if the maximum positive moments in the adjacent spans are increased by an amount equal to the numerical value of the reduction. No indication is given of how the bending moment

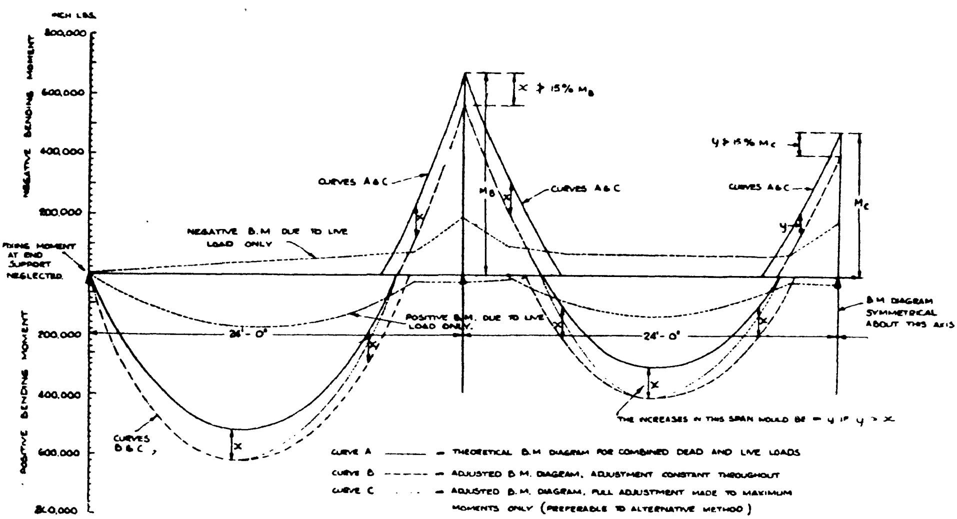


FIG. 3.—ADJUSTED BENDING MOMENT DIAGRAMS.

curves should be adjusted. Two methods of adjustment are possible: (*a*) by movement of the base line of the moment diagram, and (*b*) by retaining the points of contraflexure and adjusting the moment envelopes between these points.

These alternatives are illustrated in *Fig.* 3, which gives the moment diagrams for two spans of a four-span series of beams with a low live : dead load ratio

similar to the secondary beams in the roof of the building under consideration. The theoretical bending moment curves are given for the live load separately and for the combined dead and live loads (curve A). Comparison of the two adjusted moment diagrams (curves B and C) shows that, although the maximum moments correspond, the shape of the envelopes and the points of contraflexure differ slightly. As the reduction to the peak negative moment is a moment reduction only and is not presumably intended to affect the midspan negative bending moment, it has been recommended that the method of adjustment represented by curve C should be adopted.

Justification for making a support moment reduction can be based on consideration of the factors that tend to decrease the negative moments usually at the expense of the positive moments. These factors include continuity with intermediate supports (for example, columns), the width of the supports, and deflection of intermediate supports exceeding that of end supports. On the other hand there are factors that tend to reverse this relation between the negative and positive moments, including greater moment of inertia at the support than at midspan (for instance, haunches) and deflection at end supports being greater than that at intermediate supports. In favour of the negative moment reduction, however, is the fact that the ultimate load-carrying capacity of a continuous

TABLE 13.

CONTINUOUS BEAMS: BENDING MOMENT COEFFICIENTS.

(By-laws and Code.)

BENDING MOMENT COEFFICIENTS. CONTINUOUS BEAMS. (UNIFORM M. OF I.) 2 & 3 EQUAL SPANS. A B A — C D E D C

LOADING.								L/2 L/2					L/3 L/3 L/3				
SECTION.			TWO SPANS		THREE SPANS			TWO SPANS		THREE SPANS			TWO SPANS		THREE SPANS		
			A +	B −	C +	D −	E +	A +	B −	C +	D −	E +	A +	B −	C +	D −	E +
DEAD LOAD ONLY.	WITHOUT ADJUSTMENT.		·070	·125	·080	·100	·025	·156	·188	·175	·150	·100	·111	·167	·122	·134	·033
	WITH MAX.M ADJUSTMENT.		·089	·107	·095	·085	·040	·184	·160	·198	·128	·123	·136	·142	·142	·114	·053
LIVE LOAD ONLY.	WITHOUT ADJUSTMENT.		·096	·125	·101	·117	·075	·203	·188	·213	·175	·175	·139	·167	·144	·156	·100
	WITH MAX.M ADJUSTMENT.		·115	·107	·119	·100	·093	·231	·160	·239	·149	·201	·164	·142	·167	·133	·123
COMBINED DEAD AND LIVE LOAD R = LIVE LOAD / DEAD LOAD	WITHOUT ADJUSTMENT.	R = ½	·079	·125	·087	·105	·042	·172	·188	·188	·159	·125	·121	·167	·129	·141	·055
		= 1	·083	·125	·091	·109	·050	·180	·188	·194	·163	·138	·125	·167	·133	·145	·067
		= 2	·087	·125	·094	·111	·058	·187	·188	·200	·167	·150	·130	·167	·137	·149	·078
		= 3	·090	·125	·096	·113	·063	·191	·188	·204	·169	·156	·132	·167	·139	·151	·083
	WITH MAX.M ADJUSTMENT.	R = ½	·097	·107	·103	·089	·058	·200	·160	·212	·135	·149	·146	·142	·150	·120	·076
		= 1	·101	·107	·107	·093	·066	·208	·160	·218	·139	·162	·150	·142	·155	·123	·089
		= 2	·105	·107	·111	·094	·075	·215	·160	·225	·142	·175	·155	·142	·159	·127	·100
		= 3	·108	·107	·113	·096	·080	·219	·160	·229	·144	·181	·157	·142	·162	·128	·106
	ADJUSTED TO MINIMISE INEQUALTY IN B.M. COEFFICIENT.	R = ½	·097	·107	·096	·096	·051	·180	·180	·188	·159	·125	·144	·144	·135	·135	·061
		= 1	·101	·107	·100	·100	·059	·184	·184	·194	·165	·138	·146	·146	·139	·139	·073
		= 2	·105	·107	·103	·103	·066	·187	·188	·200	·167	·150	·149	·149	·143	·143	·084
		= 3	·108	·107	·105	·105	·071	·191	·188	·204	·169	·156	·150	·150	·145	·145	·089
	ALTERNATIVE APPROXIMATE COEFFICIENTS.		·100	·100	·100	·100	·100	-	-	-	-	-	-	-	-	-	-

NOTES. BENDING MOMENT = (COEFFICIENT) × (TOTAL LOAD) × (EFFECTIVE SPAN). COEFFICIENTS ASSUME BEAMS ARE FREELY SUPPORTED ON END SUPPORTS. SEE TABLE 16 FOR ADJUSTMENTS DUE TO END FIXING MOMENTS.

FOR MAXIMUM ADJUSTMENT DEDUCT 15% FROM SUPPORT MOMENT AND ADD TO THE POSITIVE MOMENTS IN THE TWO ADJACENT SPANS.

span depends more on the positive than on the negative moment so long as the value of the former has not been underrated. The beneficial effect in practice is to reduce the congestion of reinforcement at the supports, that is, at the intersection of the beams and columns in a normal type of building.

In order to present in a practicable manner the bending moment coefficients permissible under the Memorandum, *Tables* 13, 14 and 15 have been prepared. These relate to two, three, four, and five continuous equal spans. In the first place the theoretical coefficients without adjustment and with the full 15 per cent. adjustment are given for both dead and live loads separately. The corresponding coefficients for the combined load are also given for various ratios of live and dead load from ½ to 3, and at the foot of each table the approximate coefficients for uniformly distributed loads are tabulated. Except for the latter the coefficients are given for central and third-point loading in addition to uniformly distributed loads; coefficients for other loadings can often be determined closely enough by interpolation.

It is often convenient, especially when designing solid slabs, to have the same moment at the support as at mid-span. Although absolute uniformity of maximum moments cannot be obtained for all types of loading and for all the critical sections, advantage can be taken of the 15 per cent. reduction to regulate

TABLE 14.

CONTINUOUS BEAMS: BENDING MOMENT COEFFICIENTS.

(By-laws and Code.)

BENDING MOMENT COEFFICIENTS. CONTINUOUS BEAMS (UNIFORM M. OF I.) 4 EQUAL SPANS. A B C D C B A

LOADING.								$\frac{L}{2}$ — $\frac{L}{2}$				$\frac{L}{3}$ — $\frac{L}{3}$ — $\frac{L}{3}$			
SECTION.			A +	B −	C +	D −	A +	B −	C +	D −	A +	B −	C +	D −
DEAD LOAD ONLY	WITHOUT ADJUSTMENT.		·077	·107	·036	·071	·170	·161	·116	·107	·119	·143	·056	·096
	WITH MAXIMUM ADJUSTMENT.		·093	·091	·052	·061	·194	·137	·140	·090	·140	·122	·077	·081
LIVE LOAD ONLY	WITHOUT ADJUSTMENT.		·099	·116	·081	·107	·210	·174	·183	·161	·143	·155	·111	·143
	WITH MAXIMUM ADJUSTMENT.		·116	·099	·098	·091	·236	·148	·209	·137	·166	·132	·134	·122
COMBINED DEAD AND LIVE LOADS. $R = \frac{\text{LIVE LOAD}}{\text{DEAD LOAD}}$	WITHOUT ADJUSTMENT.	R = ½	·085	·110	·051	·083	·183	·165	·139	·125	·127	·147	·075	·112
		= 1	·088	·112	·059	·089	·190	·168	·150	·134	·131	·149	·084	·120
		= 2	·092	·113	·066	·095	·197	·170	·161	·143	·135	·151	·093	·127
		= 3	·094	·114	·070	·098	·200	·171	·166	·148	·139	·152	·097	·131
	WITH MAXIMUM ADJUSTMENT	R = ½	·101	·094	·067	·071	·208	·140	·164	·106	·149	·125	·097	·095
		= 1	·105	·095	·075	·076	·215	·143	·175	·114	·153	·127	·106	·102
		= 2	·109	·096	·083	·081	·222	·145	·186	·122	·158	·128	·116	·108
		= 3	·111	·097	·087	·083	·226	·145	·192	·126	·162	·130	·120	·111
	ADJUSTED TO MINIMISE INEQUALITY OF B. M. COEFFICIENTS.	R = ½	·098	·098	·067	·071	·183	·165	·139	·125	·137	·137	·093	·094
		= 1	·100	·100	·073	·076	·190	·168	·150	·134	·140	·140	·102	·102
		= 2	·103	·103	·081	·081	·197	·170	·161	·143	·143	·143	·110	·110
		= 3	·104	·104	·084	·084	·200	·171	·166	·148	·146	·146	·114	·114
	ALTERNATIVE APPROXIMATE COEFFICIENTS.		0·100	0·100	0·083	0·083	-	-	-	-	-	-	-	-

NOTES. BENDING MOMENT = (COEFFICIENT) × (TOTAL LOAD) × (EFFECTIVE SPAN). COEFFICIENTS ASSUME BEAMS ARE FREELY SUPPORTED ON END SUPPORTS. SEE TABLE 16 FOR ADJUSTMENTS DUE TO END FIXING MOMENTS.

FOR MAXIMUM ADJUSTMENT DEDUCT 15% FROM SUPPORT MOMENT AND ADD TO THE POSITIVE MOMENTS IN THE TWO ADJACENT SPANS.

the moments within the allowable limits with a view to minimising the inequality between peak moments; suitable coefficients are given on the tables for this purpose and are applied to the same range of live : dead load ratios as are the other coefficients. When considering tee-beams, however, it is usually advantageous to reduce the support moments as much as possible, since the area of concrete in compression in the rib at the support is much less than in the flange at mid-span. The full 15 per cent. reduction is therefore advisable when computing the bending moments for such beams. The same argument is perhaps even more forcible in the case of hollow-tile floor slabs.

The tabulated data are based on the assumption that the beams are freely supported on the end supports but, both in the case of beams framing into columns and of slabs built monolithic with heavy end-supporting beams, negative moments may occur at the end supports. These moments must be provided for, and, if relatively large, as in beams, allowance should be made for their effect on the moments throughout the adjoining spans. To assist in this modification, *Table* 16 has been prepared to show the effect of a unit moment applied at one end or at both ends of a series of continuous spans. The corresponding moment diagrams should be superimposed upon the normal free-end-support diagrams.

TABLE 15.

Continuous Beams: Bending Moment Coefficients.

(By-laws and Code.)

BENDING MOMENT COEFFICIENTS. CONTINUOUS BEAMS (UNIFORM M. OF I.) — 5 EQUAL SPANS. A B C D E D C B A

LOADING								½ ½					⅓ ⅓ ⅓				
SECTION			A +	B −	C +	D −	E +	A +	B −	C +	D −	E +	A +	B −	C +	D −	E +
DEAD LOAD ONLY.	WITHOUT ADJUSTMENT		·078	·105	·034	·079	·046	·171	·158	·112	·119	·131	·120	·141	·050	·106	·061
DEAD LOAD ONLY.	WITH MAXM ADJUSTMENT.		·094	·090	·049	·067	·058	·195	·134	·136	·101	·149	·141	·120	·071	·091	·077
LIVE LOAD ONLY.	WITHOUT ADJUSTMENT.		·100	·116	·079	·107	·086	·211	·174	·181	·160	·191	·143	·155	·108	·142	·114
LIVE LOAD ONLY.	WITH MAXM ADJUSTMENT.		·117	·099	·096	·091	·102	·237	·148	·207	·136	·215	·166	·132	·131	·121	·135
COMBINED DEAD AND LIVE LOAD. R = LIVE LOAD / DEAD LOAD	WITHOUT ADJUSTMENT.	R − ½	·085	·109	·049	·089	·059	·185	·163	·135	·133	·151	·128	·146	·069	·118	·079
		− 1	·089	·111	·057	·093	·066	·191	·166	·147	·140	·161	·132	·148	·079	·124	·088
		− 2	·093	·112	·064	·098	·078	·198	·169	·158	·146	·171	·135	·150	·089	·130	·096
		− 3	·095	·113	·068	·100	·076	·201	·170	·164	·150	·176	·137	·152	·094	·133	·101
	WITH MAXM ADJUSTMENT	R − ½	·101	·093	·065	·076	·073	·209	·139	·159	·113	·171	·150	·124	·091	·100	·097
		− 1	·106	·094	·074	·079	·080	·216	·141	·172	·119	·182	·154	·126	·101	·105	·107
		− 2	·110	·095	·081	·083	·087	·223	·144	·183	·124	·193	·157	·128	·111	·110	·116
		− 3	·112	·097	·085	·085	·091	·227	·145	·189	·127	·199	·160	·129	·117	·113	·121
	ADJUSTED TO MINIMISE INEQUALITY OF B.M. COEFFICIENTS	R − ½	·097	·097	·06	·076	·073	·185	·163	·135	·133	·151	·137	·137	·087	·100	·097
		− 1	·100	·100	·070	·080	·080	·191	·166	·147	·140	·161	·140	·140	·097	·106	·106
		− 2	·103	·103	·076	·086	·086	·198	·169	·158	·146	·171	·143	·143	·106	·113	·113
		− 3	·104	·104	·080	·088	·088	·201	·170	·164	·150	·176	·145	·145	·110	·117	·117
	ALTERNATIVE APPROXIMATE COEFFICIENTS.		·100	·100	·083	·083	·083	-	-	-	-	-	-	-	-	-	-

NOTES: BENDING MOMENT = (COEFFICIENT) x (TOTAL LOAD) x (EFFECTIVE SPAN). COEFFICIENTS ASSUME BEAMS ARE FREELY SUPPORTED ON END SUPPORTS. SEE TABLE 16 FOR ADJUSTMENTS DUE TO END FIXING MOMENTS.

FOR MAXIMUM ADJUSTMENT DEDUCT 15 % FROM SUPPORT MOMENT AND ADD TO THE POSITIVE MOMENTS IN THE TWO ADJACENT SPANS.

The application of the coefficients given on *Tables* 13, 14, 15 and 16 will be illustrated in the succeeding calculations for the design of slabs and beams.

For the purpose of calculating moments the Memorandum defines the effective span as being either the distance between the centres of the supports, or the clear distance between the faces of the support plus the effective depth of the beam or slab. This is in agreement with the early L.C.C. Regulations and

TABLE 16.

CONTINUOUS BEAMS : BENDING MOMENTS APPLIED AT END SUPPORTS.

(By-laws and Code.)

Nº OF SPANS	UNIT BENDING MOMENT APPLIED AT ONE END ONLY.	UNIT BENDING MOMENT. APPLIED AT BOTH ENDS SIMULTANEOUSLY.
2.	−1·000; +0·250	−1·000; −1·000; +0·500
3.	−1·000; −0·067; +0·267	−1·000; −1·000; +0·200
4.	−1·000; −0·071; +0·268; +0·018	−1·000; −1·000; −0·142; +0·286; +0·286
5.	−1·000; −0·072; −0·005; +0·268; +0·019	−1·000; −1·000; −0·053; +0·265; +0·265

with general practice, but the Memorandum implies that the loaded length of the beam or slab must correspond to the effective span. This abolishes the questionable practice of calculating the bending moment from the product of the effective span and the load on the clear span without modification to the moment coefficient.

Design of Roof Slab.

For the design of the roof slab the approximate moment coefficients will be taken and, since the beam widths are not yet determined, the full centre-to-centre span of 8 ft. will be used. The superimposed load has already been determined from *Table* 2, and the permissible working stresses as already selected are 18,000 and 950 lb. per square inch on the steel and concrete respectively. The design calculations would therefore proceed as shown on Calculation Sheet No. 1 and the reinforcement would be arranged as shown in the details in *Fig.* 4.

Although the roof panel between the front wall and the first line of internal secondary beams (and similarly between the back wall and the last row of internal secondaries) is apparently an end span, the cornice, parapet, and fascia beam combined are sufficiently substantial compared with the rather thin roof

slab to produce conditions of fixity at the outer support. The whole of the bays will therefore be designed for maximum positive and negative moments of $\frac{wl^2}{12}$ and reinforcement is provided to resist an end moment of this amount.

The value of the resistance-moment factor Q given on Calculation Sheet No. 1 is derived by substituting the moment (M) in the general expression

$$Q = \frac{M}{bd^2}.$$

The value 85 so obtained when using a $3\frac{1}{2}$-in. slab (effective depth assumed to be 2·75 in.) is considerably less than the value 179 taken from *Table* 8 for the quality and mix of concrete adopted and the appropriate steel stress of 18,000 lb. per square inch. This warrants the use of a slab thickness less than $3\frac{1}{2}$ in., but in the author's opinion this thickness is a practical minimum for roof work. The values of t and c given on the calculation sheet are obtained from *Table* 11 (using $Q = 85$). With this concrete stress of 575 lb. per square inch the lower part of *Table* 10 is entered and at the intersection with the "$t = 18{,}000$" curve we proceed vertically to the appropriate "m" curve ($m = 15$) and then horizontally to the "lever arm" curve. The value of a_1, the lever arm factor, is then read off the top line and substituted in the formula

$$A_T = \frac{M}{ta_1d}$$

to obtain the area of the main reinforcement.

The procedure given for determining an accurate value for a_1 is not generally justified for slab calculations, as the value corresponding to the maximum permissible stresses is sufficiently correct with any but very low or very high values of Q. If this special value for a_1 ($= 0·85$ from *Table* 8) had been adopted with the effective depth based on using $\frac{5}{16}$-in. diameter bars and $\frac{1}{2}$-in. cover the calculation would become

$$A_T = \frac{7{,}680}{18{,}000 \times 0{\cdot}85 \times 2{\cdot}84} = 0{\cdot}18 \text{ sq. in.}$$

The selection of suitable bar sizes and spacing can be controlled by adopting as nearly as practicable the maximum spacing allowed by Code, which is twice the effective depth. In the present case the maximum spacing would be

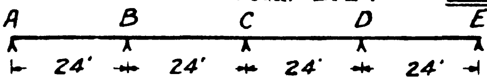
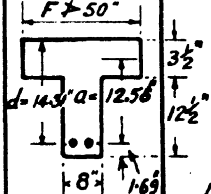
$$2d = 2 \times 2{\cdot}84 = 5{\cdot}68 \text{ in., say } 5\tfrac{1}{2} \text{ in.}$$

Calculations for the slab in accordance with the Code would be very similar to those given on Calculation Sheet No. 1 except that *Tables* 9 and 12 would be used instead of *Tables* 8 and 11 respectively, and when using *Table* 10 the curve for $m = 14$ would be referred to. The required area is given by $\frac{5}{16}$-in. diameter bars spaced at 5-in. centres.

With regard to distribution steel, By-law 110 stipulates that the area of such steel shall at least be equal to 10 per cent. of the area of the main reinforcement and according to By-law 107 the distribution bars shall be spaced at not more than four times the effective depth of the slab. To select suitable distribution steel in accordance with these requirements *Table* 17 can be used. Entering this diagram from the left at the appropriate A_T value, proceed to the right

CALCULATION SHEET No. 1.

ROOF.

ROOF SLAB	Dead Load:- ¾" Asphalt = 8 lb per ft²	
	Screeding, say = 14 " " "	
Effective Span = 8·0′	Ceiling finish = 6 " " "	
	3½" Slab = 42 " " "	
	Total Dead Load = 70 " " "	
$R = \frac{50}{70} = \cdot 7$	Live Load = 50 " " "	
	120 " " "	
	$BM = 0{\cdot}083 \times 120 \times 8^2 \times 12 = 7{,}680$ in lbs	
$d = 2{\cdot}75$	$Q = \frac{7{,}680}{12 \times 2{\cdot}75^2} = 85$ $\left\{ \begin{array}{l} t = 18{,}000 \text{ lbs/in}^2 \\ c = 575 \text{ " "} \end{array} \right.$	3½" Slab
$a_1 = \cdot 895$	$A_T = \frac{7{,}680}{18{,}000 \times \cdot 895 \times 2{\cdot}75} = \cdot 173$ ins.²	$\frac{5}{16}$" ϕ at 5" c/c (= ·184 ins²)
	Distribution Steel:—	¼" ϕ at 11" c/c
	Flange Steel (over main beams):—	¼" ϕ at 4½" c/c
INTERNAL SECONDARY BEAMS.	Dead Load:-Slab, etc. = 70 × 8 = 560 lbs/ft.	
	Beam Rib, say. = 100 " "	
	Total Dead Load = 660 " "	
$R = \frac{240}{660} = \cdot 36$	Live Load = 30 × 8 = 240 " "	
	Total Load = 900 " "	
		
	Midspans AB and DE:- in.lbs	
(4,570,000)	B.M. Dead: $\cdot 093 \times 660 \times 24^2 \times 12 = 425{,}000$	
(1,660,000)	Live: $\cdot 116 \times 240 \times 24^2 \times 12 = 192{,}500$	
	617,500	
	$n = \cdot 44 \times 14{\cdot}31'' = 6.3$ "(NA below Slab)	
	$F = \frac{2 \times 617{,}500}{3\frac{1}{2} \times 950 \times 12{\cdot}56} = 29\frac{1}{2}''$	12½" × 8" net
	$A_T = \frac{617{,}500}{18{,}000 \times 12{\cdot}56} = 2{\cdot}73$ ins²	2 - 1⅛" ϕ & 1 - 1" ϕ (= 2·77 ins²)
	[If two layers of bars: $A_T = \frac{617{,}500}{18{,}000 \times 12{\cdot}0} = 2{\cdot}86$ ins² or	2 - 1⅛" ϕ & 2 - ¾" ϕ (2·87 ins²)]

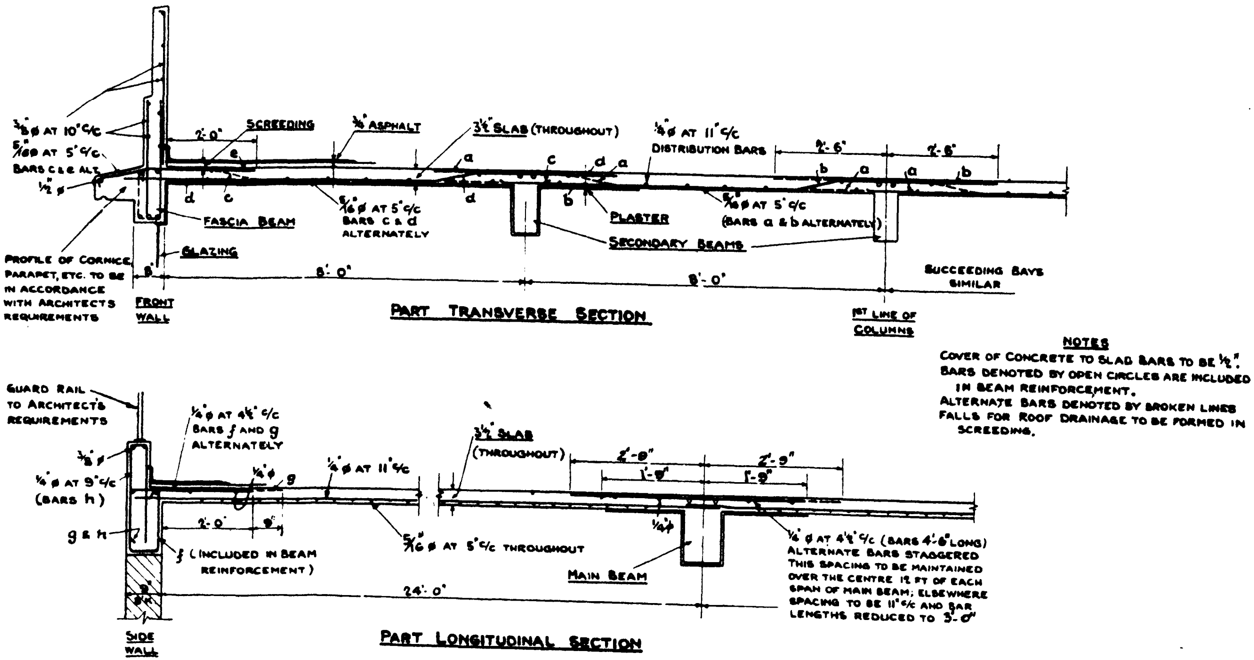


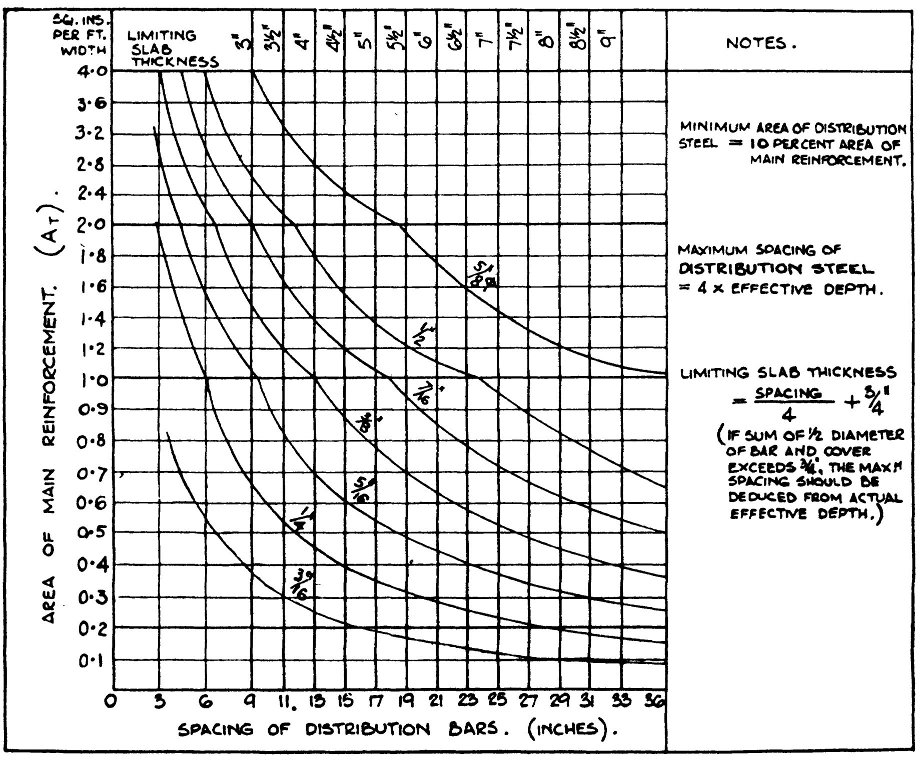
FIG. 4.—DESIGN OF ROOF SLAB : DETAILS.

until the "bar diameter" curve, immediately to the left of the vertical line corresponding to the slab thickness, is reached. The spacing at which this "bar diameter" curve is intersected is the required value if bars of the corresponding diameter are adopted. Take the case of, say, a 6-in. slab reinforced with 0·55 sq. in. of steel per foot width. The $A_T = 0{\cdot}55$ ordinate intersects the vertical line marked "6 in." between the bar diameter curves of $\frac{5}{16}$ in. and $\frac{3}{8}$ in., the former being to the left. The spacing where the $\frac{5}{16}$ in. curve is cut is 16 in.; hence appropriate distribution steel would be $\frac{5}{16}$-in. bars at 16-in. centres. According to By-law 106 bars smaller than $\frac{1}{4}$ in. in diameter are not permissible as main reinforcement in slabs or beams, but $\frac{3}{16}$-in. bars can be used for distribution steel. Therefore, if the intersection of the "A_T" and "slab thickness" ordinates falls to the left of the $\frac{3}{16}$-in. bar curve, the distribution steel can be $\frac{3}{16}$ in. diameter bars spaced at the maximum permissible spacing $4d$. Thus, in the case of the roof slab, where $A_T = 0{\cdot}173$ and the slab thickness is $3\frac{1}{2}$ in., the distribution steel could be $\frac{3}{16}$-in. bars at 11-in. centres. Generally, however, $\frac{1}{4}$-in. bars constitute the smallest practicable size, except for fabricated meshes, and this size has therefore been adopted as the minimum throughout all details.

TABLE 17.

DISTRIBUTION BARS IN SLABS.

(By-laws.)

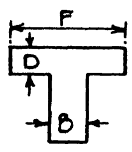
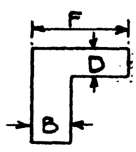


The Code recommends that the area of distribution steel should be 20 per cent. of the main steel compared with 10 per cent. specified in the By-laws. *Table* 17 can be used for Code designs if the values for twice the actual area of main reinforcement are taken.

Special recommendations are given in the Memorandum for reinforcement within the flanges of tee and ell beams. Since this reinforcement is comparable to slab reinforcement it can be appropriately considered here. The stipulations are that where the flanges of these beams are in compression the effective width of the flange shall be reinforced with bars placed transversely to the length of the beam and running the full width of the flange. The area of this reinforcement must be at least 0·3 per cent. of the total cross-sectional area of the slab forming the flange, and in cases where the slab itself is designed as spanning parallel to the beam (for example, a main beam) the flange reinforcement is to be placed

TABLE 18.

BEAM-FLANGE REINFORCEMENT.

SLAB THICK-NESS. D INS.	MINIMUM STEEL AREA $0{\cdot}036\,D$ INS.^{2} PER FT.	SUITABLE REINFORCEMENT SPACING $\not> 2(D-\tfrac{3}{4}'')$	WHEN BEAM FLANGE IS IN COMPRESSION, REINFORCEMENT TO EXTEND OVER WHOLE OF EFFECTIVE FLANGE WIDTH (=F) VALUES OF F:—	
3	·108	$\tfrac{3}{16}''$ φ AT 3" c/c.	T BEAM	Γ BEAM
3½	·126	¼" φ " 4½" "		
4	·144	$\tfrac{5}{16}''$ φ " 6½" "		
4½	·162	⅜" φ " 7½" "		
5	·180	⅜" φ " 7" "		
5½	·198	$\tfrac{7}{16}''$ φ " 9" "	$F \not> 12D+B$	$F \not> 4D+B$
6	·216	½" φ " 10½" "	OR $\not> \frac{L}{3}$	OR $\not> \frac{L}{6}$
6½	·236	½" φ " 10" "	OR $\not> C+B$	OR $\not> \frac{C}{2}+B$
7	·254	½" φ " 9" "		
7½	·270	⅝" φ " 13½" "	C = CLEAR DISTANCE BETWEEN RIBS	
8	·288	⅝" φ " 12½" "	L = SPAN OF BEAM.	

in the top of the slab. When the slab spans normally to the beam the negative moment steel over the top of the beam (secondary beam) together with the bottom steel that would be in the slab at this point, would usually be more than sufficient to comply with the flange reinforcement requirements, but the latter give a basis for determining the nominal steel that is usually placed over main beams. No indication of limitation to the spacing for the flange reinforcement is given and it is considered that the " twice the effective depth " rule is applicable, as this reinforcement is more in the nature of moment resisting than distributive steel. On this basis *Table* 18 has been compiled to give suitable flange reinforcement for various slab thicknesses. Unless the beam calculations indicate that the full permissible flange width (F) is not required to resist compression, the reinforcement should extend for a distance equal to the least of the three alternative values of F given on the table and based on the Memorandum.

The $3\frac{1}{2}$-in. slab adopted for the roof will require $\frac{1}{4}$-in. bars at $4\frac{1}{2}$-in. centres, but this close spacing need only be maintained over that length of the main beam in which the flange is in compression, say three-quarters of 16 ft. span = mid-section of 12 ft. Beyond this section there seems to be no reason why the spacing cannot be increased to, say, the maximum allowed for normal distribution steel, that is, four times the effective depth = 11 in.

Presuming that for the main beam the full effective width of flange is required to resist compression, the minimum length of the flange reintorcement would be the least of the following, the width of the beam being 9 in.:

(i) Based on slab thickness, $(12 \times 3\frac{1}{2} \text{ in.}) + 9 \text{ in.} = 4 \text{ ft. } 3 \text{ in.}$

(ii) Based on span of main beams, $\frac{16 \text{ ft.}}{3} = 5 \text{ ft. } 4 \text{ in.}$

(iii) Based on distance between beams, $= 24$ ft.

Later calculations will show that a width of flange of about 3 ft. is required to resist the compression in the midspan of the main beams; this is less than the permissible width of 4 ft. 3 in., and the required reinforcement has been provided to cover a minimum width of 3 ft. 6 in. symmetrical about the beam (see *Fig.* 4, " Part Longitudinal Section "). It is often convenient and economical to bend up the distribution bars to provide part of this top steel.

For the secondary beams the calculations on Sheet No. 1 show that in this case also the full permissible flange width is not required for resisting compression. The arrangement of the slab steel over these beams (see " Part Transverse Section," *Fig.* 4) will amply cover the effective flange width, which does not exceed 30 in.

The data given on *Table* 18 conform also to the recommendations of the Code.

The practice of staggering the points where bars are stopped off is encouraged by the Code. It will be seen from the roof slab details that in the case of the bars over the main beams and bars *f* and *g* this can be easily done, and the same effect is achieved with the top steel over the secondary beams by staggering the points of bending down and stopping-off; observe bars *a* and *d*, also bars *c* and *e*.

In a slab of this type bond does not need particular consideration, as small diameter bars are employed throughout and there is no difficulty in providing ample lengths for laps and end anchorages without providing hooks. With the low live : dead load ratio for the roof the possibility of midspan negative moments occurring does not call for investigation. The points of contraflexure are well covered if the top steel is carried slightly beyond the quarter points, which are 2 ft. from the support centres.

A minimum cover of concrete of $\frac{1}{2}$ in. for slab bars is specified in By-law 97, unless the diameter of the bars exceeds $\frac{1}{2}$ in. when the cover should not be less than the bar diameter. Since the bars are $\frac{5}{16}$ in. diameter and plaster and asphalt finishes are superimposed on the concrete, there is no reason to provide more than the minimum of $\frac{1}{2}$ in. for the roof slab. Occasions arise, for example, where there is contact with water or damp earth, where surfaces are subject to abrasion, and in marine work, when a cover considerably in excess of $\frac{1}{2}$ in. is advisable.

The reinforcement in the parapet and kerb over the external walls is only nominal and is not provided to resist any particular moments. Sufficient steel is, however, required to assist in preventing temperature and shrinkage cracks.

Although the design of the roof slab is a simple problem it has been dealt with at some length as features of the By-laws introduced in the calculations and details are worthy of more than passing consideration. The foregoing treatment will enable later explanations relative to the floor slabs to be curtailed. In practice, with the aid of the appropriate tables, no more calculation than that given in the upper part of Calculation Sheet No. 1 would be required.

Moment Calculations for Internal Secondary Roof Beams.

The essential computations for assessing the amount of moment resisting reinforcement required in the secondary beams of the roof are given in the lower part of Calculation Sheet No. 1 and on Sheets Nos. 2 and 3. Each line of these beams forms a series of four continuous spans, the effective span being taken as equal to the distance (24 ft.) between support centres. The clear span plus effective depth would exceed this value by a few inches. The live, or superimposed, load is taken from *Table* 5 (Class No. 8 = 30 lb. per square foot of roof slab supported). The dead load of 70 lb. per square foot of roof slab is taken from the preceding slab calculations, and a provisional value of 100 lb. per foot run of beam has been taken for the weight of the beam rib.

Owing to the live : dead load ratio being less than $\frac{1}{2}$ (the minimum for which combined coefficients are given on *Tables* 13 to 15 inclusive), it is proposed to employ the coefficients tabulated on *Table* 14 (four spans) for live and dead load separately. The full 15 per cent. reduction has been allowed, as the members under consideration are tee beams. On this basis the coefficients will be those given on the calculation sheets for the four critical sections. When the moments at a number of sections have to be computed it is a convenience to evaluate and record the numerical value of $wl^2 \times 12$ (in which l is in ft.) for both the live and dead loads, as this expression is common to all the moment equations; to obtain the bending moment at any section these values are multiplied by the appropriate coefficients. In the present case the values are 4,570,000 for the dead load and 1,660,000 for the live load.

Alternate lines of internal secondary beams are supported on a series of columns or a series of main beams. The calculations on Sheets Nos. 1 and 2 apply particularly to secondary beams supported on beams, in which case no definite moment is provided for at the end supports A and E. A calculation is given on Sheet No. 3 for the modification due to the fixing moment where the beams frame into columns.

From a preliminary consideration of the shearing forces it is evident that these will not control the size of the secondary beams, the depths of which are limited, however, by the specified headroom of 8 ft. 6 in. The maximum allowable overall depth of the roof beams is therefore 10 ft. — 8 ft. 6 in. = 1 ft. 6 in.* If this depth is adopted for the main beams it is convenient to make the secondaries a few inches shallower in order to avoid cranking the reinforcement at the intersection. A minimum depth : span ratio is not specified in the By-laws, but for a maximum steel stress of 18,000 lb. per square inch a suitable limiting value

* See general plan of building (*Fig.* 1).

is $\frac{1}{20}$. Thus the minimum effective depth for a beam spanning 24 ft. is about 14½ in., and this is given with a beam rib projecting 12½ in. below the 3½-in. slab. This rib will be 2 in. shallower than the maximum allowable for the main beams. The width of the secondary beam ribs is best determined from a consideration of the space required to accommodate the reinforcement (see later).

Having determined the moments the design of the sections follows the usual procedure. With the value of $n_1 = 0{\cdot}44$ taken from *Table* 8 it will be seen that for the midspan sections the neutral axis falls below the slab and the complete thickness of the latter can therefore be included in the compression area. In these conditions the approximate lever arm depth is given by $a = d - \frac{D}{2}$ where d = the effective depth and D = the slab thickness.

The width of slab required for compression is

$$F = \frac{2\ \text{B.M.}}{Dca}$$

In the case of the end spans $F = 29\frac{1}{2}$ in., which is within the maximum flange width allowed by the By-laws; the permissible width is that given by the least of the alternative values of F expressed on *Table* 18; for the secondary beams the allowable value of F will be controlled by the slab thickness and must not exceed 50 in. if a rib breadth of 8 in. is assumed.

The expression given in the Memorandum for the ratio of the length between lateral supports of a beam to the breadth of the compression flange, should be observed, although this would apply principally to rectangular beams.

The required area of reinforcement in the case of the midspan sections of the end span is given either (i) by two 1⅛-in. and one 1-in. bar arranged in one layer, or (ii) by two 1⅛-in. and two ⅞-in. bars arranged in two layers. The minimum width of the beam rib can be determined from the requirements of By-laws 97 and 107 (or 9) that (*a*) the minimum cover to beam bars is to be equal to the diameter of the bar and not less than 1 in., and (*b*) the space between adjacent horizontal bars (except at laps) is to be equal to the maximum size of the aggregate plus ¼ in. or to the bar diameter, whichever is greater. If aggregate of the usual ¾-in. maximum gauge is used the minimum horizontal distance between adjacent bars is 1 in. With bars of different sizes the larger diameter determines the spacing based on bar diameters according to By-law 107.

If the single layer of reinforcement is adopted the minimum beam width would be calculated thus:

Two side covers of 1⅛ in. each	= 2¼ in.
Two bars of 1⅛ in. diameter each	= 2¼ in.
One bar of 1 in. diameter	= 1 in.
Two intermediate spaces of 1⅛ in. each	= 2¼ in.
	7¾ in., say, 8 in.

The alternative arrangement in two layers will result in a reduced effective depth based on the bar diameter and on the requirement of By-law 107 that a minimum vertical distance of ½ in. shall be provided between horizontal main

CALCULATION SHEET NO. 2.

ROOF.

(By-laws.)

INTERNAL SECONDARY BEAMS (cont.)

Midspans B.C and C.D. in. lbs.

B.M. Dead: $\cdot 052 \times 4{,}570{,}000 = 237{,}000$

Live: $\cdot 098 \times 1{,}6660{,}000 = 163{,}000$

$400{,}000$

$$A_T = \frac{400{,}000}{18{,}000 \times 12 \cdot 75} = 1 \cdot 74 \text{ ins}^2$$

$4 - \frac{3}{4}''\phi$ ($= 1 \cdot 767 \text{ ins}^2$)

Supports B. and D. in lbs

B.M. Dead: $\cdot 091 \times 4{,}570{,}000 = 416{,}000$

Live: $\cdot 099 \times 1{,}660{,}000 = 164{,}500$

$580{,}500$

$C_c = 179 \times 8 \times 14 \cdot 25^2 = 292{,}000$ in lbs

$C_s = 580{,}500 - 292{,}000 = 298{,}500$

$d = 14 \cdot 25''$

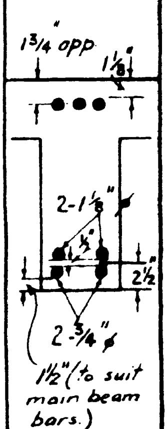
$n = 6 \cdot 25''$ $f = 2\frac{1}{2}''$

$$C_s = \frac{6 \cdot 25 - 2 \cdot 5}{6 \cdot 25} \times 950 \times 13 \cdot 0 = 7{,}000 \text{ lbs/in}^2$$

$a_s = 16 \cdot 0 - 1 \cdot 75 - 2 \cdot 5 = 11 \cdot 75''$

$a = \cdot 85 \times 14 \cdot 25 = 12 \cdot 15$

$a_2 = 12 \cdot 0$ Approx.



$$A_c = \frac{298{,}500}{7{,}000 \times 12 \cdot 0} = 3 \cdot 42 \text{ ins}^2$$

$$A_T = \frac{580{,}500}{18{,}000 \times 12 \cdot 0} = 2 \cdot 69 \text{ ins}^2$$

$A_T = 3 - 1\frac{1}{8}''\phi$ ($= 2 \cdot 98 \text{ ins}^2$)

Alternative:

$$A_T = A_c = \frac{580{,}500}{18{,}000 \times 11 \cdot 75} = 2 \cdot 75 \text{ ins}^2$$

$A_c = 2 - \frac{3}{4}''\phi$ & $2 - 1\frac{1}{8}''\phi$ ($= 2 \cdot 87 \text{ ins}^2$)

Support C. in lbs

B.M. Dead: $\cdot 061 \times 4{,}570{,}000 = 279{,}000$

Live: $\cdot 091 \times 1{,}660{,}000 = 151{,}000$

$430{,}000$

$C_s = 430{,}000 - 292{,}000 = 138{,}000$ in lbs

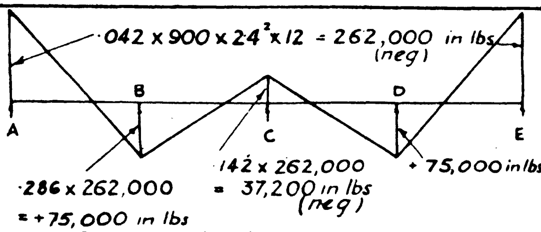
$$A_c = \frac{138{,}000}{7{,}000 \times 12 \cdot 0} = 1 \cdot 64 \text{ ins}^2$$

$$A_T = \frac{430{,}000}{18{,}000 \times 12 \cdot 0} = 1 \cdot 98 \text{ ins}^2$$

$A_T = 2 - 1''\phi$ & $1 - \frac{3}{4}''\phi$ ($= 2 \cdot 01 \text{ ins}^2$)

$A_c = 4 - \frac{3}{4}''\phi$ ($= 1 \cdot 77 \text{ ins}^2$)

CALCULATION SHEET NO. 3.
ROOF.
(By-laws.)

INTERNAL SECONDARY BEAMS FRAMING INTO COLUMNS.	 End Supports A & E :- C_c = 292,000 in lbs. ∴ A_c = 0. $A_T = \frac{262,000}{18,000 \times \cdot 85 \times 14 \cdot 5} = 1 \cdot 17$ ins² Midspans AB & DE Approx. B.M Reduction = ⅓ x 262,000 = 87,300 in lbs Reduction in $A_T = \frac{87,300}{18,000 \times 12 \cdot 56}$ = ·38 ins² A_T as before = 2·73 " Net. 2·35 " Supports B and D:- Reduction in $A_T = \frac{75,000}{18,000 \times 11 \cdot 75}$ = ·35 ins² A_T as before = 2·75 " Net. 2·40 " Reduction in A_c = similar to A_T. A_c = 2·40 ins.² Midspan BC and CD:- No change: Support C:- Additional $A_T = \frac{37,200}{18,000 \times 12 \cdot 0}$ = ·17 ins² A_T as before = 1·98" Total. 2·15" Additional $A_c = \frac{37,200}{7270 \times 12 \cdot 0}$ = ·43 ins² A_c as before = 1·68 " 2·11 "	2 - ½" ø & 1 - 1" ø (= 1·178 ins²) 2 - 1⅛" ø and 1 - ¾" ø (= 2·43 ins²) 2 - 1⅛" ø & 1 - 3/4" ø (2·43 ins²) 2 - 1⅛" ø & 1 - ¾" ø (2·43 ins²) 4 - ¾" ø (1·77 ins²) 3 - 1" ø (2·36 ins²) 5 - 3/4" ø (2·21 ins²)
SHEAR FORCE.	See Sheet No 4	

reinforcing bars. For the two-layer arrangement previously given, the minimum beam width based on the lower layer would be

Two side covers of $1\frac{1}{8}$ in. each	$= 2\frac{1}{4}$ in.
Two bars of $1\frac{1}{8}$ in. diameter each	$= 2\frac{1}{4}$ in.
One intermediate space of $1\frac{1}{8}$ in.	$= 1\frac{1}{8}$ in.
	$5\frac{5}{8}$ in., say, 6 in.

Unless an uneconomical amount of reinforcement is provided, two layers of bars will be required at midspan of the internal spans, and at the penultimate supports as well as in the end spans if a 6-in. rib is provided. The reduced effective depth and consequent congestion of bars seem to be best avoided by adopting a single layer of bars where practicable in an 8-in. rib. Thus the rib size will be $12\frac{1}{2}$ in. by 8 in. net, giving a weight of 100 lb. per foot run as assumed in the loading calculations.

The controlling factor in the design of the support sections is the compression, and the two-stage process of calculation given for these sections determines the amount of compression steel required (i) if the concrete is assumed to take part of the compression and (ii) if the section is designed on the "steel beam" theory with the area of the tensile equal to that of the compressive reinforcement. Both bases of calculation are acceptable, and it will be seen from the calculation sheets that in the case of the penultimate supports the steel-beam method gives the most economical arrangement of bars, while at the centre support the application of the alternative method is most convenient.

In accordance with By-law 111, it is necessary to restrict the spacing of the beam stirrups to 8 times the diameter of the compression bar if the steel beam theory is adopted; if the alternative method is used, the spacing of the stirrups must not exceed 12 times the bar diameter. If no other considerations controlled (for example, shear resistance), the spacing of the stirrups at the penultimate supports for the roof secondaries would be 8 times the diameter of the bar, or $8 \times \frac{3}{4} = 6$ in. and for the central support, $12 \times \frac{3}{4} = 9$ in. Limiting spacings are given on *Table* 20.

If the "equal steel" method is adopted the compressive stress in the steel can be taken at 18,000 lb. per square inch, and, if the alternative, the usual stress of m times the compressive stress in the surrounding concrete is worked to. The formulæ used in the calculation are therefore the ordinary expressions, as follows:

(i) Concrete in Compression:

Compressive resistance of concrete $= C_c = Qbd^2$

Resistance moment to be provided by compressive reinforcement $= C_s = \text{B.M.} - C_c$

Depth of neutral axis $= n = n_1 d$

Effective compressive stress in steel

$$= c_s = \frac{n - f}{n}(m - 1)c$$

where f = distance from compressed edge to centroid of compressive reinforcement.

Lever arm of concrete $= a = a_1 d$

Lever arm of steel $= a_s =$ distance between centroids of tensile and compressive reinforcements.

Values of Q, n_1, a_1, and c are taken from *Table* 8 for (in this example) Quality A concrete Mix IIIA with $m = 15$.

$$\text{Area of compressive steel} = A_C = \frac{C_s}{c_s a_s}$$

$$\text{Area of tension steel} = A_T = \frac{\text{B.M.}}{t a_2}$$

where $a_2 =$ mean value of a and a_s (this is only approximately correct but close enough for practical calculations).

(ii) Steel Beam Theory:

$$A_T = A_C = \frac{\text{B.M.}}{t a_s}$$

In assessing the value of f it appears to be necessary to allow a space of $\frac{1}{2}$-in. vertically between the two layers of compressive bars at all supports, since being fully employed as compression reinforcement these bars should be classed as "main reinforcements." If the bars in one layer had been sufficient to resist compression the two layers at the supports could have been considered as a splice, in which case, in accordance with By-law 107, the bars could have been placed in vertical contact over the length of the lap. For brevity the same factors are used in the calculation of A_C for the centre support as for the penultimate supports, although slightly conservative results are obtained thereby.

The computations on Calculation Sheet No. 3 concern the modifications required to allow for the negative bending moment at the end supports of those lines of secondary beams that are monolithic with the columns. With uniformly distributed loads the coefficient for the end fixing moment may vary from $\frac{1}{12}$ to 10 per cent. of this figure, dependent upon the relative stiffness of the beams and columns. A preliminary calculation * indicates that an approximate coefficient of $\frac{1}{24}$ can be taken for the present case and, by application of the appropriate factors given on *Table* 16 for four spans with fixing moments at both end supports, the additions and deductions indicated on the calculation sheet follow. The modification to the maximum positive moments in the interior spans is ignored as these moments will not be appreciably altered, but the reduction in the negative moment at the penultimate supports and the increase at the centre support are investigated. Adjustments should be made, if necessary, to the amount of reinforcement provided at the central support as the moment at this section is increased. If there is no risk of the end fixing moment not being realised, the reinforcement at the penultimate supports can be reduced.

With this same proviso a reduction in the positive moments in the end spans is justified, but to do more than assess roughly the amount of this reduction necessitates drawing out a bending moment diagram and making the adjustments thereon. For the roof secondary beams this reduction has been assessed conservatively at one-third of the end fixing moment. The more accurate

* Since this calculation concerns the reaction between columns and beams, it involves so many features that it cannot with justice be summarily dealt with. The calculation is therefore omitted from the present Chapter as the subject is treated fully in Chapter VI.

method of modifying the moment envelope is explained in the design of the floor beams (Chapter VII).

The negative moments calculated on the basis of *Table* 14 can be subjected to a reduction of 15 per cent. if the positive moments in the adjacent spans are increased; since, in the present instance, the additional negative moment at the centre support is comparatively small, the reduction is hardly worth while.

Adjustments to Calculation Sheet No. 3 if the design were in accordance with the Code would only affect the numerical value of the beam design factors. The resulting concrete section and the amount and arrangement of reinforcement would be identical with the design complying with the By-laws.

CHAPTER IV

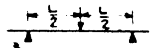
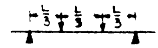
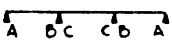
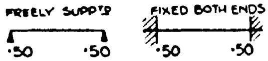
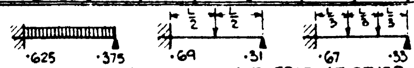
SHEAR AND BOND

Shear Force.

ALTHOUGH conditions for the calculation of moments on continuous beams are precisely expressed, permissible methods of shear force assessment are not so

TABLE 19.

CONTINUOUS BEAMS: MAXIMUM SHEAR FORCE COEFFICIENTS.

CONTINUOUS BEAMS MAXIMUM SHEAR FORCE COEFFICIENTS.			EQUAL SPANS UNIFORM M. OF I.											
TYPE OF LOADING							 L/2 L/2				 L/3 L/3 L/3			
SECTION (ADJACENT TO SUPPORT)			A.	B.	C.	D.	A.	B.	C.	D.	A.	B.	C.	D.
2. SPANS.	DEAD LOAD		·38	·63	-	-	·31	·69	-	-	·33	·67	-	-
	LIVE LOAD		·44	·63	-	-	·41	·69	-	-	·42	·67	-	-
	R	½	·40	·63	-	-	·34	·69	-	-	·36	·67	-	-
A BB A	(LIVE LD / DEAD LD)	1	·41	·63	-	-	·36	·69	-	-	·37	·67	-	-
		2	·42	·63	-	-	·37	·69	-	-	·39	·67	-	-
		3	·42	·63	-	-	·38	·69	-	-	·40	·67	-	-
3 SPANS.	DEAD LOAD		·40	·60	·50	-	·35	·65	·50	-	·37	·64	·50	-
	LIVE LOAD		·45	·62	·58	-	·43	·68	·68	-	·43	·67	·61	-
	R	½	·42	·61	·53	-	·38	·66	·56	-	·39	·65	·54	-
A BC CB A		1	·43	·61	·54	-	·39	·66	·59	-	·40	·66	·56	-
		2	·43	·61	·56	-	·40	·67	·62	-	·41	·66	·57	-
		3	·44	·61	·56	-	·41	·67	·63	-	·42	·66	·58	-
4 SPANS.	DEAD LOAD		·39	·61	·54	·46	·34	·66	·55	·45	·36	·64	·55	·45
	LIVE LOAD		·45	·62	·60	·57	·42	·68	·65	·61	·43	·66	·64	·60
	R	½	·41	·61	·56	·50	·37	·67	·59	·50	·38	·65	·58	·50
A BC DD CB A		1	·42	·61	·57	·52	·38	·67	·60	·53	·39	·65	·59	·52
		2	·43	·62	·58	·54	·39	·67	·62	·55	·41	·65	·61	·55
		3	·43	·62	·59	·55	·40	·68	·63	·57	·41	·65	·61	·56
SINGLE SPANS. (LIVE OR DEAD LOAD).	 FREELY SUPP'D ·50 ·50; FIXED BOTH ENDS ·50 ·50; ANY SYMMETRICAL LOAD.						 ·625 ·375; L/2 L/2 ·69 ·31; L/3 L/3 L/3 ·67 ·33; FIXED AT ONE END AND FREE AT OTHER.							
NOTES:- SHEAR FORCE = COEFFICIENT × TOTAL LOAD.														

clearly specified in the By-laws or Memorandum. A common practice is to consider the shear forces on a span continuous at one or both ends as identical with those on a freely-supported span. This, sometimes referred to as the "static" shear method, may be satisfactory if conservative working stresses

are adopted. In pursuance of the sound policy that increase in working stresses predicates a refinement in calculation, the shear force computation should bear some relation to the basis of the moment calculation. For this reason the author considers that when Quality A concrete is employed the shear forces should be calculated in accordance with the elastic theory for all important beams, and Clause 2 of the Memorandum seems to imply that such " elastic shears " should be adopted.

Table 19 gives the coefficients for maximum elastic shear forces, these coefficients applying to equal or approximately equal spans and to three types of loading assuming " freely-supported " conditions at the end supports. The coefficients are given separately for live and dead loads and for combined loads with live : dead ratios ranging from $\frac{1}{2}$ to 3. Other types of loading can be interpolated. The effect of end-fixing moments can often be closely assessed by inspection of the free-end and fixed-end coefficients, or alternatively *Table* 20 can be used.

In the cases of special beams or irregular loading, for which moment diagrams are prepared, the theoretical shear force at any section can be obtained directly from these diagrams by determining the slope of the moment envelope at the section under consideration.

The shear force calculations for the roof beams are given on Calculation Sheet No. 4 for the secondary beams and on No. 7 for the main beams. In view of the low live : dead ratio, the coefficients for the secondary beams without end-fixing are taken from *Table* 19 and lie between those for $R = \frac{1}{2}$ and those for dead load only. If a combined coefficient could not be assessed with reasonable accuracy, the shear force could be determined by considering the dead and live loads separately. Thus for span AB, at the section adjacent to support A,

$$\text{Total dead load on span} = 660 \times 24 = 15{,}840 \text{ lb.}$$
$$\text{Total live load on span} = 240 \times 24 = 5{,}760 \text{ lb.}$$

$$\text{Shear force: dead} = 0{\cdot}39 \times 15{,}840 = 6{,}178 \text{ lb.}$$
$$\text{live} = 0{\cdot}45 \times 5{,}760 = 2{,}592 \text{ lb.}$$
$$\text{Total} = 8{,}770 \text{ lb.}$$

This may be compared with the value of 8,640 lb. obtained by the assessed coefficient of 0·40 and with the " static shear force " of 0·50 × 21,600 = 10,800 lb.

The effect of partial end-fixing is, in the end span, to increase the shear force adjacent to the end support and to reduce that adjacent to the penultimate support. The shear in other spans will also be affected slightly, and the coefficients given on *Table* 20 have been used in the secondary beam shear calculations.

The calculations for the main beams (Calculation Sheet No. 7) involve coefficients taken from *Table* 19, allowances being made due to the facts that (i) the bulk of the load is applied as a central point load, (ii) the live load is small compared with the dead load, and (iii) a moderate amount of end-fixing is involved. In such conditions coefficients can be reasonably assessed without splitting the loading and calculating precisely the effect of end-fixing.

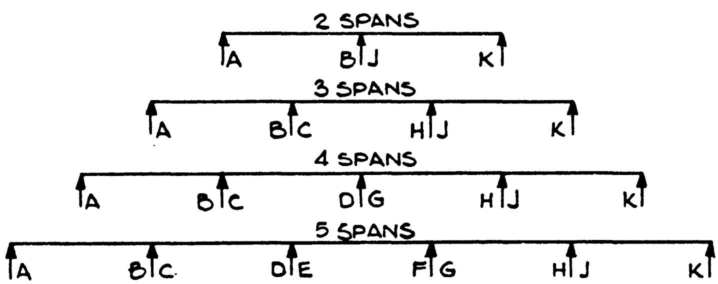
TABLE 20.

CONTINUOUS BEAMS: ADJUSTMENT TO SHEAR FORCES.

CONTINUOUS BEAMS.
ADJUSTMENT TO SHEAR FORCES DUE TO MOMENTS AT END SUPPORTS.

	SECTION	A	B	C	D	E	F	G	H	J	K
BENDING MOMENT APPLIED AT A ONLY.	2 SPANS	+·104	−·104	-	-	-	-	-	-	−·021	+·021
	3 SPANS	+·106	−·106	−·028	-	-	-	-	+·028	+·006	−·006
	4 SPANS	+·106	−·106	−·028	+·028	-	-	+·007	−·007	−·002	+·002
	5 SPANS	+·106	−·106	−·028	+·028	+·008	−·008	−·002	+·002	+·0004	−·0004
EQUAL BENDING MOMENT APPLIED AT A AND K	2 SPANS	+·125	−·125	-	-	-	-	-	-	−·125	+·125
	3 SPANS	+·100	−·100	NIL	-	-	-	-	NIL	−·100	+·100
	4 SPANS	+·107	−·107	−·036	+·036	-	-	+·036	−·036	−·107	+·107
	5 SPANS	+·105	−·105	−·026	+·026	NIL	NIL	+·026	−·026	−·105	+·105

KEY



NOTES: TABULATED FIGURES ARE VALUES OF K FOR SUBSTITUTION IN THE EXPRESSION:

$$\text{ADJUSTMENT TO SHEAR FORCE} = K\frac{M}{L}$$

WHERE M = APPLIED END FIXING MOMENT IN INCH LBS.
L = SPAN OF BEAM IN FEET.

CALCULATION SHEET No. 4.

ROOF.

SECONDARY BEAMS (cont.)	SHEAR CALCULATIONS.	
WITHOUT END FIXING	Total Load per span = 24 × 900 = 21600 lbs	
	A B C D E	
$a = 12{\cdot}5''$ approx.	**Spans AB and DE :-**	
	At A (and E) : $S = 0{\cdot}40 \times 21{,}600 = 8{,}640$ lbs	Nominal Stirrups say $\frac{3}{8}''\phi$ at $12''$ c/c
	$s = \dfrac{8{,}640}{8 \times 12{\cdot}5} = 86$ lbs/ins² (<95)	
	At B (and D) : $S = 0{\cdot}61 \times 21{,}600 = 13{,}170$ lbs	
	$s = 132$ lbs/ins² (>95)	
	$V = \dfrac{13{,}170}{12{\cdot}5} = 1054$	$\frac{3}{8}''\phi$ Stirrups at $3\frac{1}{2}''$ c/c $(V = 1130)$
	Spans BC and CD :-	
	At B (and D) : $S = 0{\cdot}55 \times 21{,}600 = 11{,}900$ lbs	
	$s = 119$ lbs/ins² (>95) $V = 950$	$\frac{3}{8}\phi$ Stirrups at $4''$ c/c $(V = 990)$
	At C : $S = 0{\cdot}48 \times 21{,}600 = 10{,}370$ lbs	
	$s = 104$ lbs/ins² (>95) $V = 832$	$\frac{3}{8}''\phi$ at $4\frac{1}{2}''$ c/c $(V = 880)$
WITH END FIXING.	**Spans AB and DE :-**	
$\dfrac{M}{L} = \dfrac{262\,000}{24} = 10{,}900$	At A (and E) S as above = 8,640 lbs	
	Additional shear $= {\cdot}106 \times 10{,}900 =$ 1,160 "	
	= 9,800 "	
	$s = \dfrac{9800}{8 \times 12{\cdot}5} = 98$ lbs/ins² $\ngtr 95$ lbs/ins² at edge of support.	Nominal Stirrups
	At B (and D) : S as above = 13,170 lbs	
	Reduction in shear $= {\cdot}106 \times 10{,}900 =$ 1,160 "	
	= 12,010 "	
	$s = 120$ lbs/ins² $V = \dfrac{12{,}010}{12{\cdot}5} = 960$	$\frac{3}{8}''\phi$ Stirrups at $4''$ c/c
	Spans BC and CD :-	
	At B (and D) : S as above = 11,900 lbs.	
	Reduction $= {\cdot}028 \times 10{,}900 =$ 310 "	
	= 11,590 "	
	$s = 116$ lbs/ins² $V = 930$	$\frac{3}{8}''\phi$ Stirrups at $4''$ c/c
	At C : S as above = 10,370 lbs.	
	Add 310 "	
	$s = 107$ lbs/ins. $V = 855$ = 10,680 "	$\frac{3}{8}''\phi$ Stirrups at $4\frac{1}{2}''$ c/c

Shear Resistance.

The Memorandum states that the shear stress on a section must be calculated from the expression

$$s = \frac{S}{ba}$$

where S = total shear force across the section,
a = the lever arm, and
b = the breadth of a rectangular beam or the breadth of the rib of flanged beams.

This expression applies theoretically to beams of constant depth only, and when the effective depth of the member varies, the expression is $s = \dfrac{S - \frac{M}{d} \tan \alpha}{ba}$ when the bending moment increases with the increase in d. When the bending moment decreases with the increase in d, the expression reads:

$$s = \frac{S + \frac{M}{d} \tan \alpha}{ba}.$$

The angle in these formulæ is that between the top and bottom edges of the beam at the section considered. With haunches adjacent to the interior supports of continuous beams the sign in the equation is negative, and is positive for haunches at free supports.

Shear reinforcement is only necessary when s exceeds one-tenth of the permissible compressive stress in bending, in accordance with By-laws 99 and 108 and if s exceeds this amount, reinforcement must be provided to take the whole of the shear force. If shear reinforcement is provided, the value of s may be increased but must not exceed four-tenths of the permissible compressive stress in bending. The permissible values of s depend on the quality and mix of the concrete and are summarised on *Table* 21.

Punching shear stresses are limited by By-law 99 to twice the permissible value of s.

The foregoing consideration of shear conforms also to the Code recommendations except that in the Code no mention is made of punching shear. With this exception *Table* 21 can be used for Code designs, the values in it for High-grade concretes being taken as those for "Quality A." For Special-grade concrete the stresses are 25 per cent. higher than for High-grade with limits of 150 lb. per square inch for cases where no shear reinforcement is provided and 600 lb. per square inch in other cases.

Shear reinforcement is usually in the form of stirrups or inclined bars or a combination of the two forms. The total shear resistance can be taken as the sum of the shear resistance of inclined bars and stirrups calculated separately if both are used.

TABLE 21.

LIMITING VALUES OF SHEAR STRESS.

(By-laws.)

Concrete Mix	Quality of Concrete	Maximum Permissible Stress (lb. per square inch) Without Shear Reinforcement	With Shear Reinforcement	Punching Shear
(1 : 1 : 2)	Ordinary . . .	98	390	196
	Quality A . . .	125	500	250
(1 : 1½ : 3)	Ordinary . . .	85	340	170
	Quality A . . .	110	440	220
(1 : 2 : 4)	Ordinary . . .	75	300	150
	Quality A . . .	95	380	190

The resistance S (lb.) of vertical stirrups is given by the expression

$$S = \frac{A_T ta}{p}$$

where A_T = cross-sectional area of a single stirrup (two arms) in sq. in.
t = permissible stress = 18,000 lb. per square inch.
a = lever arm (in.)
p = pitch (in.) or spacing between adjacent stirrups; not to exceed the value of a.

To comply with By-law 106, the minimum bar diameter for beam stirrups is $\frac{3}{16}$-in.

It is convenient to express the shear value, V, of groups of stirrups of given diameter and spacing by the value of $\frac{A_T t}{p}$. Values of V are given on *Table* 22, together with the resistance of stirrups at the spacing $p = a$. As the spacing may be controlled by the requirements of By-law 111 concerning compression steel, the minimum sizes of compression bars corresponding to any given pitch are also tabulated. To comply with By-law 109, stirrups must be anchored at both ends to develop the full working stress, and the end of the stirrup must pass around the tensile reinforcement. If sufficient anchorage length can be obtained in the compression area of the beam it is not essential for the stirrup to pass around the top bars in the beam, but if stirrups used to obtain shear resistance are also depended upon to bind compression bars, the stirrups must pass around the compression and tensile bars to comply with By-law 111. According to the Memorandum, stirrups can be considered as effectively anchored if a bend in the bar of at least 90 deg. passes around a bar and projects beyond the end of the curve for a length of at least eight diameters.

With regard to inclined bars acting as shear reinforcement, the Memorandum states that inclined bars, to be effective, must be carried through a depth of beam equal to the lever arm and be adequately anchored. By-law 108 requires

TABLE 22.

SHEAR RESISTANCE OF BEAM STIRRUPS.

(By-laws and Code.)

DIAM. OF BAR	VALUES OF V FOR VARIOUS SPACING OF STIRRUPS $\left(V = \frac{S}{a} = \frac{A_T t}{p}\right)$ 1½"	2"	3"	4"	4½"	5"	6"	7"	7½"	8"	9"	10½"	12"	14"	16"	18"	24" c/c	SHEAR RESISTANCE OF NOMINAL STIRRUPS p = a	
3/16"	682	496	331	248	221	198	166	142	132	124	110	96	82	72	63	56	41	995	LBS
1/4"	1174	882	587	441	391	355	294	252	235	221	196	168	147	126	110	98	74	1764	"
5/16"	1848	1386	924	693	616	554	462	396	370	347	308	264	231	198	173	154	115	2772	"
3/8"	2640	1980	1320	990	880	792	660	566	528	495	440	377	330	283	248	220	165	3960	"
7/16"	3600	2700	1800	1350	1200	1080	900	772	720	675	600	514	450	386	338	300	225	5400	"
1/2"	4700	3530	2350	1765	1567	1412	1175	1008	940	883	784	671	588	504	441	392	294	7060	"
MINIMUM DIAMETER OF BARS IN COMPRESSION		ORDINARY COMPN. (12d)	1/4"	3/8"	3/8"	7/16"	1/2"	5/8"	5/8"	11/16"	3/4"	7/8"	1"	1 3/16"	1 3/8"	1½"	2"	BEAM BARS:- MIN. DIA. = 1/4" MAX DIA. = 2"	
FOR GIVEN STIRRUP SPACING		$A_C = A_T$ (8d)	3/8"	1/2"	9/16"	5/8"	3/4"	7/8"	15/16"	1"	1 1/8"	1¼"	1½"	1¾"	2"	-	-	STIRRUPS:- MIN. DIA. = 3/16"	

the tensile resistance of the shear reinforcement to act in proper conjunction with the compressive resistance of the concrete. This is embodied in the principle expressed in the Memorandum, in accordance with which inclined bars form the tension members of a lattice girder while the concrete forms the compression members. The shear resistance at any section is then taken as the sum of the vertical components of the inclined tension and compression crossing the section. It seems reasonable to stipulate that the force required in the horizontal portion to balance that in the inclined portion of the shear bar shall not induce stresses in the horizontal portion in excess of the permissible stress. The permissible stress in tensile reinforcement is, according to By-law 100, 18,000 lb. per square inch, which corresponds to ordinary mild steel bars. If this maximum stress is worked to in both bending and shear it is essential that the spacing of inclined shear bars should not be greater than indicated in *Fig.* 5. This diagram also gives the corresponding spacing of the inclined bars if the horizontal portion can be stressed to 20,000 lb. per square inch, as is permissible with certain high-yield-point steels, and the inclined portion at 18,000 lb. per square inch.

If the more usual spacing (*Fig.* 6) is adopted, the tensile stress allowed for shear resistance should be decreased to limit the horizontal component of the force in the inclined members so as not to exceed the permissible force in the horizontal portion. *Fig.* 6 also gives the corresponding stress in the inclined portion if the permissible stress in the horizontal portion is 20,000 lb. per square inch.

Economic comparison of the two methods favours the spacing indicated in *Fig.* 5, and for a single system of inclined bars bent up at any angle the shear value at any vertical section would be given by the expression $S = A_T t \sin\theta$, which takes into account the inclined tension and compression and would have twice the value for a double system (see *Table* 23).

The value of d should be at least equal to the lever arm, and both ends of the inclined bar should be anchored sufficiently to develop the required stress. With a maximum stress of 18,000 lb. per square inch the shear value of a single system of bars inclined at 45 deg. is

$$S = (18{,}000 \times 0{\cdot}707)A_T = 12{,}700A_T$$

where A_T is the normal cross-sectional area of the bar.

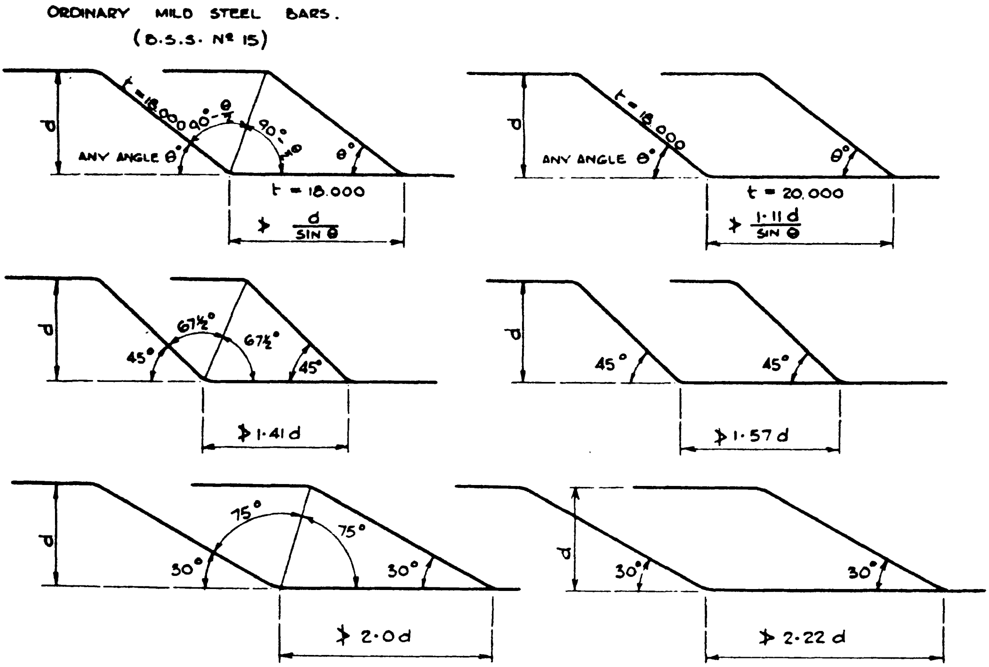


FIG. 5.—INCLINED BARS ARRANGED FOR MAXIMUM STRESS.

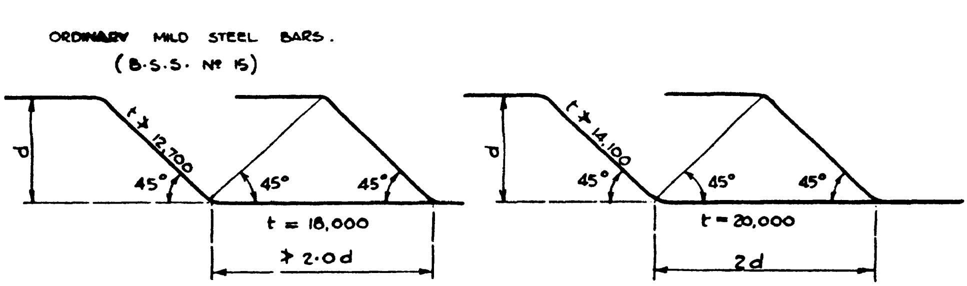


FIG. 6.—LIMITING STRESSES FOR ALTERNATIVE ARRANGEMENT OF INCLINED BARS.

Values of S for a range of different diameters at this stress are given on *Table* 23 for bars inclined at 45 deg. and 30 deg. At other stresses the shear value would be in proportion to the stress, and for intermediate angles the value will be proportional to the sine of the angle.

TABLE 23.

SHEAR RESISTANCE OF INCLINED BARS.

(By-laws and Code.)

SHEAR RESISTANCE OF INCLINED BARS.
t = 18,000 (PRO RATA FOR OTHER STRESSES)

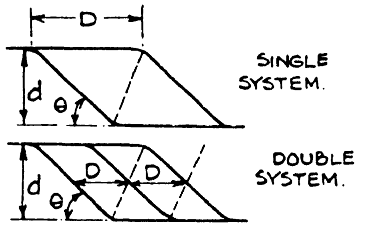
BAR DIA.	θ = 45°		θ = 30°	
	SINGLE	DOUBLE	SINGLE	DOUBLE
½"	2.500	5,000	1,750	3.500
⅝"	3,900	7.800	2.750	5.550
¾"	5.600	11,200	3.900	7.800
⅞"	7.650	15.300	5,400	10,800
1"	10,000	20,000	7,050	14,100
1⅛"	12,650	25,300	8,950	17,850
1¼"	15,600	31,200	11,050	22,050
1⅜"	18,900	37,800	13,350	26,700
1½"	22.500	45,000	15,900	31,800
MAX^M VALUE OF D.	1·41 d	0·71 d	2·00 d	1·00d

NOTES

$S = A_T t \sin\theta$ SINGLE SYSTEM.

2S FOR DOUBLE SYSTEM.

TABULATED VALUES OF SHEAR RESISTANCE ARE IN LBS. AND ARE EXPRESSED TO THE NEAREST 50LBS.



Shear Calculations for Roof Beams.

The calculations on Sheets Nos. 4 and 7 demonstrate the application of the foregoing principles and appropriate tables to the estimation of the shear resistance of the beams detailed in *Figs.* 7 and 8. The resistance of the secondary beams is provided entirely by stirrups. In each case the unit shear stress is determined first ; if this is less than the value given on *Table* 21 (95 lb. per square inch with the quality and mix of concrete adopted) shear reinforcement is not required. This is illustrated by the left-hand portion of span *AB*, and nominal stirrups only are provided. In the other instances the unit stresses exceed 95 lb. per square inch but do not exceed the maximum value of 380 lb. per square inch, and the whole shear is taken on the stirrups ; suitable diameters and spacing for the latter are obtained from *Table* 22 for the appropriate value of *V*.

The shear forces computed from the coefficients on *Table* 19 are the reactions from each span on the supports, and the shear forces at the face of the latter are, in the case of the secondary beams, somewhat less than these maximum values. There is therefore no objection to the shear value of the stirrups adjacent to the support being slightly less than that based on the value of *V* at the centre of the support.

At the section where the unit shear stress does not exceed 95 lb. per square inch the stirrups need only be nominal, and between this section and the support the spacing can be graduated.

The shear calculations for the main beams are given on Calculation Sheet No. 7, and these show that no shear reinforcement is necessary adjacent to the end supports. Elsewhere both stirrups and inclined bars are provided. The

diameter of the inclined bars, which are arranged as a single system, is dictated by the size of bars available from the reinforcement provided for bending. The shear value of these bars determined from *Table* 23 is deducted from the total shear force and the difference is made up by stirrups, using *Table* 22.

A feature of interest occurs in the centre span, where the horizontal anchorage lengths of bars *c*, *d*, and *e* are limited to avoid congestion. The stress corresponding to this length is about 12,000 lb. per square inch, and the shear value of these bars is $\frac{12,000}{18,000} \times 10,000 = 6,700$ lb., which is allowed in the shear calculation.

Details of Roof Beams.

The calculations on Sheets Nos. 5 and 6 for the main beams follow similar lines to those already given for secondary beams. The reactions from the latter beams ($= P$) are taken as equal to the total load on a single span of secondary beam, this being a reasonable value for the elastic reaction on the central string of main beams. A more refined loading calculation would consider the point load as less than the full reaction from the secondary beams, some load from the slab being transferred directly to the main beams as a distributed load. The moment calculations therefore err slightly on the safe side, thus compensating wholly or in part for any underestimation of the reactions on the strings of main beams on either side of the central string.

As all the main beams frame into columns, the effect of the end-fixing moments is taken in the initial bending moment calculation and not treated separately as in the case of the secondary beams. The bending moment coefficients are taken from *Tables* 13 and 16, the full reduction of 15 per cent. being made to the negative moments for conditions of free support on end supports, but to avoid complication no similar adjustment has been made to the negative moment due to end-fixing. A method of obtaining a close value for the relief to the positive moment in the end span due to end-fixing is indicated.

A study of details of the main beams given in *Fig.* 8 shows that the requirements of By-laws 97 and 107 with regard to the spacing of bars and cover of concrete are complied with. The maximum number of bars occurs in section 4–4, where there are four 1-in. bars in one layer. These require 1-in. side cover and, with the minimum lateral spacing of 1 in. (for 1-in. bars and ¾-in. coarse aggregate), the minimum beam width is 9 in. The specified ½-in. vertical space between the layers has been provided at the midspan sections, but the four compression bars at the internal supports are in contact (in pairs), as two 1-in. bars only are required at this section; the bars from adjacent spans are lapping only and therefore do not require the ½-in. clearance. In the case of the compression steel at the support of the secondary beams (*Fig.* 7) the full number of bars provided is required to resist compression and the ½-in. clearance is provided.

The minimum cover of 1 in. (or 1⅛ in. where 1⅛-in. bars are used) is provided throughout the secondary and main beam details unless the position of the reinforcement from intersecting beams controls the distance the bars are placed from the concrete surfaces.

The tensile and compression bars are stopped-off, bent-up, or bent-down beyond the points where they are no longer required for resisting bending moments,

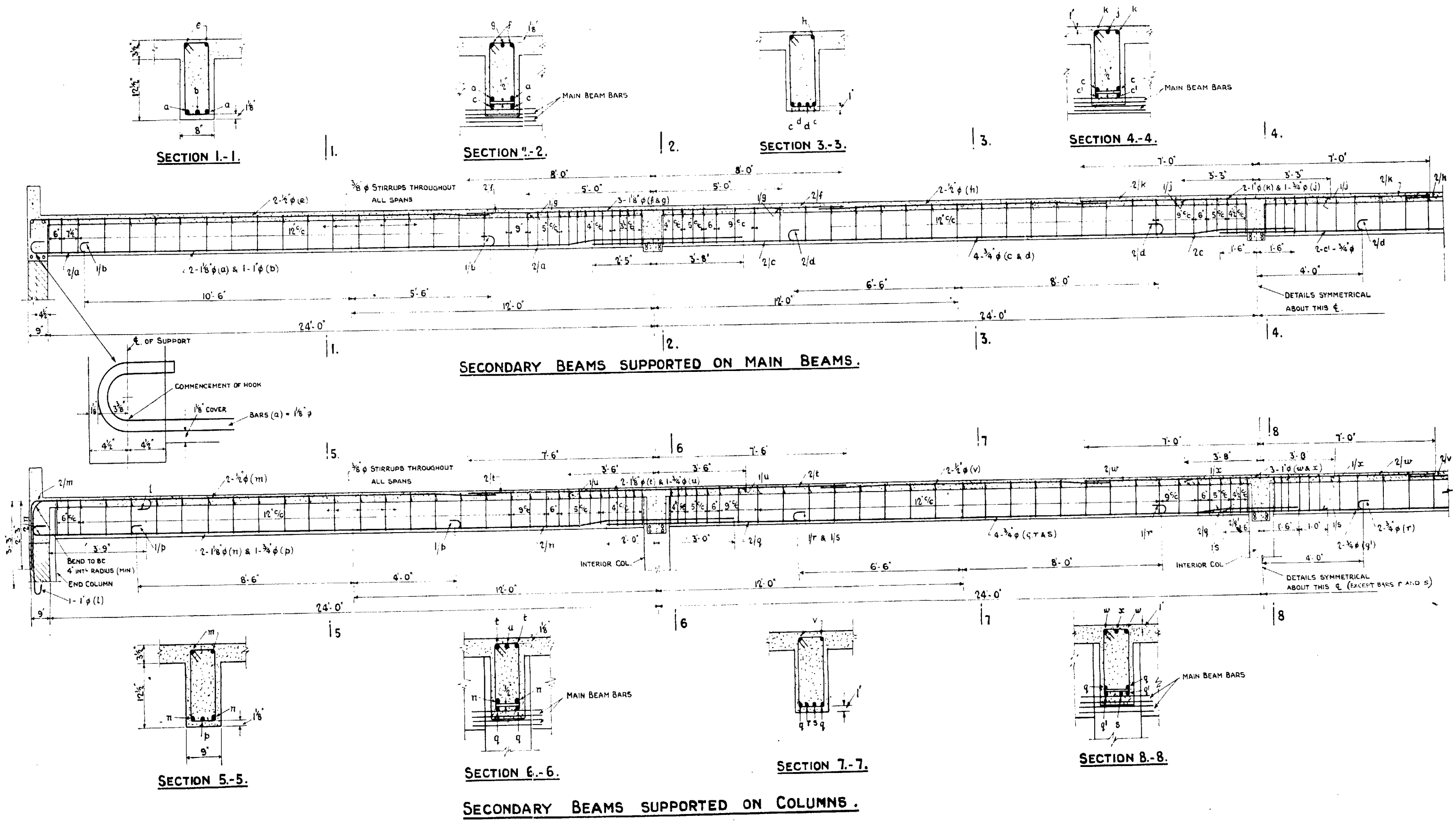


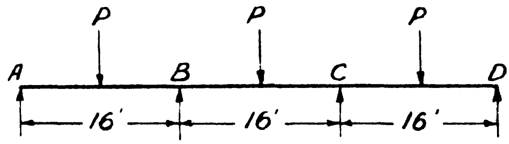
FIG. 7.—DETAILS OF SECONDARY BEAMS IN ROOF.

diameter of the inclined bars, which are arranged as a single system, is dictated by the size of bars available from the reinforcement provided for bending. The shear value of these bars determined from *Table* 23 is deducted from the total

CALCULATION SHEET NO. 5.
ROOF.
(By-laws.)

MAIN BEAMS

BENDING RESISTANCE CALCULATIONS.



Total Load on each span:

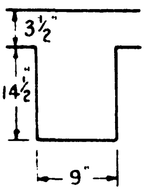
P = Reactions from Secondary Beams.

Dead $= 24 \times 660 = 15{,}840$ lbs

Live $= 24 \times 240 = 5{,}760$ "

Distributed: say $200 \times 16 = 3{,}200$ "

Total $= 24{,}800$ "



Assumed end fixing moments $= \frac{WL}{20}$

$= \frac{1}{20} \times 24{,}800 \times 16 \times 12$

say $= 240{,}000$ in lbs.

Bending Moments:-

Supports B & C :- in lbs

(3,040,000) P.- Dead: $\cdot 128 \times 15{,}840 \times 16 \times 12 = 389{,}000$

(1,105,000) Live: $\cdot 149 \times 5{,}760 \times 16 \times 12 = 165{,}000$

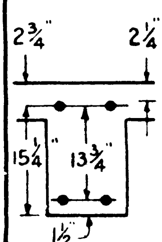
(614,000) Dist^{d}: $\cdot 085 \times 3{,}200 \times 16 \times 12 = 52{,}000$

$606{,}200$

Deduct end fixing B.M.

$= 0{\cdot}2 \times 240{,}000 = 48{,}000$

Total Bending Moment $= 558{,}200$



$C_c = 179 \times 9 \times 15{\cdot}25^2 = 375{,}000$

$C_s = 183{,}200$

$M = \cdot 44 \times 15{\cdot}25 = 6{\cdot}7$;" $f = 1{\cdot}5$"

$C_s = \frac{6{\cdot}7 - 1{\cdot}5}{6{\cdot}7} \times 950 \times 14 = 10{,}300$ lbs/ins^2

$\left.\begin{array}{l} a = \cdot 85 \times 15{\cdot}25 = 13{\cdot}0" \\ a_s = 18" - 2\tfrac{3}{4}" - 1\tfrac{1}{2}" = 13{\cdot}75" \end{array}\right\} a_2 = 13{\cdot}4"$

14½" × 9" net

$A_c = \frac{183{,}200}{10{,}300 \times 13{\cdot}0} = 1{\cdot}36$ ins^2

2 – 1" ϕ (= 1·57 ins^2)

$A_T = \frac{558{,}200}{18{,}000 \times 13{\cdot}4} = 2{\cdot}31$ ins^2

3 – 1" ϕ (= 2·36 ins^2)

Calculation Sheet No. 6.

ROOF.

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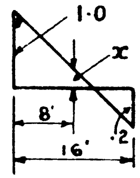
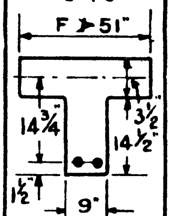
MAIN BEAMS (cont.)



$x = \frac{1{\cdot}0 - {\cdot}2}{2}$
$= 0{\cdot}40$



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Midspan AB and CD :- in.lbs.

P :- Dead : ·198 × 3,040,000 = 602,000
Live : ·239 × 1,105,000 = 264,000
Distributed : ·119 × 614,000 = 73,400
939,400

Deduct for end fixing B.M.
= 0·4 × 240,000 = 96,000
Total B.M. = 843,400

(N.A. below slab)

Width of flange required (neglecting part of rib below slab) $F = \frac{2 \times 843{,}400}{3\frac{1}{2} \times 950 \times 14{\cdot}75} = 34{\cdot}4$

$A_T = \frac{843{,}400}{18{,}000 \times 14{\cdot}75} = 3{\cdot}18 \text{ ins}^2$

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4 – 1" φ
(= 3·14 ins²)

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Midspan BC :- in.lbs.

P : Dead : ·123 × 3,040,000 = 374,400
Live : ·201 × 1,105,000 = 222,000
Distributed : ·040 × 614,000 = 24,600

Add extra due to end-fixing BM.
= 0·20 × 240,000 = 48,000
Total B.M. 669,000

$A_T = \frac{669{,}000}{18{,}000 \times 14{\cdot}75} = 2{\cdot}51 \text{ ins}^2$

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4 – 1" φ
(= 3·14 ins²)

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End Supports A and D :-

Assumed B.M. = 240,000 in.lbs.

Compression O.K. $A_c = 0$

$A_T = \frac{240{,}000}{18{,}000 \times 13} = 1{\cdot}02 \text{ ins}^2$

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2 – ⅞" φ
(= 1·21 ins²)

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<tr>
<td>SHEAR FORCE</td>
<td>See Calculation Sheet Nº 7</td>
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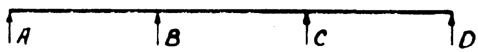
these points having been determined from a sketch bending moment diagram.* Bar e in the main beam cannot be considered as effective in resisting negative moment as it is bent down at a point too close to the support. Thus, although

CALCULATION SHEET No. 7.

ROOF.

MAIN BEAMS (cont.)	SHEAR CALCULATIONS: Total Load on each span = 24,800 lbs. (per Sheet 5)	
	 	
$a = 14 \cdot 0$ approx	Spans AB and CD:- At A (and D): $S = 0{\cdot}40 \times 24{,}800$ $= 9{,}920$ lbs. $S = \frac{9{,}920}{9 \times 14} = 79$ lbs/ins.^{2} (< 95)	Nominal Stirrups
	At B (and C): $S = 0{\cdot}60 \times 24{,}800$ $= 14{,}900$ lbs. $S = 118$ lbs/ins.^{2} (> 95) 1-1"φ inclined at 45° (18,000) = 10,000 " Stirrups to take 4,900 " $V = \frac{4{,}900}{14} = 350$	1-1"φ at 45° (at 18,000) 5/16"φ at 8" c/c $(V = 347)$
	Span BC:- At B (and C). $S = 0{\cdot}52 \times 24{,}800$ $= 12{,}900$ lbs. $S > 95$ lbs/ins.^{2} 1-1"φ at 45° (12,000) = 6,700 " 6,200 " $V = \frac{6{,}200}{14} = 443.$	1-1"φ at 45° (12,000) 3/8"φ at 8½" c/c

four bars are provided in the top at this section, only three bars can be treated as reinforcement to resist bending. The spacing of the inclined shear bars in the main beams is taken from the expression "$1{\cdot}41d$" for a single system as given

* This diagram is very similar to that shown in *Fig.* 3.

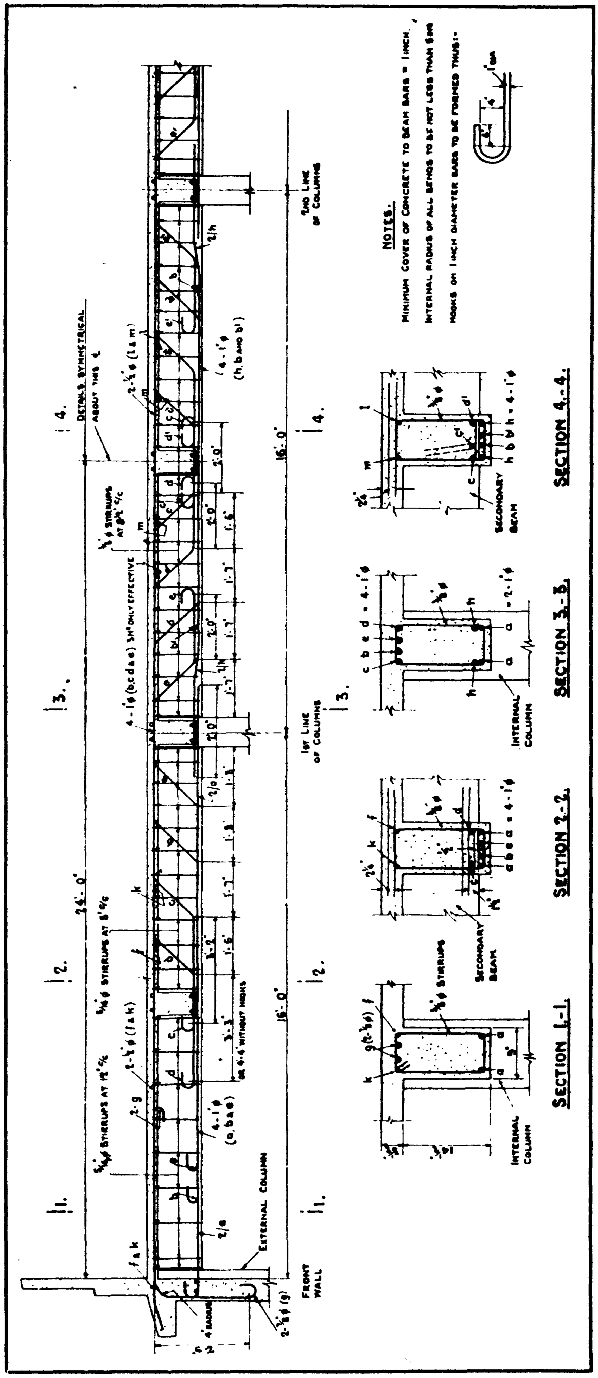


FIG. 8.—DETAILS OF MAIN BEAMS IN ROOF.

on *Table* 23; for bars *b* and *e*, for example, the maximum spacing is 1·41 (18—1½—2¾), say, 20 in., where 18 in. = total depth of beam, 1½ in. = bottom cover plus semi-diameter of bar, and 2¾ in. = top cover plus semi-diameter of bar.

Anchorage of Reinforcement.

Beyond specifying in By-law 99 the maximum permissible bond stresses and in By-law 112 that hooks or other anchorages shall be of such form that the concrete is not overstressed, the By-laws give no further assistance to the designer. The Memorandum, however, gives details of anchorages which are embodied in the following remarks and tables.

The Memorandum stipulates that every bar in tension must extend beyond any given section for a sufficient length to develop by adhesion the force in the bar at that section, and at the end of this length an end anchorage should be provided. This anchorage can be in the form of either (i) a semicircular hook of the form given on *Table* 24, or (ii) a further extension of the bar for a length of 14 diameters, or (iii) mechanical anchorage as described in the Memorandum. When a hook fits over a main reinforcing bar or an anchor bar, the internal diameter of the hook may be equal to the diameter of the anchor bar. The anchorage of stirrups has already been discussed. The minimum length L_0 (in.) required to develop by adhesion the force in the bar can be determined from the expression

$$L_0 = \frac{td}{4s_b}$$

where t = tensile stress in the bar in lb. per square inch,
d = diameter of bar (in.), and
s_b = maximum permissible bond stress in lb. per square inch.

The value of s_b depends on the quality and mix of the concrete, and permissible values have been given on *Table* 8. On *Table* 24 values of L_0 are given for a 1 : 2 : 4 mix of Ordinary and Quality A concretes for a reasonable range of tensile stresses and for various bar diameters. With L_0 are given the lengths L_1 that are the minimum required when the end anchorage takes the form of a straight length of bar; thus $L_1 = L_0 + 14d$.

Table 24 can be used for designs in accordance with the Code if for High-grade concrete the values for Quality A concrete are adopted.

The dimensions to the stopping-off points of the tensile reinforcement in the secondary beams (*Fig.* 7) have been determined from a consideration of the moment envelopes and the anchorage lengths required. The bars are carried beyond the theoretical stopping-off points a distance not less than half the effective depth of the beam (plus 14 diameters when no hook is provided) and each is then investigated for anchorage value. The section of maximum stress for the bars over the supports is the centre line of the latter, the distance from this section to the end of the bars being, with few exceptions, greater than the required minimum values of L_1 given on *Table* 24 for t = 18,000 lb. per square inch. Hooks are therefore not required on these bars.

When considering bars *f* and *g*, the latter being one of the exceptions mentioned, it was found from the moment diagram that bar *g* (1⅛ in. diameter) could be stopped off about 2 ft. 3 in. from the support, and with an addition of, say, 23 in. could terminate at a point 4 ft. 2 in. from the support. Since the value

TABLE 24.
ANCHORAGE OF REINFORCEMENT.
(Minimum Bar Lengths in Inches.)
(By-laws.)

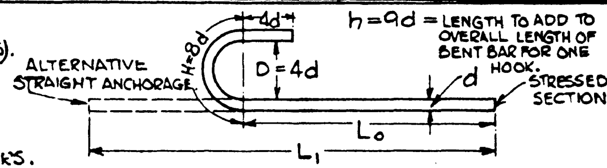
BAR DIAMETER.		½"		⅝"		¾"		⅞"		1"		1⅛"		1¼"		1⅜"		1½"	
HOOK DIMENSION.	H	6"		7½"		9"		10½"		12"		13½"		15"		16½"		18"	
	h	4½"		5⅝"		6¾"		7⅞"		9"		10⅛"		11¼"		12⅜"		15½"	
	D	2"		2½"		3"		3½"		4"		4½"		5"		5½"		6"	
CONCRETE.	STEEL STRESS	L_o	L_1	L_o	L_1	L_o	L_1	L_o	L_1	L_o	L_1	L_o	L_1	L_o	L_1	L_o	L_1	L_o	L_1
ORDINARY	9,000	12	19	15	23	(17)	28	(20)	32	(23)	37	(26)	42	(29)	46	(31)	51	(34)	55
QUALITY	10,000	13	20	16	25	19	30	22	35	25	39	29	44	32	49	35	54	38	59
	12,000	15	22	19	28	23	33	27	39	30	44	34	50	38	55	42	61	45	66
1 : 2 : 4	14,000	18	25	22	31	27	37	31	43	35	49	40	56	44	62	49	68	53	74
MIX III	16,000	20	27	25	34	30	41	35	48	40	54	45	61	50	68	55	75	60	81
s_b = 100 LBS/$_{IN.}^{2}$	18,000	23	30	29	37	34	45	40	52	45	59	51	67	57	74	62	82	68	89
	20,000	25	32	32	40	38	48	44	56	50	64	57	73	63	80	69	88	75	96
QUALITY	9,000	(10)	17	(12)	21	(15)	25	(17)	29	(19)	33	(21)	37	(24)	41	(26)	45	(29)	50
A	10,000	(11)	18	(13)	22	(16)	27	(19)	31	(21)	35	(24)	40	(26)	44	(29)	48	(32)	53
	12,000	13	20	16	25	19	30	22	35	25	39	29	44	32	49	35	54	38	59
1 : 2 : 4	14,000	15	22	19	27	22	31	26	38	30	44	33	49	37	54	40	60	44	65
MIX IIIA	16,000	17	24	21	30	26	36	30	42	34	48	38	54	42	60	47	66	51	72
s_b = 120 LBS/$_{IN}^{2}$	18,000	19	26	24	33	29	39	33	46	38	52	43	58	47	65	52	72	57	78
	20,000	21	28	27	35	32	42	37	49	42	56	47	63	53	70	58	77	63	84

L_o = MINIMUM LENGTH REQUIRED FOR TENSION BARS PROVIDED WITH HOOK OR OTHER END ANCHORAGE (INCHES).

L_1 = MINIMUM OVERALL LENGTH FOR TENSION BARS WITHOUT HOOKS OR OTHER END ANCHORAGE (INCHES).

MINIMUM BOND LENGTH FOR COMPRESSION BARS = L_o

VALUES IN BRACKETS DENOTE LENGTHS LESS THAN 24 DIAMETERS.



of L_1 is 58 in., the bar cannot stop at this point but must be extended a distance of about 5 ft. on both sides of the support.

Bar *f*, also $1\frac{1}{8}$ in. diameter, must be extended at least to the point of contraflexure about 6 ft. from the support; adding 23 in., this bar can terminate at a point 7 ft. 11 in. from the support. Although bar *f* will be fully stressed at the support, it will also be almost fully stressed at the theoretical stopping-off point of bar *g* since the latter is stopped off at the minimum permissible distance from the support. A full anchorage length of 58 in. provided for bar *f* beyond the theoretical stopping-off point of bar *g* would place the end of the bar 2 ft. 3 in. + 4 ft. 10 in. = 7 ft. 1 in. from the support centre. The length provided is in excess of this value and is therefore satisfactory.

Bar *l* (1 in. diameter) must be anchored a distance of about 3 ft. 3 in. into the column ($L_0 = 38$ in.) if a hook is provided, while bar *m* ($\frac{1}{2}$ in. diameter) must extend about 2 ft. 3 in. ($L_1 = 26$ in.) if without a hook.

Bottom bars not carried to the support have been provided with hooks, as in the author's opinion it appears to be a better method for bars in the bottom layer of narrow beams than stopping the bars off straight; a preferable method is to bend the bars up when they can be conveniently used for shear or negative moment.

In the main beam, *Fig.* 8, most bars are used for positive moment reinforcement, shear reinforcement, and negative moment reinforcement in turn, and to avoid congestion hooks are provided as anchorages, thus restricting the overall bar lengths. Although anchorage lengths of 3 ft. 2 in. with hooks are required for bars *c* and *d* in the end span, the reinforcement in this span is more open and providing an alternative straight length of 4 ft. 4 in. would not lead to unnecessary congestion. To avoid intricate threading of the main beam bars with those of the secondaries in the centre span, the anchorage length of bar *d* is limited to about 2 ft. This corresponds to a stress of about 12,000 lb. per square inch as indicated in the shear calculations. Similar anchorages are provided for bars *c* and *d*, as the additional shear resistance can be easily provided by binders.

A number of subsidiary factors relating to anchorages must be considered in designing in compliance with the Memorandum. No account can be taken of the anchorage value of a hook if the latter is in a position where there is any risk of splitting the surrounding concrete. For this reason it is suggested that a minimum thickness of concrete of at least three diameters should be provided above the hook, but at the ends and sides the minimum allowable cover (equal to the bar diameter) should be sufficient. The Memorandum allows hooks to be taken as effective if they can be wrapped to reduce the risk of splitting due to insufficient cover. As no indication is given concerning the amount of wrapping reinforcement that would be considered effective, this is apparently left to the designer's discretion. All hooks in the details in *Figs.* 7 and 8 have ample thickness of concrete over or under the hook as the case may be.

When a form of anchorage other than a hook is adopted, its anchorage value must be at least equal to that of 14 bar diameters and must not subject the concrete to a stress exceeding that given for direct compression on *Table* 8. This stress can be increased to three times the tabulated value when the cover of concrete over the anchorage is considered ample to prevent local failure, or if suitable wrapping or other reinforcement is provided to prevent failure. In

the beam designs under consideration an example does not occur where the adoption of such an anchorage is necessary.

A further clause in the Memorandum specifies that at least 25 per cent. of the main tensile reinforcement in continuous beams or slabs must be carried beyond the points of contraflexure for a distance of not less than half the effective depth of the beam or slab before the hook or other form of end anchorage begins. The points of contraflexure are determined from the moment envelopes due to the arrangement of superimposed loads specified for the calculation of the maximum bending moments. This requirement does not usually affect the arrangement of the positive moment bars in continuous beams in which, as in *Figs.* 7 and 8, it is more convenient to extend a number of the bottom bars beyond the points of contraflexure to provide compressive reinforcement at the supports. Negative moment reinforcement has already been considered in this respect, and it will be seen that considerably more than 25 per cent. of the bars are carried beyond the contraflexure point.

When the end of a beam or a slab is freely supported it is stipulated in the Memorandum that at least 25 per cent. of the main reinforcement shall extend to the centre line of the support before the end anchorage begins. With narrow supports and large diameter bars this may prohibit the use of hooks, but in the case of the free-end support of the secondary beams supported on main beams (*Fig.* 7) the prescribed conditions are just obtained as shown in the enlarged detail. Two $1\frac{1}{8}$-in. bars (1·98 sq. in.) are extended to the support, and this represents more than 70 per cent. of the area of tensile steel at midspan (2·77 sq. in.). The end cover over these bars is $1\frac{1}{8}$-in., which is allowable according to the By-laws. If the anchorage had been other than a hook, an end cover of twice the diameter of the bar and not less than 2 in. would have been required to comply with By-law 97.

Although the By-laws are not explicit concerning the anchorage of compression bars in beams, it is reasonable to assume that sufficient length of bar should be provided to develop the force in the bar by adhesion and no additional end anchorage should be required. In this case the values of L_0 given on *Table* 24 represent the total straight length required for compressive anchorage. At the penultimate support of the secondary beams supported on main beams (*Fig.* 7) the two $1\frac{1}{8}$-in. and the two $\frac{3}{4}$-in. bars are all used at 18,000 lb. per square inch as compression steel in accordance with the "steel beam" theory; the $1\frac{1}{8}$-in. bars require a length of 44 in. to develop the adhesion at this stress (*Table* 24) and the $\frac{3}{4}$-in. bars require 29 in. From the point of view of resistance moment there is no objection to stopping-off the bars at these distances from the support. At the corresponding support for the secondary beams supported on columns bars of the same number and diameter are provided, although the calculations on Sheet No. 3 indicate that one $\frac{3}{4}$-in. bar less is required. Hence the working stress will be below the maximum permissible and the anchorage lengths can be reduced accordingly. At the centre support of the secondary beams the compression steel only assists the concrete in providing adequate compressive resistance, thus the anchorage length on each side of the support centre need only be sufficient to develop a stress of less than 8,000 lb. per square inch (Calculation Sheets Nos. 2 and 3). It is considered, however, that a minimum length of 24 diameters should be provided whenever convenient, even though the working

stress requires considerably less. Thus $24 \times \frac{3}{4} = 18$ in. is the bond length provided for the $\frac{3}{4}$-in. bars at the centre supports.

At the interior supports of the main beams (*Fig.* 8) only two 1-in. bars are required in compression at a stress of 9,500 lb. per square inch (Calculation Sheet No. 5). Since four 1-in. bars are provided, the total lap length need only be sufficient to produce the working stress, that is $L_0 = 20$ in.; as this is less than 24 diameters a length of 2 ft. is provided. In these conditions the lap length, not being required on both sides of the support, is placed symmetrically about the latter.

In accordance with the Memorandum, beam bars should be investigated for bond stress due to variation in tensile stress by applying the formula

$$s_b = \frac{S}{ao}$$

where S = maximum shear force (lb.) at the section under consideration,
a = lever arm (in.), and
o = total perimeter (in.) of the tension bars at the section.

The Memorandum requires that the effect of change of depth of the beam should be taken into account when calculating the bond stress. For beams that are not of constant depth the expression becomes

$$s_b = \frac{S \pm \frac{M}{d} \tan \alpha}{ao}.$$

The symbols have the same signification, and the negative and positive signs the same application as for the corresponding shear stress formulæ given earlier in this chapter.

According to By-law 99, the value of s_b must not exceed twice the allowable bond stress tabulated on *Table* 8 for the different concrete mixes and qualities. On *Table* 25 data referring to the perimeter of numbers of bars of various diameters are given for substitution in the formula. The following examples of the applica-

TABLE 25.

BEAM REINFORCEMENT—BOND.

(By-laws.)

GENERAL FORMULA :-

$\frac{S}{ao} \ngtr s_b$ or $o \nless \frac{S}{a s_b}$

S = SHEAR FORCE. O = PERIMETER OF BARS. a = LEVER ARM. s_b = 2 (MAX. BOND STRESS).

VALUES OF S_b

MIX OF CONC.	QUALITY ORD.	QUALITY A.	BY-LAW REFCE.
1:2:4	200	240	III
1:1⅔:3⅓	214	260	-
1:1½:3	220	270	II
1:1⅓:2⅔	236	290	-
1:1:2	246	300	I

PERIMETER OF ROUND BARS. VALUES OF 'O'

NUMBER OF BARS \ DIAMETER OF BARS.	¼"	5/16"	⅜"	½"	⅝"	¾"	⅞"	1"	1⅛"	1¼"	1⅜"	1½"
1.	0·79	0·98	1·18	1·57	1·96	2·36	2·75	3·14	3·53	3·93	4·32	4·71
2.	1·57	1·96	2·36	3·14	3·93	4·71	5·50	6·28	7·07	7·85	8·64	9·42
3.	2·36	2·95	3·53	4·71	5·89	7·07	8·25	9·43	10·60	11·78	12·96	14·14
4.	3·14	3·93	4·71	6·28	7·86	9·42	11·00	12·57	14·14	15·71	17·28	18·85
5.	3·93	4·91	5·89	7·86	9·82	11·78	13·75	15·71	17·67	19·64	21·60	23·56
6.	4·71	5·89	7·07	9·43	11·78	14·14	16·49	18·85	21·20	23·56	25·92	28·27
7.	5·50	6·87	8·25	11·00	13·75	16·49	19·27	21·99	24·74	27·49	30·24	32·98
8.	6·28	7·86	9·42	12·57	15·70	18·85	21·98	25·14	28·27	31·42	34·56	37·70
9.	7·07	8·84	10·60	14·14	17·68	21·20	24·74	28·28	31·81	35·34	38·88	42·41
10.	7·85	9·82	11·78	15·71	19·64	23·56	27·49	31·42	35·34	39·27	43·20	47·12

tion of this formula relate to two typical sections of the secondary beam details (*Fig.* 7) and demonstrate the procedure.

(1) End span AB : section adjacent to end support A (freely supported). From Calculation Sheet No. 4, $S = 8{,}640$ lb. ; $a = 12{\cdot}5$ in. approximately.

From *Table* 25, $s_b = 240$ lb. per square inch (Quality A, 1 : 2 : 4 mix).

$$\text{Minimum value of } o = \frac{8640}{12{\cdot}5 \times 240} = 2{\cdot}88 \text{ in.}$$

Tensile steel (bottom bars) provided = two $1\frac{1}{8}$-in. bars, having a value of $o = 7{\cdot}07$ in. (*Table* 25) which is sufficient.

(2) End span AB : section adjacent to interior support B (continuous). $S = 13{,}170$ lb. ; $a = 12{\cdot}5$ in. ; $s_b = 240$ lb. per square inch.

$$o \nless \frac{13{,}170}{12{\cdot}5 \times 240} = 4{\cdot}39 \text{ in.}$$

Tensile steel (top bars) provided = three $1\frac{1}{8}$-in. bars for which $o = 10{\cdot}60$ in.

TABLE 26.

RADIUS OF BENDS IN REINFORCEMENT.

(By-laws.)

VALUES OF K.									
CONCRETE MIX			1 : 2 : 4 (III)		1 : 1⅔ : 3⅓		1 : 1½ : 3 (II)		1 : 1 : 2 (I)
QUALITY			ORD.	A.	ORD.	A.	ORD.	A.	ORD.	A.
TENSILE STRESS IN BARS AT BEGINNING OF BEND.	WITH MINIMUM COVER.	10.000	4·2	3·3	3·8	3·0	3·7	2·8	3·2	2·5
		12.000	5·0	4·0	4·6	3·6	4·4	3·4	3·8	3·0
		14.000	5·8	4·6	5·4	4·2	5·1	4·0	4·5	3·5
		16.000	6·7	5·3	6·1	4·8	5·9	4·6	5·1	4·0
		18.000	7·5	5·9	6·9	5·4	6·6	5·1	5·8	4·5
		20.000	8·3	6·6	7·7	5·9	7·4	5·7	6·4	5·0
	WHEN NO RISK OF SPLITTING.	10.000	2·8	2·2	2·6	2·0	2·5	1·9	2·1	1·7
		12.000	3·3	2·6	3·1	2·4	3·9	2·3	2·6	2·0
		14.000	3·9	3·1	3·6	2·8	3·4	2·7	3·0	2·3
		16.000	4·4	3·5	4·1	3·2	3·9	3·0	3·4	2·7
		18.000	5·0	3·9	4·6	3·6	4·4	3·4	3·8	3·0
		20.000	5·6	4·4	5·1	4·0	4·9	3·8	4·3	3·3
MINIMUM INTERNAL RADIUS = K (DIA. OF BAR). VALUES OF K PRO RATA FOR OTHER STRESSES.									

The minimum internal radii of bends are expressed in the Memorandum in terms of the tensile stress in the bar at the bend and the permissible direct stress in the concrete (*Table* 8). The requirements are covered by the data tabulated on *Table* 26, where the two cases are given, namely, when the minimum cover

is allowed, and when the bar is so held by the surrounding concrete that there is no risk of splitting the concrete. The bends in the secondary beam bars *l* and main beam bars *g* are examples of the second case, and the inclined bars in the main beams can be considered as in the first case. The radii in the second case are two-thirds of those required for the first case.

The previous consideration of anchorage and bond conforms both to the By-laws and the Code and therefore *Tables* 25 and 26 can be applied to Code designs if Quality A values are used for High-grade concretes. Consequently the details given in *Figs.* 7 and 8 and the appropriate calculation represent designs in accordance with the Code except that beam design factors would be slightly modified.

Welding.

Under By-law 114, electric welding of mesh reinforcement for solid slabs is permitted but other reinforcement may only be welded under conditions prescribed for individual cases. The London County Council issue as a general guide regulations applicable to electric arc welding (Appendix II), which, although principally relating to structural steelwork, would also control the welding of reinforcement.

In tests authorised by the London County Council it was found that for bars larger than $1\frac{1}{2}$ in. in diameter a double-V or double-U butt weld is preferable, and for bars of $1\frac{1}{8}$-in. to $1\frac{1}{2}$-in. diameter either a single-V or double-V butt weld could be used, while a single-V butt weld is definitely preferable for bars 1 in. or less in diameter. A double-V or U joint can only be made when the bar can be rotated. When bars of unequal diameter are connected by butt welds, single bevel butt welds appear to be suitable.

CHAPTER V

COLUMNS SUBJECT TO AXIAL LOAD

Columns with Lateral Ties.

ACCORDING to By-law 100 the safe load on reinforcement in axially loaded columns is calculated in the customary manner, the permissible stress on the longitudinal reinforcement being considered equal to the stress on the surrounding concrete multiplied by the modular ratio ($m = 15$).

The safe load in a short column with lateral ties which fulfils the requirements of the By-laws is therefore

$$P = Ac[1 + 0{\cdot}14p]$$

where A = the cross-sectional area of the concrete including the "cover,"
c = the permissible direct compressive stress on the concrete, and
p = the percentage of reinforcement.

The limiting percentages of longitudinal reinforcement are 0·8 per cent. (minimum) and 8 per cent. (maximum), according to By-law 104.

The value of P (lb.) for rectangular columns B in. by D in. in cross section can be put in the form

$$P = KBD \text{ where } K = c(1 + 0{\cdot}14p).$$

Values of K are given on *Table* 27 for the full range of steel percentages and for various qualities and mixes of concrete. For convenience, the requirements of the By-laws regarding limiting bar diameters, concrete cover, bar arrangements, anchorage, and similar matters relating to the longitudinal reinforcement are summarised in *Table* 27.

The major departure in the Code from conventional column design is the adoption of a maximum compressive stress on the longitudinal reinforcement without direct reference to the modular ratio. The value of this stress is 13,500 lb. per square inch where ordinary mild steel bars are employed and is increased to 15,000 lb. per square inch if rolled mild steel with a higher yield point is used. The carrying capacity of a short column subjected to axial load alone and with lateral ties (as distinct from helical binding) is given by

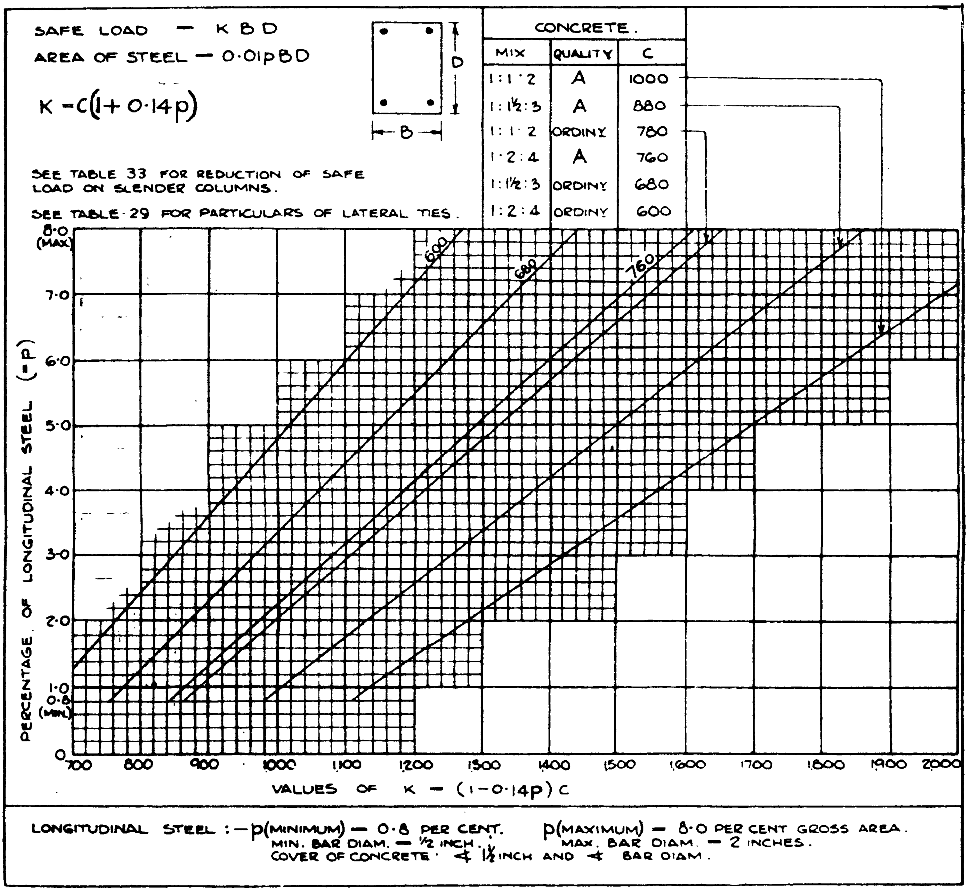
$$P = cA + c_sA_c$$

where c = the permissible direct compressive stress on the concrete (see *Table* 9), and is unaffected by the amount of lateral binding provided in excess of the minimum permissible volume; A = the cross-sectional area of the concrete, including the "cover" (but presumably omitting the area occupied by the reinforcement); c_s = the permissible direct compressive stress on the longitudinal

TABLE 27.

RECTANGULAR COLUMNS WITH LATERAL TIES.

(By-laws.)



reinforcement (13,500 or 15,000 lb. per square inch) ; and A_c = the cross-sectional area of the longitudinal reinforcement.

The limiting percentages (p) of longitudinal reinforcement are 0·8 per cent. (minimum) and 8 per cent. (maximum). The following expression for P (lb.) can be derived for rectangular columns B in. by D in. in cross-section :

$$P = KBD$$

where $K = c + p\ (135 - 0{\cdot}01c)$ when $c_s = 13{,}500$ lb. per square inch
and $K = c + p\ (150 - 0{\cdot}01c)$ when $c_s = 15{,}000$ lb. per square inch.

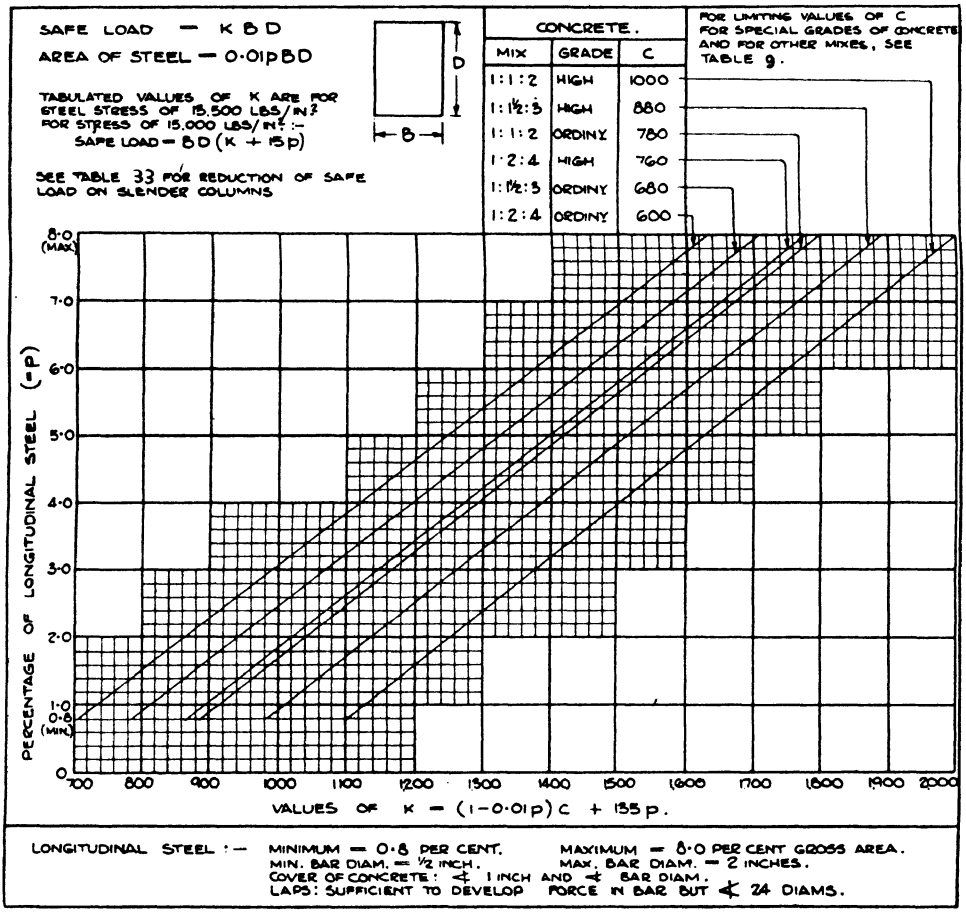
Table 28 gives values of K (with $c_s = 13{,}500$) for the full range of steel percentages and for the various grades and mixes of concrete. For convenience the Code requirements regarding limiting bar diameters, concrete cover, bar arrange-

ment, anchorage, and similar matters relating to the longitudinal reinforcement are summarised on *Table* 28.

TABLE 28.

RECTANGULAR COLUMNS WITH LATERAL TIES.

(Code.)



The procedure for designing a column is either to assume B and D, calculate the required value of K, and determine p from the *Table* 27 for By-law designs or 28 for Code designs; or to assume p, read off the corresponding value of K from the table, and determine the value of the product BD.

The requirements of By-law 105 regarding separate column binders (lateral ties) can be summarised as follows. The diameter of the tie must not be less than ¼ in.; the pitch of the ties must not exceed 12 in., and may not exceed the least lateral dimension of the column or twelve times the diameter of the smallest longitudinal bar.

The Code agrees with these stipulations (except that it allows $\frac{3}{16}$-in. column binders to be used) and additional recommendations as follows are put forward. The minimum volume of lateral ties is 0·4 per cent. of the gross volume of the column ; the spacing need not be less than 6 in. ; when the maximum spacing is employed the diameter of the tie or binder must not be less than one-quarter of the diameter of the longest longitudinal bar secured by the tie.

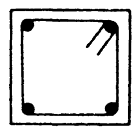
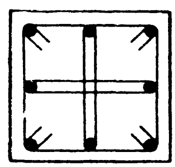
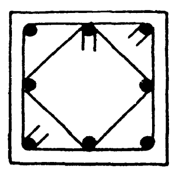
Since the By-laws specify no minimum percentage for the volume of lateral ties, a value of 0·4 per cent. of the gross volume is adopted here as it is in close

TABLE 29.

Lateral Ties in Square Columns (Based on 0·4 per cent. of Volume). If 0·2 per cent. is Admissible the Equivalent can be found by Increasing the Tabulated Column Sizes by $\sqrt{2}$.

(By-laws and Memorandum.)

DIAM. OF LONGITUDINAL BAR	1/2"	5/8"	3/4"	7/8"	1"	OVER ONE INCH
MAXIMUM PITCH BASED ON DIAM. OF LONGL BARS.	6"	7½"	9"	10½"	12"	MAX. PITCH OF 12" EXCEEDED.

MAXIMUM SIZES OF SQUARE COLUMNS (INCHES) FOR TIES OF VARIOUS DIAMETERS AND PITCH.	DIAM. OF TIES.	PITCH OF TIES. 6"	7"	8"	9"	10"	11"	12" (MAX)	ARRANGEMENT OF TIES.
	5/16"	6	-	-	-	-	-	-	
	3/8"	12	9	8	(6)	-	-	-	
	7/16"	21	16	12	10	(9)	-	-	
	1/2"	29	24	21	17	13	11	(10)	
	5/16"	22	16	13	10	(9)	-	-	
	3/8"	33	28	23	21	17	14	12	
	7/16"	-	40	34	30	26	23	21	
	1/2"	-	-	-	-	36	32	29	
	5/16"	16	12	10	(8)	-	-	-	
	3/8"	28	23	19	15	13	(10)	-	
	7/16"	39	34	28	25	22	19	15	
	1/2"	-	-	39	35	30	26	24	

TABULATED VALUES ASSUME:— VOLUME OF TIES ≮ 0·4% GROSS VOLUME OF COL.
1½ INCH COVER TO LONGITUDINAL BARS.

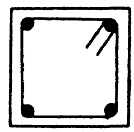
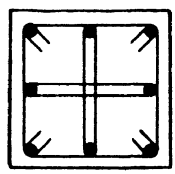
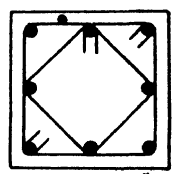
VALUES IN BRACKETS ARE INADMISSIBLE AS SPACING EXCEEDS COLUMN SIZE.

TABLE 30.

COLUMNS WITH LATERAL TIES—PARTICULARS OF TIES.

(Code.)

DIAM. OF LONG^L BARS.	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	1 3/4"	2"
MINIMUM DIAM. OF TIE IF AT MAXIMUM PITCH.	3/16"	3/16"	3/16"	1/4"	1/4"	5/16"	5/16"	3/8"	3/8"	7/16"	1/2"
MAXIMUM PITCH BASED ON DIAM. OF LONG^L BARS.	6"	7 1/2"	9"	10 1/2"	12"	MAX. PITCH OF 12" EXCEEDED.					

MAXIMUM SIZES OF SQUARE COLUMNS (INCHES) FOR TIES OF VARIOUS DIAMETERS AND PITCH.	DIAM. OF TIES.	PITCH OF TIES.							ARRANGEMENT OF TIES.
		6"	7"	8"	9"	10"	11"	12" (MAX.)	
	5/16"	10	8	(6)	-	-	-	-	 (COLUMNS 6" TO 19" SQUARE).
	3/8"	16	13	11	10	(8)	-	-	
	7/16"	-	19	16	14	12	11	(10)	
	1/2"	-	-	-	19	17	15	14	
	5/16"	23	19	16	14	-	-	-	 (COLUMNS 14" TO 37" SQUARE).
	3/8"	36	30	26	22	20	18	16	
	7/16"	-	-	35	31	27	25	22	
	1/2"	-	-	-	-	37	33	30	
	5/16"	19	16	-	-	-	-	-	 (COLUMNS 14" TO 35" SQUARE).
	3/8"	29	24	21	18	16	-	-	
	7/16"	-	34	30	26	23	21	19	
	1/2"	-	-	-	35	31	28	25	

TABULATED VALUES ASSUME:- VOLUME OF TIES $\not<$ 0·4% GROSS VOLUME OF COL.
1 INCH COVER TO LONGITUDINAL BARS.

VALUES IN BRACKETS ARE INADMISSIBLE AS SPACING EXCEEDS COLUMN SIZE.

SPACING NEED NOT BE LESS THAN 6" (FOR ANY COLUMN NOT LESS THAN 6" SIDE) PROVIDED VOLUME OF TIES IS NOT LESS THAN 0·4%.

TIES 3/16" AND 1/4" DIAM. CAN BE USED BUT SPACING WOULD BE LESS THAN 6" FOR ALL COLUMN SIZES.

LIMITS OF COLUMN SIZES FOR EACH ARRANGEMENT OF TIES ARE RECOMMENDATIONS ONLY, NOT BEING SPECIFIED IN THE CODE.

agreement with the value of 0·5 per cent. of the core volume specified in the London County Council Regulations of 1915.

Combining the By-laws with this limiting percentage, *Table* 29 has been compiled to show the limiting sizes of square columns for ties of given diameter, spacing, and arrangement with 1½-in. cover over the main bars. The limiting spacings relative to the diameter of the longitudinal bars are also tabulated.

Table 30 is similar to *Table* 29, but gives the limiting sizes of columns for ties when designed strictly in accordance with the Code, that is with 1-in. minimum cover. This table also summarises the Code's recommendations relating to lateral ties.

The maximum spacing of the bars is not specified either in the Code or By-laws. The limiting column sizes given on *Tables* 29 and 30 for four-bar and eight-bar columns are the author's recommendations.

For the calculations on Sheets 8 and 9, *Tables* 27 and 29 have been used, while for Sheets 10 and 11 *Tables* 28 and 30 relating to the Code have been used.

Columns with Helical Binding.

The limitations regarding the area and diameter of the longitudinal reinforcement are common to columns with lateral ties or with helical binding. By-law 104 requires that with helical binding at least six bars shall be provided equally spaced within the hooping.

The cover on the longitudinal bars must not be less than the bar diameter or less than 1½ in., to comply with By-law 97, but in the case of hexagonal columns with six bars and octagonal columns with eight bars, where the bars are placed opposite the corners of the section (as in Section 9–9, *Fig.* 9), adequate cover over the binding may control the overall size of the column or fix the effective core diameter.

The safe load on a helically-bound column is the sum of the three components: (1) The load P_C carried on the concrete in the core of the column; (2) the load P_T carried by the longitudinal reinforcement; and (3) the additional load P_B due to the helical binding. Therefore $P = P_C + P_T + P_B$.

Expressions for these components can be determined as follows:

$$P_C + P_T = 0{\cdot}7854 D_c^2 c\ (1 + 0{\cdot}14 p_c)$$

where c = the permissible direct compressive stress on the concrete (*Table* 8); D_c = the core diameter of the column; and p_c = the percentage of longitudinal reinforcement expressed in terms of the core area; it should be noted that the limiting percentages of longitudinal reinforcement are expressed in terms of the gross cross-sectional area; values of p_c (based on core area) between 8·0 and, say, 7·0 would possibly lie outside the limits based on the gross cross section.

The expression can be written

$$P_C + P_T = 0{\cdot}7854 D_c^2 K$$

and K can be read from *Table* 27 by substituting p_c for p.

According to the Memorandum the value of the helical binding can be expressed as

$$P_B = 2 t_b A_B$$

in which A_B = the volume of helical binding per unit length of column and t_b = the permissible tensile stress in the helical binding; this stress is limited to 13,500 lb. per square inch in By-law 100.

If A_b is the cross-sectional area of helical binding per foot length of column (for example, with $\frac{3}{8}$-in. binding at 3-in. pitch, $A_b = 0{\cdot}442$ sq. in.) the expression for P_B can be reduced to

$$P_B = 0{\cdot}523\ D_c A_b \times 13{,}500 = 7{,}080 D_c A_b.$$

By-law 105, controlling the helical binding, calls for the spacing to be even and not to exceed 3 in. or $\frac{D_c}{6}$ (whichever is the less); on the other hand, the spacing must not be less than 1 in. or three times the diameter of the bar forming the helical (whichever is the greater). From these limitations it follows that the maximum diameter of bar that could be used for helicals is 1 in., since the minimum spacing of three diameters coincides with the maximum allowable, 3 in. Such helicals could not be used in a column having a core diameter of less than 18 in. (22 in. minimum overall across the flats), but the practical difficulties of forming bars of large diameter into helicals of small radius will usually restrict the use of helicals exceeding $\frac{1}{2}$ in. in diameter.

The values of the component P_B within the limits of the requirements of the By-laws and Memorandum are given on *Table* 31.

TABLE 31.

COLUMNS WITH HELICAL BINDING.

(By-laws.)

LOAD CARRIED DUE TO HELICAL BENDING = P_B		SPACING OF HELICALS	DIAMETER OF HELICALS.									MIN.	
			3/16"	1/4"	5/16"	3/8"	1/2"	5/8"	3/4"	7/8"	1"	D_c	$P_B = k_2 D_c$
	k_2	1"	2,340	4,140	6,510	-	-	-	-	-	-	6"	$k_2 = 0{\cdot}523 A_B C_s$
		1½"	1,560	2,760	4,350	6,250	11,080	-	-	-	-	9"	C_s = 13,500 lb/in²
		2"	1,170	2,070	3,265	4,700	8,300	13,000	-	-	-	12"	MINIMUM D_c
		2½"	936	1,660	2,610	3,750	6,640	10,400	15,000	-	-	15"	= 6 × SPACING.
		3"	780	1,380	2,170	3,120	5,540	8,670	12,500	17,000	22,200	18"	

An important limitation is imposed in the Memorandum on the relative values of P_C and P_B. In no case may $P_C + P_B$ exceed the product of the cross-sectional area of the concrete and one-half the specified concrete crushing strength required from the works test. Reducing this to an expression involving the permissible direct compressive stress (see *Table* 8), and the total cross-sectional area of the concrete (A) we get

$$P_C + P_B \not> 1{\cdot}875 Ac.$$

Applying this condition to the calculation given on Calculation Sheet No. 9 for the 18-in. octagonal section, $A = 0{\cdot}828 \times 18^2 = 268$ sq. in. (ignoring the reinforcement).

From *Table* 8, c = 1,000 lb. per square inch (1 : 1 : 2 mix, Quality A.)

$P_C + P_B = 0{\cdot}7854 \times 16^2 \times 1000 + 60{,}000 = 261{,}000$ lb., which is less than $1{\cdot}875\ Ac = 1{\cdot}875 \times 268 \times 1000 = 503{,}000$ lb.

For the 21-in. octagon, $P_C + P_B = 0{\cdot}7854 \times 19^2 \times 1000 + 157{,}000 = 440{,}000$ lb., which is less than $1{\cdot}875 \times 0{\cdot}828 \times 21^2 \times 1000 = 685{,}000$ lb.

The difference between the design of columns with helical bound cores in accordance with the Code and the By-laws is that the recommendations of the former specify 1-in. cover on the main longitudinal bars which are stressed to 13,500 lb. per square inch (or 15,000 lb. if high yield point steel is used) irrespective of the modular ratio. *Table* 32 has therefore been prepared to give data enabling the values of P_C, P_T and P_B to be determined readily for designs in accordance with the Code. The calculations on Sheet No. 10 are based on this table.

The minimum amount of helical binding, according to the Code, is presumably 0·4 per cent. of the gross volume. Any arrangement of such binding conforming to the other conditions of diameter and spacing will generally give volumes well in excess of this minimum. As an example, however, the volume of binding in the 18-in. octagonal section (Calculation Sheet No. 11 and Section 8–8, *Fig.* 10) will be checked.

The volume per complete turn of $\frac{3}{8}$-in. helical, 16 in. internal diameter, is $3{\cdot}14 \times 16{\cdot}375 \times 0{\cdot}110 = 5{\cdot}65$ cb. in.

The volume of $1\frac{1}{2}$ in. length of 18-in. octagon is $0{\cdot}828 \times 18^2 \times 1\frac{1}{2} = 400$ cb. in. Thus the percentage volume of helical binding $= \dfrac{5{\cdot}65 \times 100}{400} = 1{\cdot}44$. As anticipated, this is in excess of the minimum of 0·4 per cent.

Long Columns.

According to By-law 101 if the ratio of "effective" column length to least radius of gyration exceeds 50, the safe load on the column must be reduced. Coefficients for the reduction of the permissible load on axially loaded slender columns, as tabulated in this By-law, can be derived from the general expression

$$R_L = 1{\cdot}5 - \frac{0{\cdot}12 L_E}{g}$$

where L_E = the effective column length (ft.), and
g = the least radius of gyration (in.).

The numerical values of the ratios of effective length to radius of gyration as given in the By-laws are for both terms in the same units, but for practical application the units given here are more convenient. *Table* 33, which gives values of R_L for slenderness factors within the permissible limits, can be used for units expressed in either form. In accordance with By-law 101 no reinforced concrete column may have a ratio of effective length to least radius of gyration in excess of 120.

The permissible load on a long column is given by

$$P_L = R_L P$$

where P is the permissible load on a short column.

The effective column length is given in By-law 102 as equal to the actual length if the column is adequately restrained at both ends in position but not in direction, that is, if end conditions are equivalent to "hinged both ends."

TABLE 32.

COLUMNS WITH HELICAL BINDING.

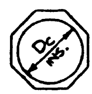
(Code.)

LOAD CARRIED ON CONCRETE = P_C — values of $\frac{P_c}{k_1}$

CONCRETE MIX.	1 : 2 : 4		1 : 1½ : 3		1 : 1 : 2	
CONCRETE GRADE	ORD.	HIGH	ORD.	HIGH	ORD.	HIGH
D_c						
10"	47,100	59,700	53,400	69,100	61,300	78,500
12"	67,900	86,000	77,000	99,400	88,200	113,100
14"	92,300	117,000	105,000	135,500	120,000	154,000
16"	121,000	153,000	137,000	177,000	157,000	201,000
18"	153,000	193,000	173,000	224,000	198,000	255,000
20"	188,500	239,000	214,000	276,000	245,000	314,200
22"	228,000	289,000	259,000	334,000	296,000	380,000
24"	272,000	344,000	308,000	398,000	353,000	455,000
30"	424,000	587,000	481,000	622,000	551,000	707,000
36"	611,000	774,000	693,000	895,000	794,000	1,018,000

P_c	(0·7)	0·8	1·0	2·0	3·0	4·0	5·0	6·0	(7·0)	(8·0)
k_1	0·993	0·992	0·99	0·98	0·97	0·96	0·95	0·94	0·93	0·92

NOTES.



$P_c = 0{\cdot}7854\, c\, k_1 D_c^2$ (LBS)

$k_1 = 1 - 0{\cdot}01\, p_c$

p_c = % OF LONG^{L} STEEL BASED ON CORE.

VALUES IN BRACKETS MAY BE OUTSIDE LIMITING % BASED ON GROSS SECTION.

LOAD CARRIED ON STEEL = P_T

DIAM. OF BARS / Nº OF BARS	6	8	10	12	14	16
½"	15,900	21,200	26,500	31,800	37,100	42,300
⅝"	24,900	33,100	41,400	49,600	58,000	66,300
¾"	35,800	47,700	59,600	71,600	83,500	95,400
⅞"	48,700	65,000	81,100	97,500	114,000	130,000
1"	63,600	85,000	106,000	127,000	148,500	170,000
1⅛"	80,500	107,500	134,000	161,000	188,000	215,000
1¼"	99,500	132,500	166,000	199,000	232,000	265,000
1½"	143,000	191,000	238,000	286,000	334,000	382,000

TABULATED VALUES OF P_T AND k_2 ARE FOR STRESS OF C_s = 13,500 LBS/IN².

FOR STRESS OF 15,000 LBS/IN². MULTIPLY VALUES BY 1·11.

$P_T = C_s A_c$ (LBS)

LOAD CARRIED DUE TO HELICAL BENDING = P_B — values of k_2

SPACING OF HELICALS	DIAMETER OF HELICALS. 3/16"	¼"	5/16"	⅜"	½"	⅝"	¾"	⅞"	1"	MIN. D_c
1"	2,340	4,140	6,510	-	-	-	-	-	-	6"
1½"	1,560	2,760	4,350	6,250	11,080	-	-	-	-	9"
2"	1,170	2,070	3,265	4,700	8,300	13,000	-	-	-	12"
2½"	936	1,660	2,610	3750	6,640	10,400	15,000	-	-	15"
3"	780	1,380	2,170	3,120	5,540	8,670	12,500	17,000	22,200	18"

$P_B = k_2 D_c$

$k_2 = 0{\cdot}523 A_B C_S$

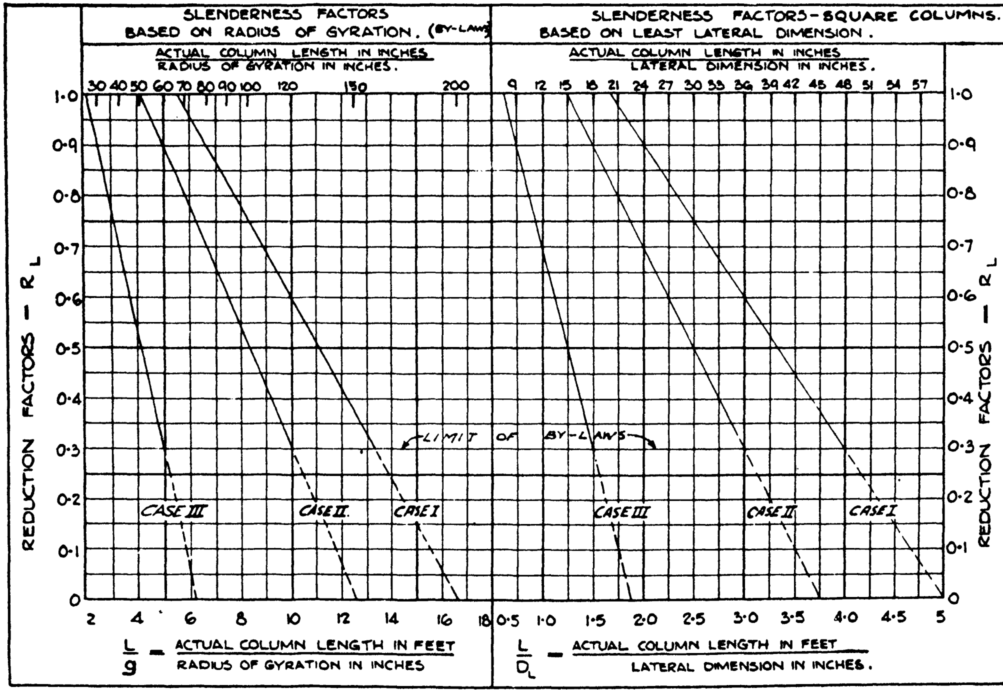
MINIMUM D_c = 6 × SPACING.

TOTAL LOAD = $P = P_C + P_T + P_B$.

This is Case II on *Table* 33. For conditions of end restraint other than "hinged" the actual length of the column must be modified to determine the equivalent length upon which R_L is based. An interpretation of the requirements of By-law 105 in this respect is embodied in *Table* 33, which includes curves for typical extreme cases of end restraint. In particular cases interpolation may be made according to the conditions existing.

TABLE 33.

LONG COLUMNS: REDUCTION FACTORS FOR SLENDERNESS AND DEGREE OF END RESTRAINT.



CONDITIONS OF END RESTRAINT.		
COLUMNS OF ONE STORY	FIXED AT BOTH ENDS	CASE I
	HINGED AT BOTH ENDS	CASE II
	FIXED AT ONE END IMPERFECTLY RESTRAINED IN DIRECTION AND POSITION AT OTHER END.	INTERMEDIATE BETWEEN CASES II & III DEPENDING ON EFFICIENCY OF IMPERFECT RESTRAINT.
COLUMNS CONTINUING THROUGH TWO OR MORE STORIES	FIXED AT BOTH ENDS	CASE I
	BOTH ENDS RESTRAINED IN POSITION. ONE OR BOTH ENDS IMPERFECTLY RESTRAINED IN DIRECTION.	INTERMEDIATE BETWEEN CASES I & II DEPENDING ON EFFICIENCY OF DIRECTIONAL RESTRAINT.
	FIXED AT ONE END IMPERFECTLY RESTRAINED IN DIRECTION AND POSITION AT OTHER END.	INTERMEDIATE BETWEEN CASES II & III DEPENDING ON EFFICIENCY OF IMPERFECT RESTRAINT.

"FIXED" — ADEQUATELY RESTRAINED IN POSITION AND DIRECTION.

"HINGED" — ADEQUATELY RESTRAINED IN POSITION BUT NOT IN DIRECTION.

According to By-law 102, the actual length of a column should be taken as the distance between adjacent floor levels. In accordance with the Memorandum the least radius of gyration is based on the gross section for columns with lateral ties and on the core section for columns with helical binding when the latter is taken into account in determining the load-carrying capacity. No stipulations are made regarding the calculation of the radius of gyration, but from consideration of the recommendations concerning the calculation of the moment of inertia of sections it appears that allowance can be made for the reinforcement using the appropriate modular ratio (= 15, see By-law 99).

For column sections that are symmetrical about two axes mutually at right angles the slenderness can be more conveniently based on the least lateral dimension, provided the section has no re-entrant angles. Thus a square section could be considered on this basis, the approximate reduction coefficients being derived from the expression

$$R_L = 1{\cdot}5 - \frac{0{\cdot}4L_E}{D_L}$$

where D_L = the lateral dimension (in.) of the column.

Curves are given on *Table* 33 for values of R_L calculated according to this expression, but it will be realised that values of R_L based on the latter take no account of variations in the amount of longitudinal reinforcement.

The recommendations in the Code conform to the foregoing remarks on slender columns, except that the limiting value of length to radius of gyration of 120 is omitted and the alternative bases of radius of gyration or least lateral dimension are recognised. Thus *Table* 33 can be used for Code designs, the lower parts of the curves shown by broken lines being applicable to cases where the ratio of length to radius of gyration exceeds 120.

The columns detailed in *Figs.* 9 and 10 are " short columns," since the most slender section is between the fourth and fifth floors where a 12-in. square column has an actual length of 12 ft. with end conditions intermediate between Cases I and II.

As an example of a " long column," consider a 12-in. square section reinforced with four 1-in. bars, the actual length being 16 ft. with end conditions as prescribed for Case II (*Table* 33). The slenderness of this column will be considered from three points of view.

(1) Radius of gyration based on 12-in. square concrete section only.

$$g = 0{\cdot}288 \times 12 = 3{\cdot}46 \text{ in.}$$

$$R_L = 1{\cdot}5 - \frac{0{\cdot}12 \times 16}{3{\cdot}46} = 0{\cdot}94.$$

(2) Radius of gyration based on combined section of concrete and reinforcement.

Area of section :	Concrete = 12^2	= 144	sq. in.	
	Reinforcement = $14 \times 3{\cdot}14$ =	44	,,	(four 1-in. bars)
		188	,,	

Moment of Inertia:

$$\text{Concrete} = \frac{12^4}{12} = 1728 \text{ in.}^4$$

$$\text{Reinforcement} = 44 \times 4^2 = 704 \text{ ,,}$$

$$2432 \text{ ,,}$$

$$g = \sqrt{\frac{2432}{188}} = 3{\cdot}58.$$

$$R_L = 1{\cdot}5 - \frac{0{\cdot}12 \times 16}{3{\cdot}58} = 0{\cdot}965.$$

This shows a small advantage over the previous calculation, but the calculation is more involved.

(3) Slenderness based on lateral dimension:

$$R_L = 1{\cdot}5 - \frac{0{\cdot}4 \times 16}{12} = 0{\cdot}967.$$

The result is substantially the same as that obtained by (2) but with much less calculation.

Design of Interior Columns.

The details on *Fig.* 9 show a design, with alternatives, for the interior columns A and B, in accordance with the By-laws. The calculations for the column A are given on Calculation Sheets Nos. 8 and 9; those for column B, with slightly different loadings, would be similar. As these internal columns support a symmetrical arrangement of beams, they are designed for axial load only. (See Chapter VI for other conditions.)

The superimposed loads on the roof and upper floors (Offices, Class No. 2, *Table* 1) are taken from *Table* 5, as these will be the same as for beams in the same class, the alternative minimum superimposed load not having to be applied for columns. The live load on the ground floor has been taken as 200 lb. per square foot (Class No. 5), which is approximately the load (including impact) obtained by considering motor coaches closely parked over the whole area of the floor. The superimposed load reductions for the first to fourth floors inclusive have been made in accordance with *Table* 7, but no reductions are made for the ground floor since the live load specified in By-law 4 exceeds 100 lb. per square foot (paragraph "d" of By-law 4). Dead loads for all floors, including for partitions where necessary, have been approximately computed, as precise calculations have not yet been prepared.

In each of the floor load calculations a factor to allow for the elastic reactions from the beams has been introduced; this factor is determined by inspection of the data on *Tables* 19 and 20. The adoption of elastic reactions from beams on columns usually leads to an increase in load on internal columns compared with the load derived from the "static" reaction, but an advantage is gained when designing the external columns since the bending moments will be combined with direct loads less than those given by "static" calculations.

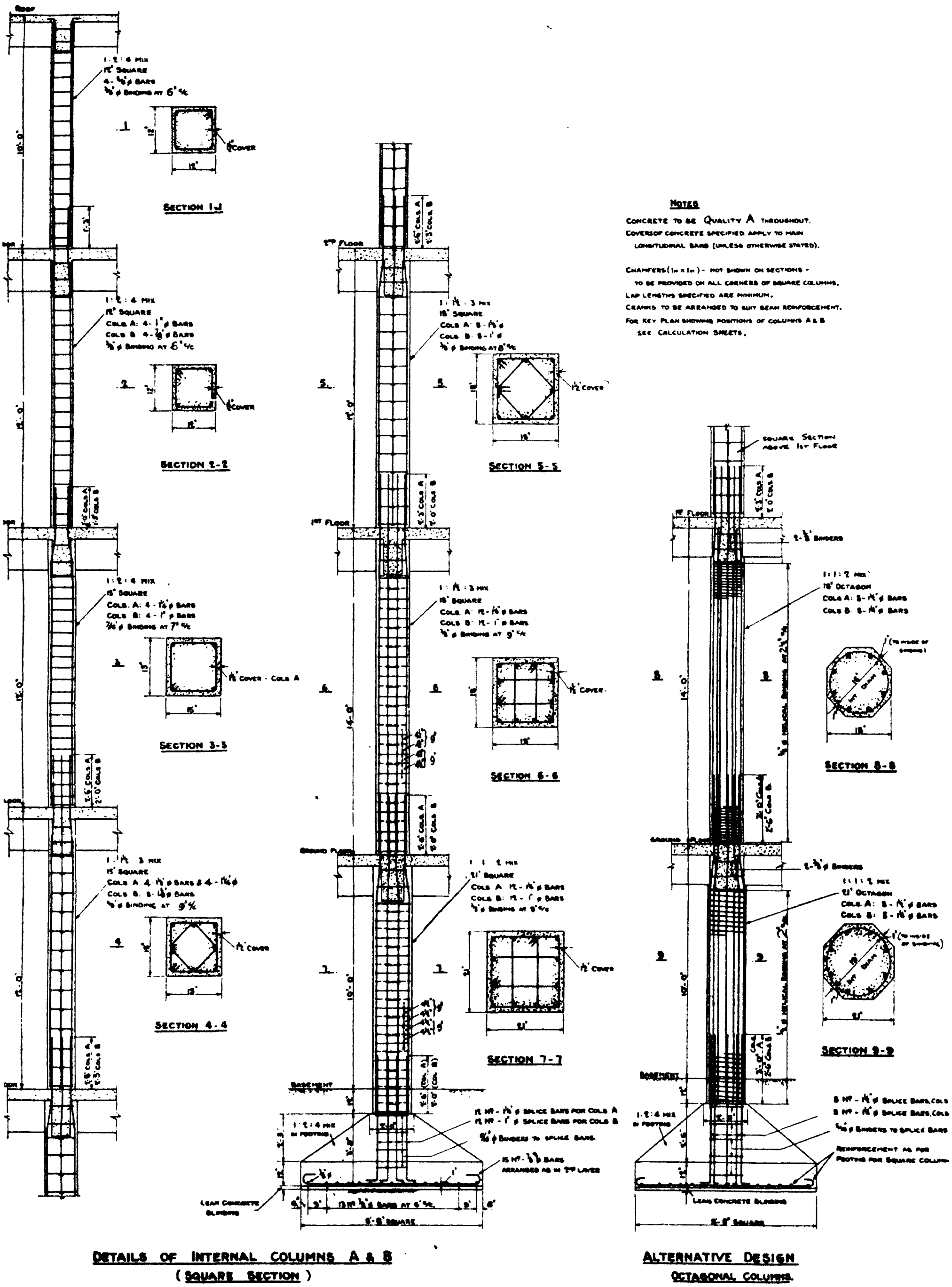
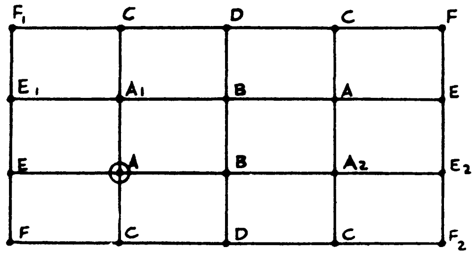


FIG. 9.—DETAILS OF INTERNAL COLUMNS.
(By-laws.)

CALCULATION SHEET No. 8.
INTERNAL COLUMNS.
(By-laws.)

COLUMN KEY PLAN



QUALITY A CONCRETE THROUGHOUT.

INTERNAL COLUMNS A.		lbs.		
5TH FLOOR TO ROOF.	Roof Dead Load = 16 x 24 x 90 lbs x 1·12 Roof Live Load = 16 x 24 x 30 lbs. x 1·39 Col. = 8·5 x 144 lbs Total	 = 38,700 = 16,000 = 1,225 55,925	$K = \frac{55{,}925}{12^2}$ = 388 $p < 0\cdot8\%$ $A_T = \frac{0\cdot8}{100} \times 12^2$ = 1·15 ins²	MIX III A. (1 : 2 : 4) 12" SQUARE 4 - 5/8" ϕ 3/8" ϕ Binding at 6" c/c
4TH FLOOR TO 5TH FLOOR	5th Floor Dead Load = 16 x 24 x 130 lbs. x 1·12 5th. Floor Live Load = 16 x 24 x 50 lbs. x 1·39 Col. = 10 x 144 lbs. Total	 = 56,000 = 26,700 1,440 140.065	$K = \frac{140{,}065}{12^2}$ = 974 $p = 2\cdot0\%$ $A_T = \frac{2\cdot0}{100} \times 12^2$ = 2·88 ins²	MIX III A (1 : 2 : 4) 12" SQUARE 4 - 1" ϕ 3/8" ϕ at 6" c/c
3RD. FLOOR TO 4TH FLOOR	4th Floor Dead Load 4th Floor Live Load = 90% x 26,700 lbs. Col. = 10 x 225 lbs. Total	= 56,000 = 24,000 = 2,250 222,315	$K = \frac{222{,}315}{15^2}$ = 994 $p = 2\cdot2\%$ $A_T = \frac{2\cdot2}{100} \times 15^2$ = 4·95 ins²	MIX III A (1 : 2 : 4) 15" SQUARE 4 - 1¼" ϕ 7/16" ϕ at 7" c/c
2ND FLOOR TO 3RD FLOOR	3RD Floor Dead Load 3rd Floor Live Load = 80% x 26,700 Col. = 10 x 225 lbs. Total	56,000 = 21,400 = 2,250 301,965	$K = \frac{301{,}965}{15^2}$ = 1335 $p = 3\cdot7\%$ $A_T = \frac{3\cdot7}{100} \times 15^2$ = 8·3 ins²	MIX II A (1 : 1½ : 3) 15" SQUARE 4 - 1¼" ϕ & 4 - 1⅛" ϕ 3/8" ϕ at 9" c/c

Calculation Sheet No. 9.

INTERNAL COLUMNS.

(By-laws.)

INTERNAL COLUMNS A. (cont.) 1st. FLOOR TO 2ND. FLOOR	lb. Load from 3rd Floor and above = 301,965 2nd. Floor Dead Load = 56,000 2nd. Floor Live Load = 70% × 26,700 = 18,700 Col. = 10 × 324 lbs = 3,240 Total = 379,905	$K = \frac{379,905}{18^2}$ $= 1,171$ $p = 2 \cdot 3\%$ $A_T = \frac{2 \cdot 3}{100} \times 18^2$ $= 7 \cdot 46$ ins²	MIX II A (1 : 1½ : 3) 18" SQUARE 8 - 1⅛" φ ⅜" φ at 8" c/c
GRD. FLOOR TO 1st. FLOOR	1st. Floor Dead Load = 56,000 1st. Floor Live Load = 60% × 26,700 = 16,000 Col. = 12 × 324 lbs = 3,890 Total 455,795	$K = \frac{455,795}{18^2}$ $= 1410$ $p = 4 \cdot 3\%$ $A_T = \frac{4 \cdot 3}{100} \times 18^2$ $= 13 \cdot 9$ ins²	(1 : 1½ : 3) MIX II A 18" SQUARE 12 - 1¼" φ ⅜" φ at 9" c/c
BASEMENT TO GRD. FLOOR	Grd. Floor Dead Load = 16 × 24 × 145 lbs × 1·12 = 60,300 Grd. Floor Live Load = 16 × 24 × 200 lbs. × 1·39 = 106,700 Col. = 9 × 441 lbs 3,970 625,765	$K = \frac{625,765}{21^2}$ $= 1420$ $p = 3 \cdot 0\%$ $A_T = \frac{3 \cdot 0}{100} \times 21^2$ $= 13 \cdot 1$ ins²	(1 : 1 : 2) MIX I A 21" SQUARE 12 - 1¼" φ ⅜" φ at 9" c/c
INTERNAL COLUMNS A. (cont.) GRD. FLOOR TO 1st FLOOR	ALTERNATIVE DESIGN OCTAGONAL COLUMNS Total Load (as above) = 455,795 lbs. $D_c = 16"$ $p_c = \frac{14 \cdot 14 \times 100}{\cdot 7854 \times 16^2} = 7 \cdot 04\%$ $K = 1990$ $P_c + P_T = 0 \cdot 7854 \times 16^2 \times 1990$ $= 415,000$ lb. $P_B = 3750 \times 16 = 60,000$ " Total Safe Load = 475,000 "		MIX I A (1 : 1 : 2) 18" OCTAGON 8 - 1½" φ (14·14 ins²) ⅜" φ Helical Binding at 2½" c/c
BASEMENT TO GRD. FLOOR	Total Load (as above) = 625,765 lbs $D_c = 19"$ $p_c = \frac{14 \cdot 14 \times 100}{\cdot 7854 \times 19^2} = 5 \cdot 0\%$ $K = 1700$ $P_c + P_T = 0 \cdot 7854 \times 19^2 \times 1700$ $= 480,000$ lb. $P_B = 8,300 \times 19 = 157,500$ " Total Safe Load = 637,500 "		(1 : 1 : 2) MIX I A 21" OCTAGON 8 - 1½" φ ½" φ Helical Binding at 2" c/c

The column sizes are controlled in some instances by considerations other than the loading. Between the fifth floor and the roof the columns must be large enough to accommodate the roof beams although the section adopted is considerably understressed. One size of column should be retained for as many lifts as possible (at least two) to ensure re-use of formwork. Additional load carrying capacity for a given size can be obtained by increasing the steel area, although it is usually more economical to increase the cement content. It is advantageous to retain the same concrete mix and column size for all interior columns in one story ; variations can be made in the longitudinal steel to provide for fluctuations of load from column to column (compare the details of columns A and B, *Fig.* 9).

Between ground and first floor levels the columns should be kept as small as possible. Thus a fairly rich mix and high steel percentage have been adopted to enable an 18-in. square column to be used. For comparison, an 18-in. octagonal column has also been designed for the same lift ; this would be more expensive as the small saving in concrete volume would be more than offset by the additional cost of the formwork, helical binding, and richer mix. The same applies to the 21-in. octagonal basement columns which are given as an alternative to the 21-in. square columns.

The lateral ties in the square columns are selected by using *Table* 29. The arrangement of the ties in each section depends on the number of bars in order to comply with the By-law 105 that every longitudinal bar must be held against buckling. This arrangement might be modified where the bars are restrained by the beams and slabs. Thus, above the soffits of the beams, ties are only provided to hold the corner bars. The number of such ties is kept to a minimum as their presence may cause inconvenience in placing the beam steel, and By-law 105 should be satisfied, if the spacing does not exceed twelve times the diameter of the longitudinal bar.

A design for columns A and B in accordance with the Code is illustrated in *Fig.* 10 and the accompanying calculations on Sheets Nos. 10 and 11. The loading calculations are identical with those for the By-law design, but the column sections differ due to the two methods of assessing the value of the longitudinal reinforcement. The fact that the By-laws require 1½-in. minimum cover of concrete over the longitudinal bars compared with 1 in. recommended by the Code also affects the design. Thus if in the two designs the concrete sizes are identical (as in *Figs.* 9 and 10), the design in accordance with the Code will be more lightly reinforced than the design in accordance with the By-laws except when rich concrete mixes are used. With the rich mix used in the column of octagonal section, the helical binding is less heavy in the By-law design, since the stress in the longitudinal bars based on a modular ratio of 15 exceeds the maximum stress of 13,500 lb. per square inch specified in the Code. The longitudinal bars will therefore take a larger share of the load in the By-law design, leaving less to be provided for by helical binding.

With regard to the bond length of column bars, By-law 104 requires the provision of a length of twenty-four diameters or sufficient to develop the force in the bar, whichever is the less. If the longitudinal bars are stressed to the maximum, the bond length required to develop the fc : in the bar is twenty-five diameters, which occurs when Quality A, 1 : 1 : 2 (Mix IA) concrete is used,

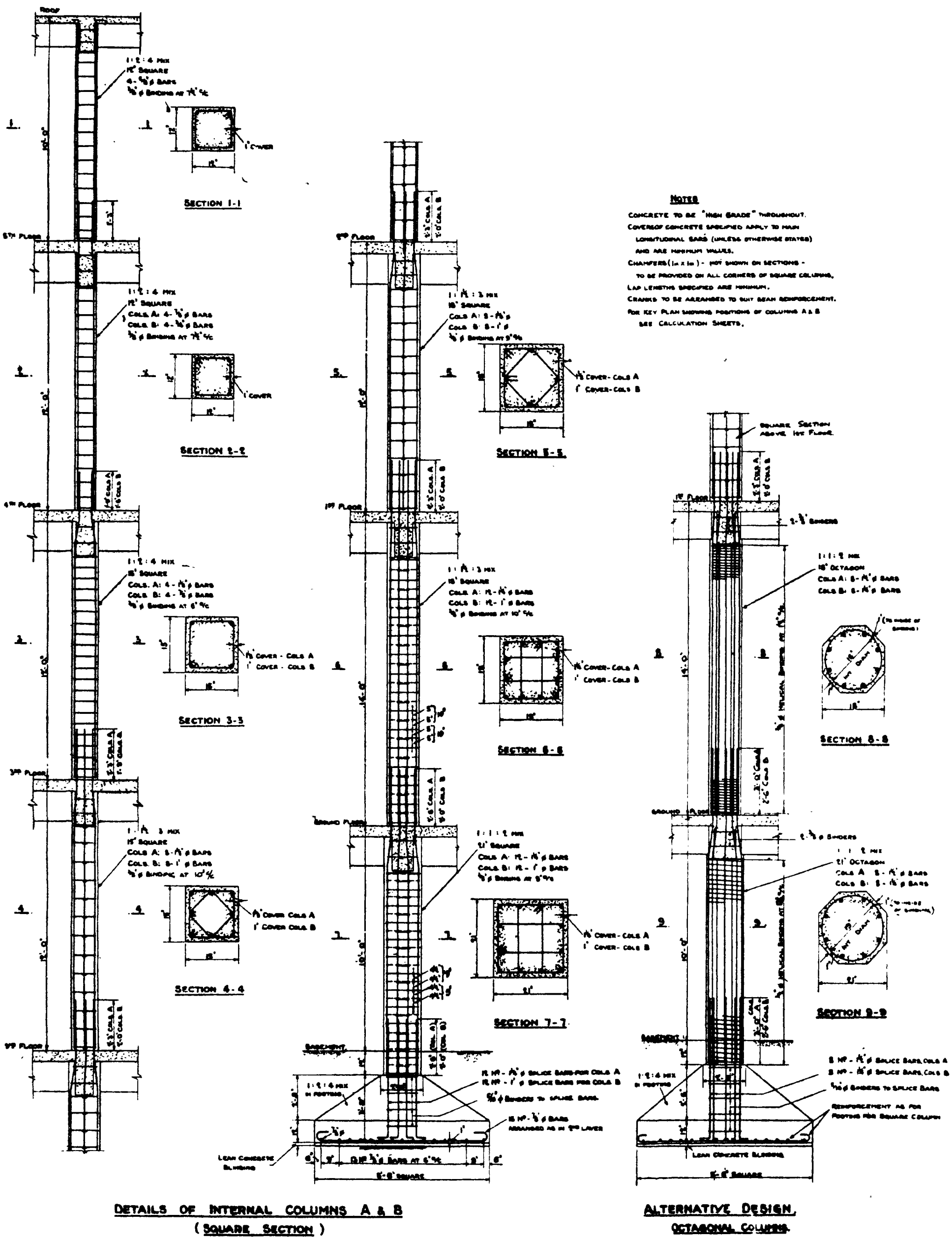
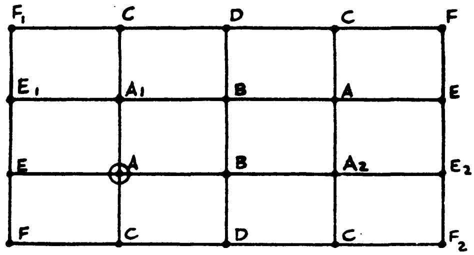


FIG. 10.—DETAILS OF INTERNAL COLUMNS.
(Code.)

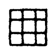
CALCULATION SHEET No. 10.
INTERNAL COLUMNS.
(Code.)

COLUMN KEY PLAN			HIGH GRADE CONCRETE
INTERNAL COLUMNS A.	lbs.		
5TH. FLOOR TO ROOF.	Roof Dead Load $= 16 \times 24 \times 90\text{lbs} \times 1{\cdot}12 = 38{,}700$ Roof Live Load $= 16 \times 24 \times 30\text{lbs.} \times 1{\cdot}39 = 16{,}000$ Col. $= 8{\cdot}5 \times 144$ lbs $= 1{,}225$ Total 55.925	$K = \frac{55{,}925}{12^2}$ $= 388$ $p < 0{\cdot}8\%$ $A_T = \frac{0{\cdot}8}{100} \times 12^2$ $= 1{\cdot}15\ \text{ins}^2$	1 : 2 : 4 MIX. 12" SQUARE 4 - 5/8" ϕ 3/8" ϕ Binding at 7½" %c
4TH FLOOR TO 5TH FLOOR	5th. Floor Dead Load $= 16 \times 24 \times 130\text{lbs.} \times 1{\cdot}12 = 56{,}000$ 5th. Floor Live Load $= 16 \times 24 \times 50\text{lbs.} \times 1{\cdot}39 = 26{,}700$ Col. $= 10 \times 144$lbs. $1{,}440$ Total 140,065	$K = \frac{140{,}065}{12^2}$ $= 974$ $p = 1{\cdot}65\%$ $A_T = \frac{1{\cdot}65}{100} \times 12^2$ $= 2{\cdot}38\ \text{ins}^2$	1 : 2 : 4 MIX. 12" SQUARE 4 - 7/8" ϕ 3/8" ϕ at 7½" %c
3RD. FLOOR TO 4TH. FLOOR	4th. Floor Dead Load $= 56{,}000$ 4th Floor Live Load $= 90\% \times 26{,}700\text{lbs.} = 24{,}000$ Col. $= 10 \times 225$lbs. $= 2{,}250$ Total 222, 315	$K = \frac{222{,}315}{15^2}$ $= 994$ $p = 1{\cdot}8\%$ $A_T = \frac{1{\cdot}8}{100} \times 15^2$ $= 4{\cdot}05\ \text{ins}^2$	1 : 2 : 4 MIX. 15" SQUARE 4 - 1⅛" ϕ 3/8" ϕ at 6" %c
2ND FLOOR TO 3RD FLOOR	3RD. Floor Dead Load 56,000 3rd Floor Live Load $= 80\% \times 26{,}700 = 21{,}400$ Col. $= 10 \times 225$lbs. $= 2{,}250$ Total 301, 965	$K = \frac{301{,}965}{15^2}$ $= 1335$ $p = 3{\cdot}6\%$ $A_T = \frac{3{\cdot}6}{100} \times 15^2$ $= 8{\cdot}10\ \text{ins}^2$	1 : 1½ : 3 MIX 15" SQUARE 8 - 1⅛" ϕ 3/8" ϕ at 10" %c

CALCULATION SHEET No. 11.

INTERNAL COLUMNS.

(Code.)

INTERNAL COLUMNS A. (cont.) 1ST. FLOOR TO 2ND. FLOOR	Load from 3rd Floor and above = 301,965 lb. 2nd. Floor Dead Load = 56,000 2nd Floor Live Load = 70% x 26,700 = 18,700 Col. = 10 x 324 lbs = 3,240 Total = 379,905	$K = \frac{379,905}{18^2}$ = 1,171 p = 2·3% $A_T = \frac{2\cdot3}{100} \times 18^2$ = 7·46 ins²	1 : 1½ : 3 MIX. 18" SQUARE 8 - 1⅛" ϕ ⅜" ϕ at 9" c/c 
GRD. FLOOR TO 1ST. FLOOR	1st. Floor Dead Load = 56,000 1st. Floor Live Load = 60% x 26,700 = 16,000 Col. = 12 x 324 lbs = 3,890 Total 455,795	$K = \frac{455,795}{18^2}$ = 1410 p = 4·2% $A_T = \frac{4\cdot2}{100} \times 18^2$ = 13·6 ins²	1 : 1½ : 3 MIX. 18" SQUARE 12 - 1¼" ϕ ⅜" ϕ at 10" c/c 
BASEMENT TO GRD. FLOOR	Grd. Floor Dead Load = 16 x 24 x 145 lbs x 1·12 = 60,300 Grd. Floor Live Load = 16 x 24 x 200 lbs. x 1·39 = 106,700 Col. = 9 x 441 lbs 3,970 625,765	$K = \frac{625,765}{21^2}$ = 1420 p = 3·3% $A_T = \frac{3\cdot3}{100} \times 21^2$ = 14·6 ins²	1 : 1 : 2 MIX. 21" SQUARE 12 - 1¼" ϕ ⅜" ϕ at 9" c/c 

INTERNAL COLUMNS A (cont.) GRD. FLOOR TO 1ST FLOOR	ALTERNATIVE DESIGN OCTAGONAL COLUMNS Total Load (as above) = 455,795 lbs. $D_c = 16"$ $p_c = \frac{14\cdot14 \times 100}{\cdot7854 \times 16^2} = 7\cdot04\%$ $k_1 = 0\cdot93$ P_C = 0·93 x 201,000 = 187,000 lb P_T for 8 - 1½" ϕ = 191,000 " P_B = 6250 x 16 = 100,000 " Total Safe Load = 478,000 "	1 : 1 : 2 MIX. 18" OCTAGON 8 - 1½" ϕ ⅜" ϕ Helical Binding at 1½" c/c
BASEMENT TO GRD. FLOOR	Total Load (as above) = 625,765 lbs $D_c = 19"$ $p_c = \frac{14\cdot14 \times 100}{\cdot7854 \times 19^2} = 5\cdot0\%$ $k_1 = 0\cdot95$ P_C = 0·95 x 284,000 = 270,000 lb P_T for 8 - 1½" ϕ = 191,000 " P_B = 8,300 x 19 x $\frac{2"}{1\frac{3}{4}"}$ = 180,000 " Total Safe Load = 641,000 "	1 : 1 : 2 MIX 21" OCTAGON 8 - 1½" ϕ ½" ϕ Helical Binding at 1¾" c/c

for which $c = 1{,}000$ lb. per square inch, modular ratio $m = 15$, and bond stress $= 150$ lb. per square inch. The bond length is not affected when column stresses are increased due to wind loads, as the bond stress can also be increased. The minimum bond length is $22\frac{1}{2}$ diameters and occurs with ordinary quality 1 : 2 : 4 (Mix III) concrete, for which $c = 600$ lb. per square inch, modular ratio $m = 15$, and bond stress is 100 lb. per square inch. For other than the stronger concretes, a shorter bond length than twenty-four diameters could be used, but as the saving is only one or two diameters a practical rule would be to adopt twenty-four diameters so long as the reinforcement is approximately fully stressed.

The By-law stipulations regarding the lapping of column bars conform to the Code, but when columns are being designed in accordance with the Code, the maximum bond length required to develop the force in the bar is $37\frac{1}{2}$ diameters, which occurs when $c_s = 15{,}000$ lb. per square inch and Ordinary-grade 1 : 2 : 4 concrete is used. The minimum bond length is $22\frac{1}{2}$ diameters, corresponding to $c_s = 13{,}500$ lb. per square inch and the maximum bond stress of 150 lb. per square inch. A length of 24 diameters corresponds to a bond stress of 141 lb. per square inch with 13,500 lb. per square inch ; thus when concretes weaker than $1 : 1\frac{1}{5} : 2\frac{2}{5}$, High-grade, are used with ordinary mild steel bars the alternative of 24 diameters need only be provided. For stronger concretes a shorter length could be used. When high yield point mild steel is used the length of 24 diameters represents a bond stress of 156 lb. per square inch ; thus in all cases where this steel is provided at maximum stress a bond length of 24 diameters should be acceptable.

When a length of 24 diameters cannot be provided, for example, at the heads of top-story columns, a reduced stress should be adopted.

In the present design the reinforcement is fully stressed in all cases except the top lifts. Therefore lap lengths of twenty-four diameters have been specified throughout. When the bars in consecutive lifts are of different diameters to comply with By-law 104 the lap length should be calculated on the diameter of the bar in the upper of the two lifts. Between the fourth and fifth floors, column A (*Fig.* 9), the bars are 1 in., and above the fifth floor $\frac{5}{8}$ in. Since $24 \times \frac{5}{8} = 15$ in. is sufficient to transfer the load from one $\frac{5}{8}$-in. bar to a bar of like diameter, this length is all that is required to transfer the same load from a $\frac{5}{8}$-in. bar to one of larger diameter. It is necessary to ensure that the larger diameter bars have sufficient bond length above the highest section of maximum load, which is usually the level of the beam soffits. For this reason it is not possible for any of the eight $1\frac{1}{2}$-in. bars ($= 14{\cdot}14$ sq. in.) in section 8–8 to stop immediately below the first floor level, although less than 8 sq. in. of steel is required above this level. At 24 diameters the $1\frac{1}{2}$-in. bars require 3 ft. lap, and the distance from floor level to beam soffit is only 2 ft. The bars in excess of 8 sq. in. could therefore be stopped off at 12 in. above first floor level, but for practical reasons all bars in each lift are made the same length.

Column Footings.

The areas of the bases for the interior columns are determined by the load and the nature of the ground. A footnote to By-law 30 indicates safe ground pressures, but it is pointed out that these are given as a guide only. Doubtful

cases ought to be subject to investigation to determine the safe load. According to the Code the maximum pressures can, however, be exceeded by an amount "equal to the weight of the material in which the foundation is bedded and which is displaced by the foundation itself, measured downward from the final finished lowest adjoining floor or ground level." This is equivalent to ignoring the weight of the earth refilled over the top of the foundation and including part only of the weight of the concrete footing, or other foundation, in the loading calculation.

In the case of columns A (*Figs.* 9 and 10) the total load on the ground, including part of the weight of the base, would be about 630,000 lb., and in accord-

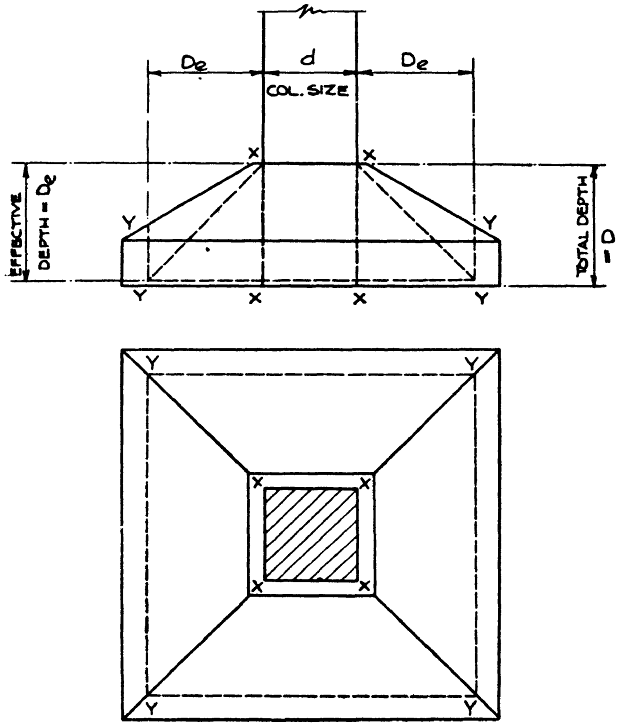


FIG. 11.—SHEAR ON COLUMN FOOTINGS.

ance with the information given on the general plan of the building (*Fig.* 1) the bases are to be founded on compact gravel. Taking a maximum pressure of 4 tons per square foot the area of the base is

$$\frac{630{,}000}{4 \times 2240} = 70 \text{ sq. ft., say 8 ft. 6 in. square.}$$

The thickness of the base is determined principally from the shear.

By-law 99 stipulates that the punching shear stress in a reinforced concrete footing shall not exceed twice the permissible shear stress for plain concrete (see *Table* 8). This stress would be calculated on the planes $X—X$ (*Fig.* 11). If the method specified in the Memorandum of considering the shear around the

heads of columns supporting flat slabs is adapted to column bases, the shear on planes $Y-Y$ without shear reinforcement should not exceed the shear stress on plain concrete (see *Table* 8 or *Table* 21).

The column base detailed on *Fig.* 9 for the 21-in. square column will be investigated for both these conditions. The concrete mix being Mix IIIA, 1 : 2 : 4 Quality A, the maximum shear stress is 95 lb. per square inch and the permissible punching shear stress is $2 \times 95 = 190$ lb. per square inch.

(*a*) Shear on X–X :

$$\text{Total shear on planes } X\text{–}X = 625{,}765 \left(1 - \frac{1\cdot75^2}{8\cdot50^2}\right)$$
$$= 600{,}000 \text{ lb.}$$

Total depth of section = 39 in.

$$\text{Stress} = \frac{600{,}000}{4 \times 21 \times 39} = 183 \text{ lb. per square inch.}$$

Allowable stress = 190 lb. per square inch.

(*b*) Shear on Y–Y :

$$d + 2D_e = 21 + (2 \times 36) = 93 \text{ in.} = 7 \text{ ft. } 9 \text{ in.}$$

$$\text{Total shear on planes } Y\text{–}Y = 625{,}765 \left(1 - \frac{7\cdot75^2}{8\cdot50^2}\right)$$
$$= 106{,}500 \text{ lb.}$$

Effective depth of section at Y–Y = 13 in. approximately.

$$\text{Stress} = \frac{106{,}500}{4 \times 93 \times 13} = 22 \text{ lb. per square inch,}$$

which is within the allowable stress of 95 lb. per square inch.

The reinforcement and resistance to bending are determined by one of the usual methods applicable to pyramidal footings. An example of the calculations for resistance to bending is given in Chapter XI. The base must also be thick enough to develop the bond in the longitudinal bars in the column. In the case of the square column (*Fig.* 9), the $1\frac{1}{4}$-in. bars at twenty-four diameters require a length of 30 in.; the available length is about 36 in. The $1\frac{1}{2}$-in. bars in the octagonal column require 36-in. bond length, the available length being about 39 in.

This consideration of column footings will also apply to Code designs, except that the Code does not give any recommendation concerning punching shear.

CHAPTER VI

COLUMNS SUBJECT TO BENDING MOMENTS

MANY engineers allow for bending moments when designing external columns of building frames, and most Continental regulations require provision to be made for these moments. Hitherto, however, regulations in this country have not enforced this method of design, the adoption of which is now required by both the Memorandum and the Code. The Memorandum requires bending actions applied to external columns and columns that are loaded in a similar manner to be taken into account. It may not be necessary to calculate the bending moments on internal columns in buildings of large areas when the columns support symmetrical arrangements of beams and loadings, although even in these cases the designer should assure himself, as described later, that the stresses caused by eccentricities due to variation in the incidence of the superimposed load do not exceed the permissible stresses.

Bending Moments on External Columns.

The Memorandum gives the following formulæ for the estimation of the bending moments on external columns:

Frames of one bay only—

B.M. at head of lower column, $\left(\dfrac{K_L}{K_L + K_U + \dfrac{K_B}{2}}\right)M_e$

B.M. at foot of upper column, $\left(\dfrac{K_U}{K_L + K_U + \dfrac{K_B}{2}}\right)M_e$

Frames of two or more bays—

B.M. at head of lower column, $\left(\dfrac{K_L}{K_L + K_U + K_B}\right)M_e$

B.M. at foot of upper column, $\left(\dfrac{K_U}{K_L + K_U + K_B}\right)M_e$

where K_U = stiffness of upper column
K_L = stiffness of lower column
K_B = stiffness of beam.

The factor M_e is defined in the Memorandum as " the bending moment at the end of the beam framing into the column assuming fixity at the connection." No reference is made to the degree of fixity at the end of the beam remote from

the column under consideration, and this may affect the value of M_e and consequently the value of the bending moment in the column.

The Code defines M_e in a similar manner to the Memorandum, but definitely states that both ends of the beam are assumed fixed.

Since the beam in any normal building frame will span from an external to an internal column it seems reasonable to suppose that the By-laws would be satisfied by a value of M_e based on the assumption that both ends of the beam are fixed. For this condition expressions for M_e are as follows:

$$\text{Any loading on beam: } M_e = \frac{2A}{L^2_B}(2L_B - 3Z)$$

$$\text{Symmetrical loading: } M_e = \frac{A}{L_B}$$

where A = area of the bending moment diagram for the beam assumed freely supported at both supports,
L_B = span of the beam,
Z = distance from column of centroid of free bending moment diagram.

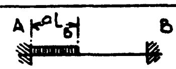
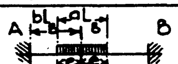
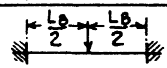
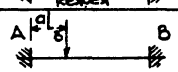
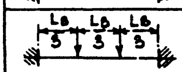
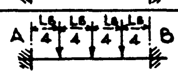
For any given loading the foregoing expressions can be reduced to the form

$$M_e = C_e W L_B$$

where W = the total load on the beam. Values of C_e for various common arrangements of loading are given on *Table* 34.

TABLE 34.

END MOMENTS FOR FIXED BEAMS.

	LOADING	C_e BOTH SUPPORTS	LOADING	C_e SUPPORT A	C_e SUPPORT B	NOTES.
VALUES OF M_e.		1·00		$a(3a^2-8a+6)$	$a^2(4-3a)$	$M_e = C_e W L_B$
		1·25		$12b(1-b)^2 - a^2(2-3b)$	$12b^2(1-b) - a^2(3b-1)$	W = TOTAL LOAD (LB)
		1·50		$12a(1-a)^2$	$12a^2(1-a)$	TABULATED VALUES OF C_e GIVE M_e IN INCH-LB. UNITS FOR L_B IN FEET.
		1·33		1·25	1·25	L_B = SPAN OF BEAM

The moments at the end of the beam equal the sum of the moments in the upper and lower columns. Thus

B.M. at end of beam—

$$\text{Frames of one bay only, } \left(\frac{K_L + K_U}{K_L + K_U + \frac{K_B}{2}}\right) M_e$$

$$\text{Frames of two or more bays, } \left(\frac{K_L + K_U}{K_L + K_U + K_B}\right) M_e.$$

When there is no upper column, as when roof beams frame into the top lift of columns, $K_U = 0$ and the expressions for the moments become :—

B.M. at head of column and at end of beam—

Frames of one bay only, $\left(\dfrac{K_L}{K_L + \dfrac{K_B}{2}}\right) M_e$

Frames of two or more bays, $\left(\dfrac{K_L}{K_L + K_B}\right) M_e$.

These formulæ only give approximate values for the moments at the junctions of columns and beams, and the Memorandum acknowledging that more exact analyses are possible will accept these in place of the approximate formulæ. In certain conditions the alternative and more precise method may lead to more economical design, but the present calculations (Calculation Sheets Nos. 12 to 16 inclusive) adopt expressions based on those given in the Memorandum. For practical use it is more convenient to express the factors in these formulæ in

TABLE 35.

BENDING MOMENTS IN EXTERNAL COLUMNS.

DATA

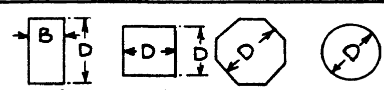
L_B = SPAN OF BEAM (FT.)
L_U = HEIGHT OF UPPER COL. (FT.)
L_L = HEIGHT OF LOWER COL. (FT.)
I_B = MOMENT OF INERTIA OF BEAM (IN^4)
I_U = DITTO UPPER COL. (IN^4)
I_L = DITTO LOWER COL. (IN^4)

M_e = B.M. AT END OF BEAM FRAMING INTO EXTERNAL COLUMN ASSUMING BOTH ENDS OF BEAM FIXED. (SEE BELOW).

$$C_U = \frac{k I_U L_B}{I_B L_U}$$

$$C_L = \frac{k I_L L_B}{I_B L_L}$$

M. OF I. OF CONCRETE SECTIONS ABOUT CENTROID IGNORING REINFORCEMENT. (SEE ALSO TABLE 31)



$\dfrac{BD^3}{12}$	$\dfrac{D^4}{12}$	$0{\cdot}055D^4$	$0{\cdot}049D^4$

END CONDITIONS: BEAM	END CONDITIONS: COLUMN	k
FIXED	FIXED	1·00
FIXED	HINGED	0·75
HINGED	FIXED	1·33
HINGED	HINGED	1·00

BENDING MOMENT FORMULÆ.

MEMBER		FRAMES OF ONE BAY.	FRAMES OF TWO OR MORE BAYS
TOP STORY	B.M. IN COLUMN AND BEAM.	$\dfrac{C_L}{C_L+0{\cdot}5} M_e$	$\dfrac{C_L}{C_L+1} M_e$
INTERMEDIATE STORIES.	B.M. AT HEAD OF LOWER COLUMN.	$\dfrac{C_L}{C_L+C_U+0{\cdot}5} M_e$	$\dfrac{C_L}{C_L+C_U+1} M_e$
	B.M. AT FOOT OF UPPER COLUMN.	$\dfrac{C_U}{C_L+C_U+0{\cdot}5} M_e$	$\dfrac{C_U}{C_L+C_U+1} M_e$
	B.M. AT END OF BEAM	$\dfrac{C_L+C_U}{C_L+C_U+0{\cdot}5} M_e$	$\dfrac{C_L+C_U}{C_L+C_U+1} M_e$

terms of the ratio of stiffness between the members concerned, and the ratios

$$C_U = \frac{\text{stiffness of upper column}}{\text{stiffness of beam}}$$

and

$$C_L = \frac{\text{stiffness of lower column}}{\text{stiffness of beam}}$$

introduced in the fundamental expressions result in the working formulæ given on *Table* 35.

Bending Moments on Internal Columns.

If the arrangement of beams or loading supported by an internal column is unsymmetrical it is necessary to consider the effect of the bending moment induced in the columns. The magnitude of this moment can be approximately estimated by considering the column as being an end column of two separate frames, one ignoring the beam on the left-hand side and the other ignoring that on the right-hand side. The difference between the moments calculated in accordance with the external column formulæ for these two cases can be taken as the bending moment on the column, if for one frame dead load only is taken.

The Memorandum implies that internal columns supporting symmetrical beam arrangements should be investigated for bending moments due to eccentricity of live loading. This can be approximately done by estimating the bending moment as for an external column, but introducing the live load only instead of the total load in the calculation for M_e. When studying the stresses due to this bending moment, it should be remembered that the direct load on the column is reduced by the amount of live load omitted from one of the beams at the floor level under consideration.

Stiffness of Columns and Beams.

The stiffness of a member of constant cross section is defined as the value obtained by dividing the moment of inertia by the length. The relative stiffness of a beam and a column is therefore expressed by

$$\frac{(\text{moment of inertia of beam}) \times (\text{height of column})}{(\text{moment of inertia of column}) \times (\text{span of beam})} \text{ or } \frac{I_B L_C}{I_C L_B}$$

This is only true when the end fixing conditions of both members are identical. If the end of the column remote from the junction with the beam is equivalent to "hinged," while the remote end of the beam is "fixed," the ratio of stiffness would be $\frac{I_B L_C}{I_C(\frac{3}{4}L_B)}$. If the end fixing conditions were reversed the ratio of stiffness would be given by $\frac{I_B(\frac{3}{4}L_C)}{I_C L_B}$.

The factor $\frac{3}{4}$ corresponds to the coefficient k in *Table* 35, where values are given for other combinations of end conditions. In normal building frames k would be unity, but it is tabulated here for use in any special cases that may arise in practice.

It is presumably acceptable that the lengths of columns and beams incorporated in the stiffness calculation are the height from floor to floor for the column and the effective span, as defined in the Memorandum, for the beam.

Three bases are indicated in the Memorandum for the calculation of the moment of inertia of a section:

(1) From the whole concrete section, neglecting the reinforcement;

(2) From the whole concrete section, allowing for the reinforcement on the basis of the modular ratio,

(3) From the compression area of the concrete only, allowing for the reinforcement on the basis of the modular ratio.

The Code recommends methods (1) and (2) for stiffness calculation, but does not mention method (3).

Method (1) is usually simpler and, being of sufficient accuracy (compared with the bending moment formulæ), has been adopted throughout the accompanying calculations. It is imperative that the method employed in determining the moment of inertia of all members in a single calculation should be identical.

Although the three methods of calculating the moment of inertia in the Memorandum give widely differing results, when expressed as ratios of moments of inertia the differences in practical cases are less marked, being usually less than 10 per cent.

When calculating the moment of inertia for a beam of **T** or **Γ** section, the Code recommends that the breadth of the flange should be taken in accordance with the maximum permissible widths given on *Table* 18. *Table* 36 (reproduced by permission from Messrs. Scott and Glanville's "Explanatory Handbook" *) gives data for calculating the moment of inertia of flanged beams when the reinforcement is ignored. On the same basis the moment of inertia of a rectangular section about its centroid is $I = 0{\cdot}083bD^3$, that is $C = 0{\cdot}083$ (outside the range of *Table* 36).

The inclusion of the flange in moment of inertia calculations for beams seems a rational method and is embodied in Calculation Sheets 12, 13, etc.

Since the moments in external columns depend on the stiffness ratios rather than on the actual stiffness, it is, as explained, more convenient to rewrite the formulæ in the Memorandum in terms of $C_U = \frac{K_U}{K_B}$ and $C_L = \frac{K_L}{K_B}$. It is then only essential that the units in which the moments of inertia are expressed shall be identical and that those for the beam spans and column lengths are consistent (but not necessarily the same as for the moment of inertia). As it is usually more convenient to consider the moment of inertia in inch^4 units, the spans and lengths in lineal feet, and the bending moment in in.-lb. units, the formulæ and data on *Tables* 34 and 35 are so expressed that the terms can be applied in these units to determine the bending moment directly in in.-lb. units.

Tables 34 to 36 are employed for the appropriate calculations on Sheets Nos. 12 to 16. Since in the present example the ends of the columns and beams remote from any joint have approximately the same degree of fixity (that is, continuous, but not necessarily absolutely fixed), the value of k (*Table* 35) in the stiffness-ratio calculation is unity throughout.

* "Explanatory Handbook on the Code of Practice for Reinforced Concrete." By W. L. Scott and W. H. Glanville.

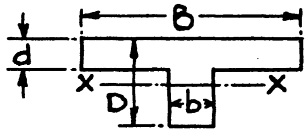
TABLE 36.

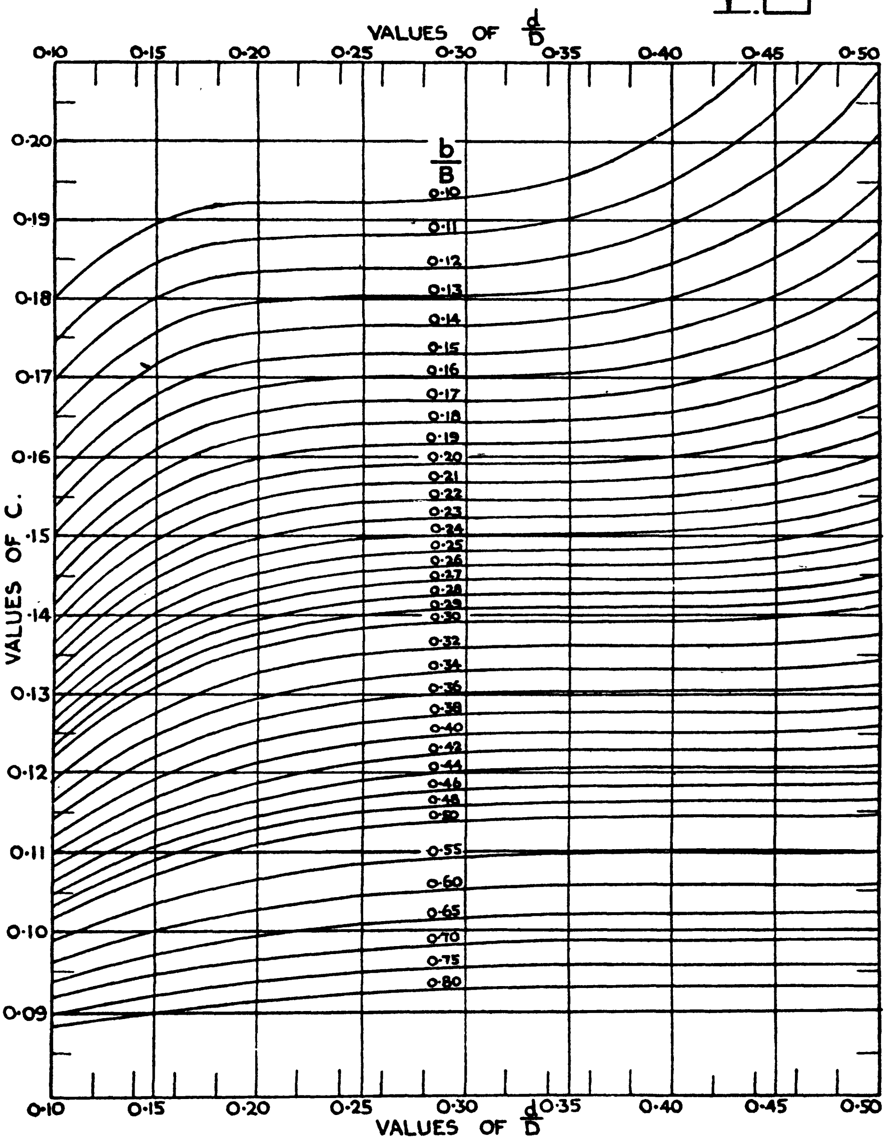
Moment of Inertia of Tee Sections.

(Code.)

M. OF I. ABOUT X.X. = CbD^3

X.X = AXIS THRO' CENTROID.





Combining Moments and Direct Loads.

The calculation of stresses due to non-axial loads is tedious unless the designer possesses suitable charts. At the same time the variables in the problem are so numerous that a comprehensive set of charts would be too bulky for easy reference. Here calculation and reference to charts, data, and formulæ are combined in a convenient form for practice.

The problem of combining a moment M with a direct thrust N falls into two classes, (I) when the stresses are wholly compressive, and (II) when tensile and compressive stresses are simultaneously induced. When considering a given section the equivalent eccentricity $e = \frac{M}{N}$ determines which case applies. For an unreinforced rectangular section the limiting value of $e_1 = \frac{e}{D}$ is $\frac{1}{6}$ (D being the total depth). For values of e_1 in excess of this, tensile stresses are produced. For reinforced sections the limiting value of e_1 exceeds $\frac{1}{6}$ and with high percentages of reinforcement e_1 may approach $\frac{1}{2}$ before tension is produced. With values of e_1 less than, say, 0·25 it is suggested that the stress should be determined in accordance with Case (I); if it is shown that appreciable tensile stresses are produced, the stresses should be determined again in accordance with Case (II). With the curves given on *Tables* 37, 38 and 39, border-line cases are immediately identified.

General formulæ for the two cases can be expressed as follows.

CASE (I).—STRESSES WHOLLY COMPRESSIVE.—The maximum and minimum compressive stresses are determined by applying the formula

$$c_m = \frac{N}{A} \pm \frac{My}{I}$$

where A = the gross effective cross-sectional area (sq. in.),
I = the moment of inertia (in.^{4}) about the centroid, and
y = distance to the extreme fibres from the centroid (in.).

For the symmetrically reinforced rectangular section shown in *Fig.* 12 (a usual case for external columns of buildings)

$$A = BD + (m - 1)(A_C + A_T)$$

and

$$\frac{I}{y} = Z = \frac{\frac{BD^3}{12} + (m - 1)(A_C + A_T)\left(\frac{D}{2} - f_1D\right)^2}{\frac{D}{2}}$$

If the steel area $(A_C + A_T)$ is replaced by an expression involving the percentage area, p, the formulæ for A and Z can be reduced to

$$A = BD[1 + 0{\cdot}01p(m - 1)]$$
$$Z = BD^2[\tfrac{1}{6} + 0{\cdot}02p(m - 1)(0{\cdot}5 - f_1)^2].$$

If the expressions for A and Z are substituted in the concrete stress formula and if $E = 0{\cdot}01p(m - 1)$ we obtain by reduction

$$c_m = \frac{N}{BD}\left(\frac{1}{1 + E} \pm \frac{6e_1}{1 + 3Eh_0^2}\right)$$

which is the concrete stress expression in a practicable form.

With $m = 15$, in accordance with the By-laws, $E = 0{\cdot}14p$ and

$$c_m = \frac{N}{BD}\left(\frac{1}{1 + 0{\cdot}14p} \pm \frac{6e_1}{1 + 0{\cdot}42ph_0^2}\right).$$

Note that p applies only to the reinforcement adjacent to the extreme edges of the section. Any bars provided along the vertical faces in *Fig.* 12 are neglected although they will decrease the compressive stress by increasing A. If these bars are on the centre line of the section they will not affect the value of Z. For example, there are six $1\frac{1}{8}$-in. and two $\frac{3}{4}$-in. bars in column D between ground and first-floor levels; only the $1\frac{1}{8}$-in. bars are allowed for in calculating the combined stress. Bars extending from the beams and turning down into the column cannot be taken into account as column reinforcement if they only extend sufficiently far to develop by adhesion the tension required for the moment at the end of the beam.

CASE (II).—TENSILE AND COMPRESSIVE STRESSES.—When e is such that both tensile and compressive stresses are induced, a method of working can be deduced from the fundamental principles (*a*) that the algebraic sum of the forces on the

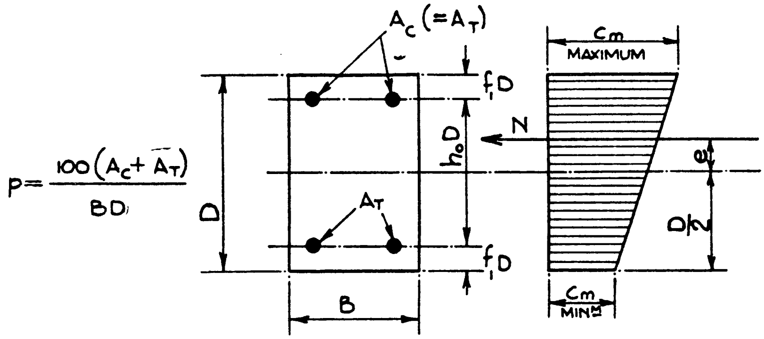


FIG. 12.—COMBINED MOMENT AND THRUST. COMPRESSIVE STRESS ONLY.

section is zero, and (*b*) that the algebraic sum of the moments on the section is zero. Referring to the symmetrically reinforced rectangular section in *Fig.* 13, the equations of equilibrium become

$$N = C - T = C_C + C_S - T$$

$$N\left(e + \frac{D}{2} - f_1D\right) = C_C\left(d - \frac{n_1d}{3}\right) + C_S(D - 2f_1D).$$

By expressing the internal forces in terms of the dimensions of the section and the maximum concrete stress c, these can be reduced to

$$\frac{N}{BD} = c\left[\frac{n_1}{2}(1 - f_1) + \frac{p\ (m - 1)(n_1 - f_1n_1 - f_1)}{200n_1(1 - f_1)} - \frac{pm}{200n_1}(1 - n_1)\right]$$

$$\frac{N}{BD}(e_1 + 0{\cdot}5 - f_1) = c\left[\frac{n_1}{2}(1 - f_1)^2\left(1 - \frac{n_1}{3}\right) + \frac{p\ (m-1)(n_1 - f_1n_1 - f_1)(1 - 2f_1)}{200n_1(1 - f_1)}\right]$$

In any given problem, the factors N, B, D, e_1, f_1, m, and p are known and by substitution two expressions involving c and n_1 are evolved. These, however, are too cumbersome to be practicable for everyday design, but the use of charts simplifies the calculation. If, however, available charts do not cover the problem in hand, the following comprehensive analytical method gives an accurate solution. With the notation given in *Fig.* 13, this method can be applied to rectangular

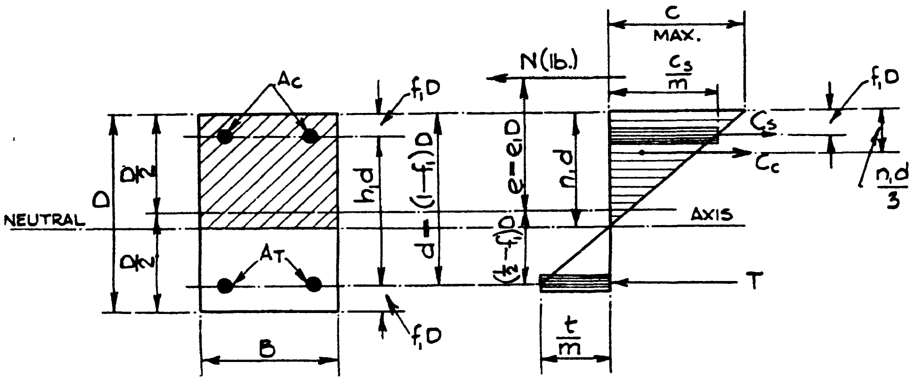


FIG. 13.—COMBINED MOMENT AND THRUST. COMPRESSIVE AND TENSILE STRESSES

sections with any cover ratio (f_1), unequal proportions of compressive and tensile reinforcement, any modular ratio, and any concrete mix. The calculations accompanying this article do not call for the adoption of this method, but the procedure is as follows.

The first step is to take a trial depth for the neutral axis by assuming a value for the factor n_1. The maximum concrete stress can then be calculated from the formula

$$c_m = \frac{NF}{GBd + Kh_1}$$

in which

$$F = 1 + \frac{D}{d}(e_1 - 0{\cdot}5)$$

$$G = \frac{n_1}{2}\left(1 - \frac{n_1}{3}\right)$$

$$K = \frac{A_C}{n_1}(m - 1)(n_1 + h_1 - 1)$$

The ratio of the stresses is given by

$$r = \frac{J + K - \dfrac{N}{c_m}}{A_T}$$

where

$$J = \frac{Bdn_1}{2}.$$

The value of n_1 based on this value of r and found from the formula

$$n_1 = \frac{1}{\frac{r}{m} + 1}$$

(or from *Table* 10) should correspond to the assumed value. At the first trial there may be a discrepancy, in which case a second trial value of n_1 will usually give satisfactory results. When the calculated and assumed values of n_1 tolerably correspond (not differing by more than 5 or 10 per cent.), c_m can be considered as the actual maximum concrete stress. The tensile stress in the steel can be determined directly from

$$t = rc.$$

Problems in design of sections are best solved by selecting what appears to be a suitable section and investigating the stresses in accordance with the preceding methods; adjustments then usually suggest themselves whereby a more satisfactory section can be obtained. It is rarely economical to design column sections subject to bending with the tension steel stressed to the maximum permissible stress (for example, 18,000 lb. per square inch); thus direct methods of design based on the permissible stresses are not of great value in this instance.

The labour entailed in obtaining accurate stresses by the preceding analytical method depends upon the accuracy of the first trial position of the neutral axis. From a preliminary consideration of the section and forces it is often possible to select a suitable value in the first instance. In other cases of a given section, the maximum stresses for which the section has been designed may indicate a reasonable value, and for large values of e_1 consideration can be given to the neutral-axis factor for pure bending corresponding to the percentage of tensile reinforcement. The trial value of n_1 should be greater than the value of n_1 for bending alone; the former exceeds the latter more and more as e_1 decreases. When the discrepancy between the assumed and calculated values of n_1 is such that re-calculation is necessary, the second trial value should be intermediate between the first trial value and the calculated value and should be nearer to the latter than the former. If the problem is merely to assure that a given maximum stress is not exceeded and if the stress derived from the first trial value is within this limit, re-calculation is not necessary if the calculated value of n_1 exceeds the first trial value.

For large values of $\frac{e}{d}$ the method of calculating the stresses given in Chapter XI can be used with practical accuracy.

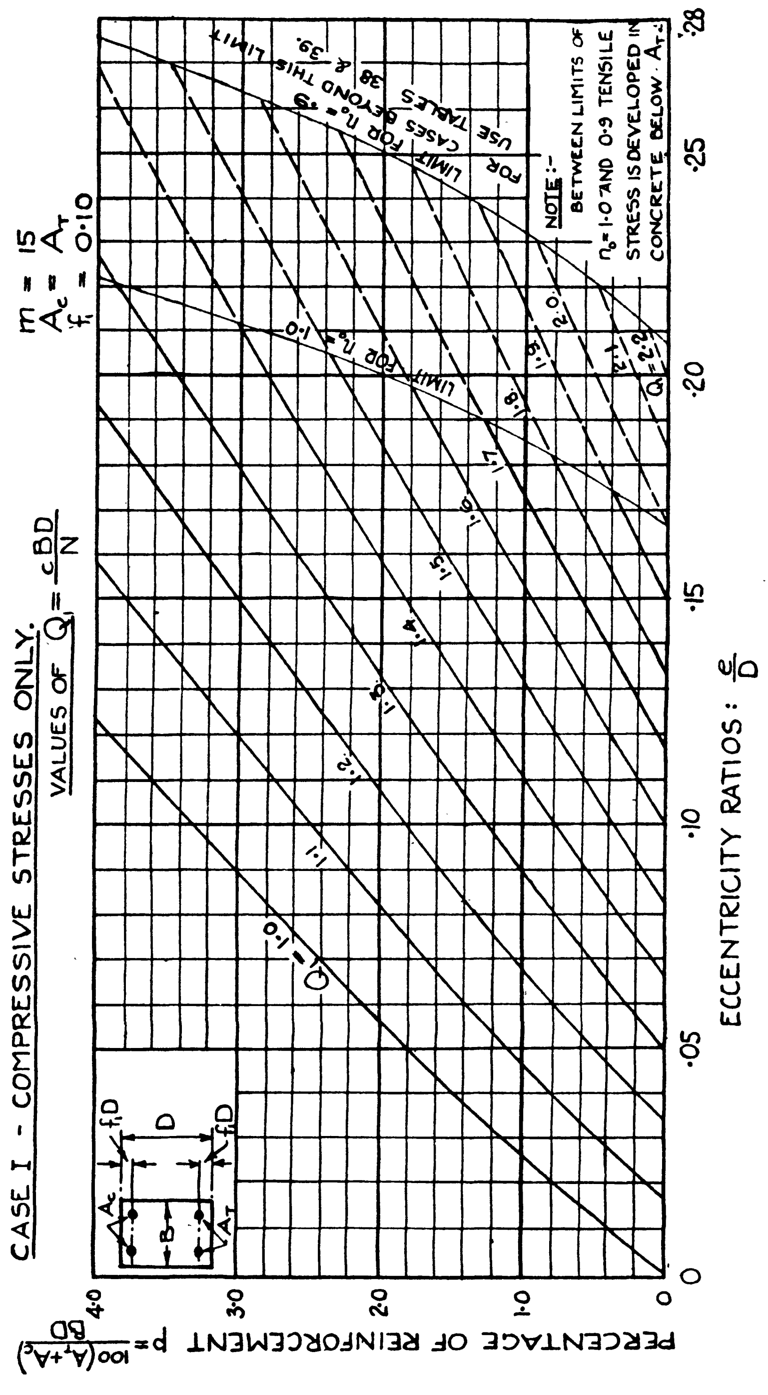
Charts for Combined Stresses.

The curves given on *Tables* 37, 38, and 39 are based on charts prepared by Mr. Leslie Turner, B.Sc., M.Inst.C.E., M.I.Struct.E., for all cases of bending and compression with a modular ratio of 15, and are therefore applicable to design under the By-laws for any mix of concrete. The curves are limited to cases where $A_T = A_C$ and for values of $f_1 = 0{\cdot}1$. *Table* 37 covers Case I, where compression extends over the whole area of the section, and this table also covers border-line cases between Case I and Case II, that is, when the neutral axis falls between the centre of A_T and the tension edge of the section and n_0 is between

TABLE 37.

BENDING AND COMPRESSION.

(By-laws.)



0·9 and 1·0. Within this range it is assumed that the concrete in the worst case (that is with $n_0 = 0{\cdot}9$) resists a tensile stress equal to one-ninth of the maximum compressive stress.

Table 37 is used as follows. For the section under consideration determine $e = \frac{M}{N}$, $e_1 = \frac{e}{D}$, and $p = \frac{100(A_T + A_C)}{BD}$. Read off the value of Q_1 at the intersection of the appropriate ordinates of e_1 and p. Substitute this value of Q_1 in the expression

$$c_m = \frac{NQ_1}{BD}$$

which gives the maximum compressive stress in the concrete. The minimum stress, if required, would be given by

$$c_m(\text{min.}) = \frac{2N}{(1 + 0{\cdot}14p)BD} - c_m(\text{max.}).$$

This minimum stress in border-line cases may be tensile, and if it is imperative that tensile stress on the concrete should be neglected, the maximum compressive stress will be increased and its approximate value can be determined from

$$c_m = \frac{NQ_1}{0{\cdot}9BD}.$$

When Case (II) applies, that is, when the stresses are such that n_0 is not greater than 0·9, *Tables* 38 and 39 can be used. As before, the values of e_1 and p are determined and are used to read from *Table* 38 the value of n_0. With the latter and the appropriate value of p_1 the value of Q_2 is read from *Table* 39. This is substituted in the expression

$$c = \frac{M}{Q_2 BD^2}$$

to give the maximum compressive stress in the concrete. The tensile stress in the reinforcement is

$$t = 15c\left(\frac{0{\cdot}9}{n_0} - 1\right).$$

It should be noted that on the charts the neutral-axis depth is expressed in relation to the total depth of the section; that is, $n = n_0 D$. Elsewhere it is expressed in terms of the effective depth, that is $n = n_1 d = n_1 D(1 - f_1)$.

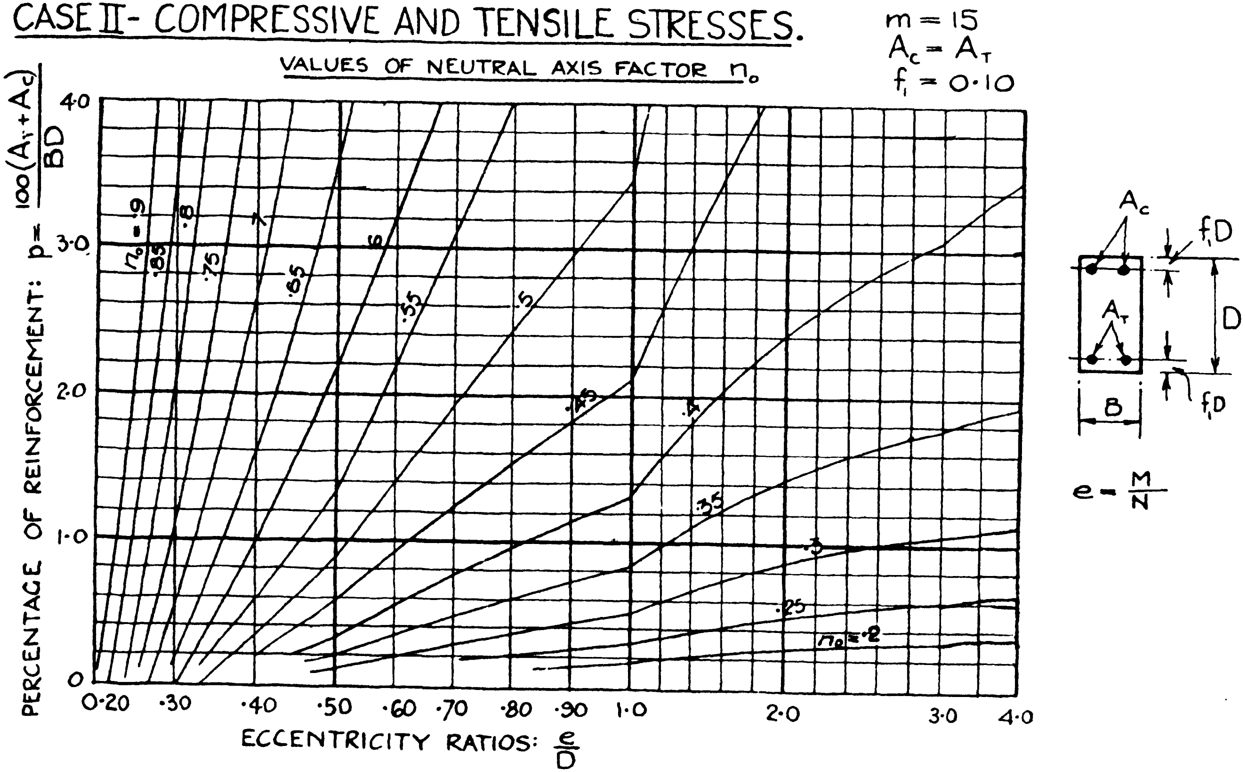
As pointed out, the charts on *Tables* 37, 38, and 39 are limited to cover ratios of one-tenth, that is $f_1 = 0{\cdot}1$, but approximate adjustments can be made for other ratios. If f_1 is less than 0·1, the value of the concrete stress calculated in accordance with the charts will be slightly higher than the actual stress; therefore the charts can be used to determine a value which will not be exceeded. If f_1 is greater than 0·1, the stress will be higher than that given by the charts, and a fair approximation can be made if for D, the total depth of the section, a value D_1 equal to $(1{\cdot}1 - f_1)D$ is substituted. Generally if f_1 exceeds 0·10 and Case (I) applies, it is more accurate and just as simple to use the formula for c_m instead of *Table* 37.

Tables 37, 38, and 39 were used in the calculations for the external columns given on Sheets Nos. 12 to 16 inclusive.

TABLE 38.

BENDING AND COMPRESSION.

(By-laws.)

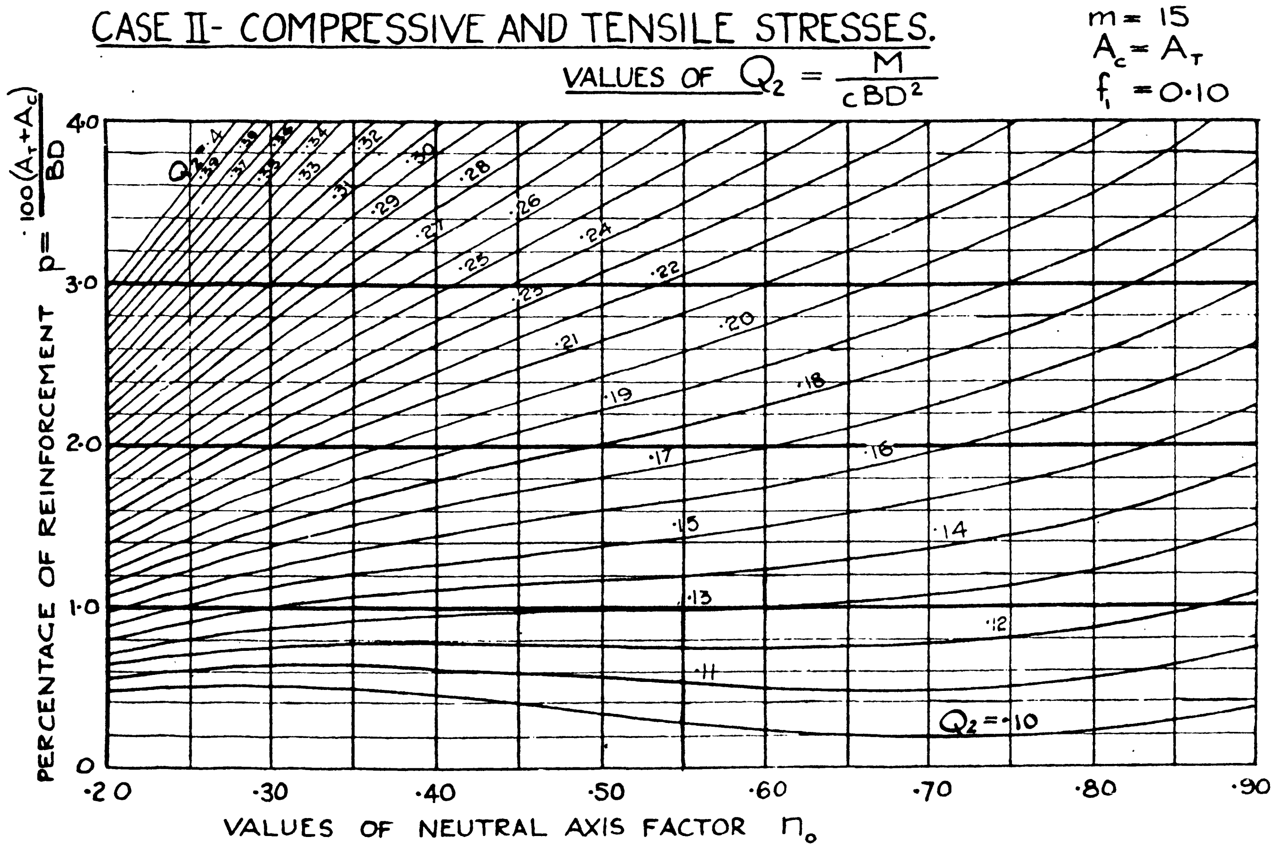


Note.—For cases where $n_o > 0{\cdot}9$ use Table 37.

TABLE 39.

BENDING AND COMPRESSION.

(By-laws.)



Owing to the difference in the modular ratio adopted in the By-laws and the Code, the charts given on *Tables* 37, 38, and 39 are not applicable to calculations in accordance with the Code. Alternative aids to design can be evolved as follows.

The general expressions already given can be written

$$\frac{N}{BD} = Q_1$$

$$\frac{N}{BD}(e_1 + 0{\cdot}5 - f_1) = Q_2$$

where $Q_1 = c\left[\frac{n_1}{2}(1 - f_1) + \frac{p\ (m-1)(n_1 - f_1n_1 - f_1)}{200n_1(1-f_1)} - \frac{pm}{200n_1}(1 - n_1)\right]$

and $Q_2 = c\left[\frac{n_1}{2}(1 - f_1)^2\left(1 - \frac{n_1}{3}\right) + \frac{p\ (m-1)(n_1 - f_1n_1 - f_1)(1 - 2f_1)}{200n_1(1-f_1)}\right]$

For any given grade and mix of concrete, c has a maximum value and m is a specified ratio (see *Table* 9). If a particular concrete is selected a series of curves can be prepared for various values of Q_1 in relation to p. Similarly another series can be prepared for Q_2. If these curves are superimposed, as on *Tables* 40, 41, and 42, working charts are available. Since the cover ratio (f_1) enters into the calculation, separate series of curves are required for various values of f_1. In the present instance curves are drawn for $f_1 = 0{\cdot}05$ (*Table* 40) $f_1 = 0{\cdot}10$ (*Table* 41) and $f_1 = 0{\cdot}15$ (*Table* 42) and are applicable only to 1 : 2 : 4 High-grade concrete. Similar curves may be drawn for other mixes and grades.

TABLE 40.

COMBINED BENDING AND COMPRESSION.

(Code.)

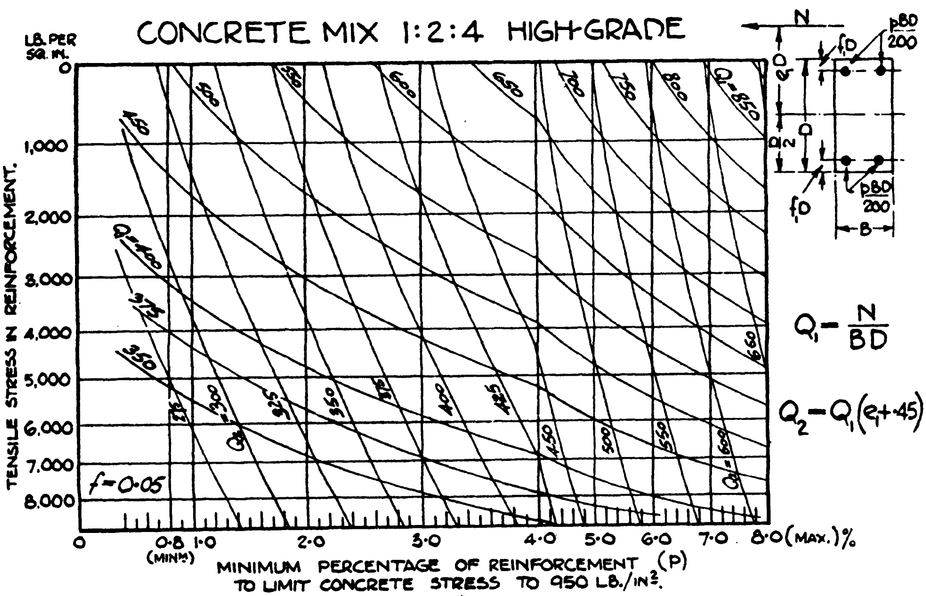


TABLE 41.

COMBINED BENDING AND COMPRESSION.

(Code.)

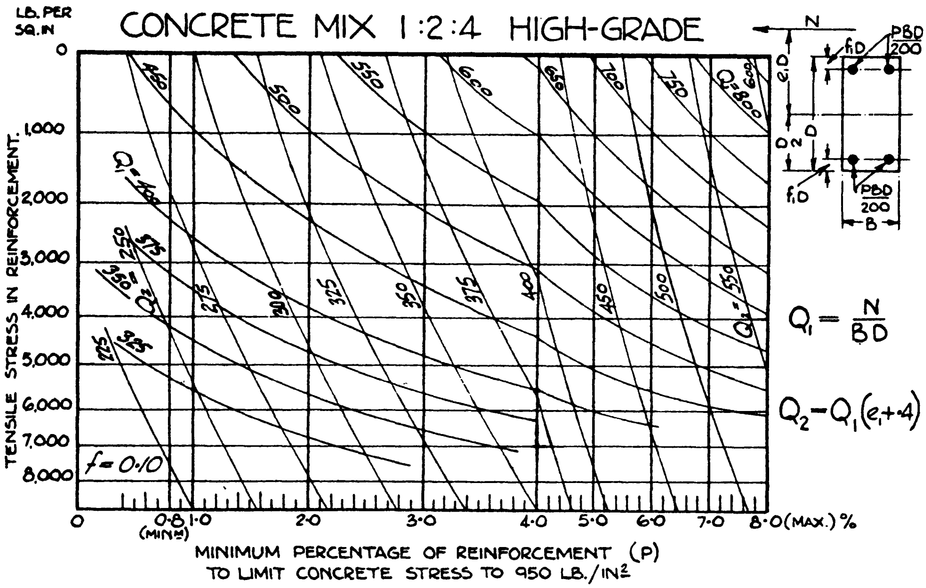
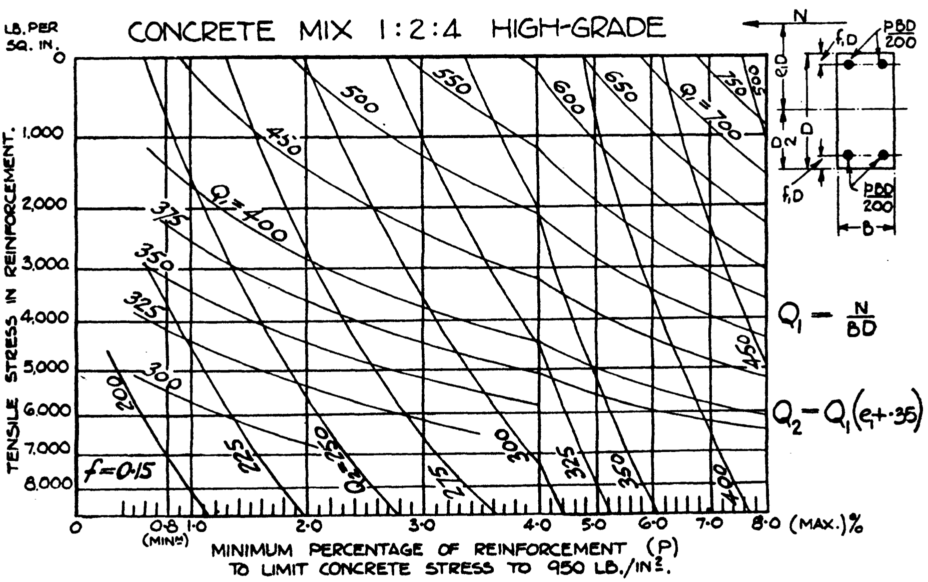


TABLE 42.

COMBINED BENDING AND COMPRESSION.

(Code.)



To apply these curves it is assumed that a column section has been selected. For this section determine the values of $Q_1 = \frac{N}{BD}$ and $Q_2 = Q_1(e_1 + 0{\cdot}5 - f_1)$. If a trial section based on the assumption that the load acts axially is derived by using *Table* 28, then $Q_1 = K$ and $Q_2 = K(e_1 + 1{\cdot}5 - f_1)$. The intersection of the corresponding Q_1 and Q_2 curves gives the minimum value of p to keep the maximum concrete stress within the permissible limit for the grade and mix of concrete adopted. If the value of p for the axial load alone is insufficient, it is necessary to increase the amount of steel, adjust the dimensions of the section, or increase the richness of the mix. The total amount of reinforcement should not exceed 8 per cent. of the total concrete section.

For values of f_1 between 0·05 and 0·15 it is sufficient to determine p for the two adjacent values of f_1 (that is, either $f_1 = 0{\cdot}05$ and 0·10, or $f_1 = 0{\cdot}10$ and 0·15) and adjust in proportion to the real value of f_1.

When the problem falls outside the range of the charts on *Tables* 40–42 the comprehensive analytical method explained previously can be adopted.

Design of External Columns.

The working stresses to be adopted in the design of columns subjected to moment and direct thrust should apparently be those specified in the By-laws for bending, due account being taken of the modular ratio in assessing the stress in the compression reinforcement. As will be seen from *Table* 8 the allowable direct compressive stress is 80 per cent. of the allowable compressive stress in bending, and a rational method of design for small ratios of moment to load is as follows.

(1) Select a column section suitable for the direct load alone assuming the latter to be axially applied; the design should be in accordance with the requirements for axially-loaded columns as regards maximum stresses on the concrete and reinforcement, limiting percentages, and arrangement of longitudinal steel and lateral ties and slenderness (see Chapter V).

(2) Determine the stresses on this section due to moment and direct force combined; these should not exceed the permissible bending stresses. If the latter are exceeded the section should be revised by adjusting either the dimensions, the steel percentage (but not in excess of 8 per cent.), or the concrete mix. Alteration to the dimensions will affect the stiffness and consequently the magnitude of the bending moment.

Where a 1 : 2 : 4 (Mix IIIA) Quality A concrete is employed with ordinary mild steel bars the maximum stresses in accordance with By-law 99 and *Table* 8 for the initial design (1) would be 760 lb. per square inch on the concrete, and for combined moment and thrust the stresses should not exceed 950 lb. per square inch compression on the concrete and 18,000 lb. per square inch tension on the steel. A short column is assumed in each case. For slender columns subject to combined stress, the maximum compressive stress must not exceed that for short columns multiplied by the coefficient R_L on *Table* 33.

If designing in accordance with the Code using 1 : 2 : 4 High-grade concrete with ordinary mild steel bars, the stresses would be identical with those previously given, but in the axial load calculation the maximum stress on the longitudinal reinforcement would be 13,500 lb. per square inch.

An external column with a helically-bound core (for example, an octagonal column) would be designed for axial load in accordance with the rules given in the last chapter, but the effect of the helical binding would be ignored when computing the combined stress. The latter should be calculated on the total area, not on the core area only.

For large ratios of moment to direct load the combined stress will usually control the design of the column section, in which case the procedure previously explained can only be followed if a stress considerably less than the maximum allowable for direct compression is adopted in the axial load calculation. Alternatively a section can be designed for the combined moment and direct thrust and checked to ensure agreement with the requirements for axially-loaded columns.

For the design of the external columns for the building under consideration in accordance with the By-laws, Calculation Sheets Nos. 12, 13, 14, 15 and 16 have been prepared and should be read in conjunction with the reinforcement details shown in *Fig.* 14.

Since calculations in accordance with the Code differ as regards modular ratio, cover of concrete, stress on longitudinal steel, etc., from the requirements of the By-laws and Memorandum, a corresponding set of Calculation Sheets Nos. 17, 18, 19, 20, and 21 has been prepared together with appropriate details of the external columns, *Fig.* 15.

The accompanying calculations contain one or other of the methods outlined —depending on the circumstances of each problem—and should to a large extent be self-explanatory. The lift of column D between ground and first floor (Calculation Sheet No. 16) is the best example of a column section designed for axial load alone and found to be satisfactory when checked for combined stress ; the moment : load ratio is very low in this case. The design of the sections for the lifts of column E between the second and fourth floors, given on Calculation Sheets Nos. 13 and 14, is controlled by the combined stress, with the result that the compression on the concrete for the load assumed to be acting axially is much less than the permissible maximum.

In all the stress calculations on Sheets Nos. 12 to 16 inclusive the value of f_1 exceeds 0·10, and adjustment is made in the calculations, when *Tables* 37, 38, and 39 are used, by replacing the actual depth D by the reduced total depth D_1. The accuracy of this adjustment is demonstrated by the alternative calculations given for column E between the third and fourth floors. In some instances where Case (I) is used, the conditions are such that the neutral axis falls between the centre of A_T and the tension edge of the section ; thus a tensile stress is developed in the concrete. As this is quite low, it has been ignored.

The stresses in any particular column lift should be investigated for the maximum moment which may occur either at the head or at the foot of the column. The moments at these two sections are determined from the separate calculations for the upper and lower junctions with the beams. It is usually more convenient in column design to proceed downwards from the top of the building ; thus the stresses due to the moment at the head of any lift of the column are determined before those at the foot of the same lift. When the moment at the foot has subsequently been determined it should be compared with the moment at the head, and if it is greater than the latter the stresses should be investigated again. An example of this occurs on Calculation Sheets

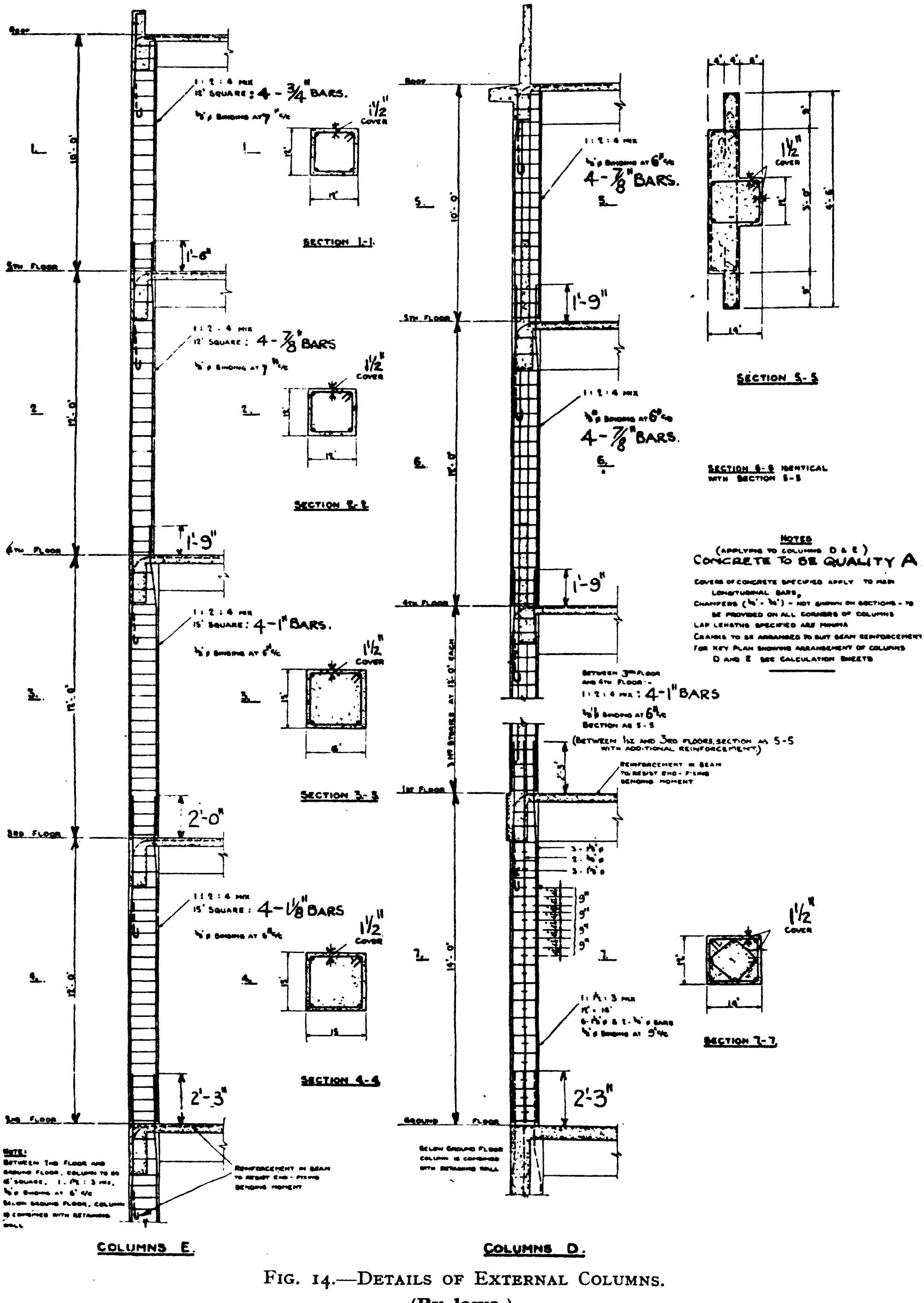
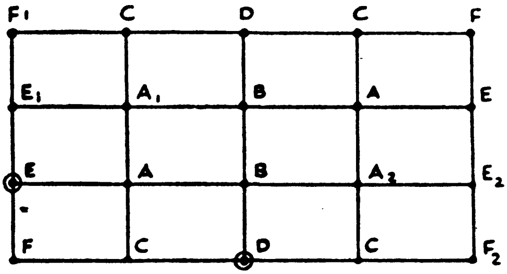


FIG. 14.—DETAILS OF EXTERNAL COLUMNS.

(By-laws.)

CALCULATION SHEET No. 12.
EXTERNAL COLUMNS.
(By-laws.)

COLUMN KEY PLAN



QUALITY A CONCRETE THROUGHOUT

EXTERNAL COLUMNS E.

5th Floor To Roof.

Loading:- lb.

Roof Dead Load $= 16 \times 24 \times 90 lb \times \cdot 42 = 14{,}500$

" Live " $= 16 \times 24 \times 30 lb \times \cdot 53 = 6{,}100$

Parapet: say $50 lb \times 16 = 800$

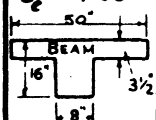
Column: $8 \cdot 5 \times 144 lb = 1{,}225$

Total. $22{,}625$

Trial Section: 12" sq. $4 - \frac{3}{4}$" ϕ — $p = 1 \cdot 22\%$

Axial Load only: $K = \frac{22{,}625}{12^2} = 158$ $(p < 0 \cdot 8\%)$

$L_B = 24'$
$L_L = 10'$
$C_e = 1 \cdot 00$



$k = 1 \cdot 00$

Bending Moment: (per Tables 34 & 35)

$W = 24 \times 900 lb$ (approx) $= 21{,}600 lb.$

$M_e = 1 \cdot 00 \times 21{,}600 \times 24 = 518{,}000$ in. lb.

I_B:- $\frac{d}{D} = \frac{3\frac{1}{2}}{16} = \cdot 219$; $\frac{b}{B}$ $\frac{8}{50} = \cdot 16$

$C = \cdot 169$ (per Table 36)

$I_B = \cdot 169 \times 8 \times 16^3 = 5530$ in^4

$I_L = \frac{12^4}{12} = 1728$ in.^{4}

$C_L = \frac{1728 \times 24}{5530 \times 10} = \cdot 75$

B.M. at Col. and Beam Junction

$= \left(\frac{\cdot 75}{\cdot 75 + 1}\right) 518{,}000 = 222{,}000$ in. lb.
(cf. 262,000 per Sheet No. 3) — 1 : 2 : 4 Mix

$f_1 = \frac{1 \cdot 81}{12} = \cdot 15$

$D_1 = (1 \cdot 1 - \cdot 15) 12 = 11 \cdot 4"$

$e = \frac{222{,}000}{22{,}625} = 9 \cdot 8"$ $e_1 = \frac{9 \cdot 8}{11 \cdot 4} = 0 \cdot 86$ — 12" SQUARE $4 - \frac{3}{4}$" ϕ

Apply Case III:- $p_1 = \frac{1 \cdot 76 \times 100}{11 \cdot 4 \times 12} = 1 \cdot 28\%$

From Tables 38 & 39:— $n_0 = \cdot 42$; $Q_2 = \cdot 15$

$c = \frac{222{,}000}{\cdot 15 \times 12 \times 11 \cdot 4^2} = 935 lb./in.^2$ Trial Section Correct

CALCULATION SHEET No. 13.
EXTERNAL COLUMNS.
(By-laws.)

<u>EXTERNAL COLUMNS E (cont.)</u>

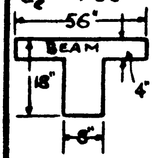
4TH FLOOR TO 5TH FLOOR

		lb.
<u>Loading</u>: From above 5th. Floor:		22,625
5th. Fl. Dead Load:	16 x 24 x 13 dlb x ·42 =	21,000
" " Live "	16 x 24 x 50 lb x ·53 =	10,180
Brick Wall ·	15 x 8·5 x 90 lb. =	11,500
Column :	10 x 144 =	1,440.
	Total =	66,745

Trial Section : As above 5th. Floor.

$K = \dfrac{66,745}{12^2} = 464$; $p < 0.8\%$ (Axial Load only) — $p = 1.67\%$

$L_B = 24'$ $\quad L_U = 10'$ $\quad L_L = 12'$ $\quad C_e = 1.00$



<u>Bending Moment</u>: $W = 8 \times 24 \times 170$ lb. (app) = 32,700 lb

$M_e = 1.00 \times 32{,}700 \times 24 = 784{,}000$ in lb.

I_B :- $\frac{d}{D} = \frac{4}{18} = \cdot 222$; $\frac{b}{B} = \frac{8}{56} = \cdot 143$; $C = \cdot 174$

$I_B = \cdot 174 \times 8 \times 18^3 = 8120 \text{ in}^4$

$I_L = I_U = 12^4/12 = 1728 \text{ in}^4$

$C_L = \dfrac{1728 \times 24}{8120 \times 12} = \cdot 426 \qquad C_U = \dfrac{1728 \times 24}{8120 \times 10} = \cdot 511$

BM (Upper Col.) $= \dfrac{\cdot 511}{1\cdot 937} \times 784{,}000 = 206{,}500$ in.lb ($< 222{,}000$)

$k = 1.00$

B.M. (Lower Col.) $= \dfrac{\cdot 426}{1\cdot 937} \times 784{,}000 = 172{,}000$ in lb.

$e = \dfrac{172{,}000}{66{,}745} = 2\cdot 58''$ $\qquad e_1 = \dfrac{2\cdot 58}{11\cdot 3} = 0\cdot 237$

1 : 2 : 4 MIX.
12" SQUARE
4 - 7/8"

Apply Case I :— $p_1 = \dfrac{2\cdot 41 \times 100}{12 \times 11\cdot 3} = 1\cdot 77\%$

$f_1 = \frac{1\cdot 94}{12} = 0\cdot 16$, $D_1 = 11\cdot 3''$

From Table 32 : $Q_1 = 1\cdot 78$

$c = \dfrac{66{,}745 \times 1\cdot 78}{12 \times 11\cdot 3} = 880 \text{ lb/in}^2$ Trial Section Correct

3RD FLOOR TO 4TH FLOOR

		lb
<u>Loading</u>:– From above 4th Floor:		66,745
4th Fl. Dead Load:	=	21,000
" " Live " : 90% of 10,180	=	9,150
Brick Wall: 15 x 10 x 90 lb.	=	13,500
Column : 10 x 225	=	2,250
	Total =	112,645

Trial Section : 15" x 15" – 4 – 1" ϕ

$K = \dfrac{112{,}645}{15^2} = 500$. (Correct for axial load) — $p = 1\cdot 40\%$

<u>Bending Moment</u>: I_B, L_B and M_e as for beam at 5th Fl.

$C_U = C_L$ for 4th to 5th Floors $= \cdot 426$

CALCULATION SHEET No. 14.
EXTERNAL COLUMNS.
(By-laws.)

<table>
<tr><td colspan="2">EXTERNAL COLUMNS E (cont.)</td><td></td></tr>
<tr><td>3RD TO
4TH. Floor
(CONT.)

$L_L = 12'$
$k = 1.00$

$f_1 = \frac{2.00}{15}$
$= .133$
$D_1 =$
$(1.1 - .133)15$
$= 14.5"$</td><td>$I_L = 15^4/12 = 4220\ in^4$ $C_L = \frac{4220 \times 24}{8120 \times 12} = 1.04$
B.M. (Upper Col.) = $\frac{.426}{2.47} \times 784{,}000 = 135{,}000$ in.lb.
($\not> 172{,}000$)
B.M. (Lower Col.) = $\frac{1.04}{2.47} \times 784{,}000 = 330{,}000$ in lb
$e = \frac{330{,}000}{112{,}645} = 2.93"$ $e_1 = \frac{2.93}{14.5} = .202$
$p_1 = \frac{3.14 \times 100}{14.5 \times 15} = 1.44\%$
Apply Case I:- From Table 32: $Q_1 = 1.70$
$c = \frac{112{,}645 \times 1.70}{15 \times 14.5} = 880\ lb/in^2$
Alternative Method by direct calculation:-
$p = 1.40$ $h_o = \frac{15"-3"-1"}{15"} = 0.735$
$e_1 = \frac{2.93}{15} = .196$
$c_m = \frac{112{,}645}{15 \times 15}\left[\frac{1}{1+(0.14 \times 1.4)} \pm \frac{6 \times .196}{1+(0.42 \times 1.40 \times .735^2)}\right] = +870\ lb/in^2$ & -27 "</td><td>1 : 2 : 4 MIX.
15" SQUARE
4 - 1"φ</td></tr>
<tr><td>2ND TO
3RD Floor

$f_1 = \frac{2.06}{15}$
$= .137$
$D_1 = (1.1 - .137)15$
$= 14.4"$</td><td>Loading: From above 3rd. Floor — 112,645 lb.
3rd Floor Dead Load = 21,000
" " Live " = 80% of 10,180 = 8,150
Brick Wall = 13,500
Column. = 2,250
Total: 157,545

Trial Section: 15"×15" - 4 - 1⅛"φ
$K = \frac{157{,}545}{15^2} = 700$ Sufficient for axial load
Bending Moment: $C_L = C_U = 1.04$ (= C_L for 3rd to 4th)
B.M. (Both Cols.) = $\frac{1.04}{3.08} \times 784{,}000 = 264{,}500$ in.lb
($< 333{,}000$)
$e = \frac{264{,}500}{157{,}545} = 1.68"$ $e_1 = \frac{1.68}{14.4} = .116$
Apply Case I:- $p_1 = \frac{3.98 \times 100}{15 \times 14.4} = 1.84\%$
$Q_2 = 1.27$
$c = \frac{157{,}545 \times 1.27}{15 \times 14.4}$
$= 920$ lb. per sq. in. Trial Section Correct</td><td>p = 1.77%

1 : 2 : 4 MIX.
15" SQUARE
4 - 1⅛"</td></tr>
<tr><td>1ST TO 2ND AND GRD. TO 1ST FLOORS.</td><td colspan="2">Retain 15" Sq. Column throughout, adjusting concrete mix and longitudinal steel as required by load and moment.</td></tr>
</table>

Calculation Sheet No. 15.

EXTERNAL COLUMNS.

(By-laws.)

EXTERNAL COLUMNS D.

5th Floor to Roof

	lb
Loading:- Parapet and Cornice, say	7,000
Roof Dead Load 16 x 24 x 90 lb. x ·41	14,200
" Live " 16 x 24 x 30 lb x ·55	6,340
Wall panel, Column, Column facing, Windows, wall finishes, etc.	9,000
Total	36,540

$L_B = 16'$

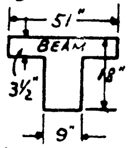
$L_L = 10'$

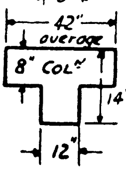
$C_e = 1{\cdot}50$

Trial Section (rib only) 14" x 12", - 4 - ⅞" φ

$K = \frac{36{,}540}{12 \times 14} = 217$ Sufficient for axial load

$p = \frac{2{\cdot}41 \times 100}{14 \times 12} = 1{\cdot}42\%$





Bending Moment · $W = 24 \times 8 \times 120 = 23{,}040$ lb

$M_e = 1{\cdot}5 \times 23{,}040 \times 16 = 552{,}000$ in lb

I_B :- $\frac{d}{D} = \frac{3\frac{1}{2}}{18} \doteq {\cdot}194$; $\frac{b}{B} = \frac{9}{51} = {\cdot}176$, ∴ $C = {\cdot}163$

$I_B = {\cdot}163 \times 9 \times 18^3 = 8560\ in^4$

I_L :- $\frac{d}{D} = \frac{8}{14} = {\cdot}571$; $\frac{b}{B} = \frac{12}{42} = {\cdot}286$; $C = {\cdot}15$ approx.

$I_L = {\cdot}150 \times 12 \times 14^3 = 4940\ in^4$

$C_L = \frac{4940 \times 16}{8560 \times 10} = {\cdot}922$

BM (Col. and beam) $= \frac{{\cdot}922}{1{\cdot}922} \times 552{,}000 = 265{,}000$ in lb.

(cf. 240,000 per Sheet No. 5)

$k = 1{\cdot}00$

$f_1 = \frac{1{\cdot}875}{14} = {\cdot}134$

$D_1 = (1{\cdot}1 - {\cdot}134)14 = 13{\cdot}5"$

$e = \frac{265{,}000}{36{,}450} = 7{\cdot}25"$ $e_1 = \frac{7{\cdot}25}{13{\cdot}5} = {\cdot}54$

Apply Case II:- $p_1 = \frac{2{\cdot}41 \times 100}{13{\cdot}5 \times 12} = 1{\cdot}5\%$

$n_0 = {\cdot}53$; $Q_2 = {\cdot}154$

$c = \frac{265{,}000}{{\cdot}154 \times 12 \times 13{\cdot}5^2} = 780$ lb./in.² Correct

1 : 2 : 4 Mix.

14" x 12" (per Section)

4 - ⅞" φ

4th to 5th Floors (Partial Calc.ⁿ only).

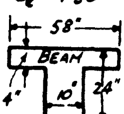
	lb.
Loading:- From above 5th. Floor =	36,540
5th. Floor Dead Load = 16 x 24 x 130 lb. x ·41 =	20,500
" " Live " = 16 x 24 x 50 lb. x ·55 =	10,600
Wall Panel, Column, etc	11,000
Total	78,640

$L_B = 16'$

$L_L = 12'$

$L_U = 10'$

$C_e = 1{\cdot}50$



$C = {\cdot}165$

$k = 1{\cdot}00$

Bending Moment: $W = 8 \times 24 \times 180 = 34{,}600$ lb.

$M_e = 1{\cdot}50 \times 34{,}600 \times 16 = 833{,}000$ in lb.

$I_B = 22{,}800\ in^4$; $I_L = I_U = 4940\ in^4$; $C_U = {\cdot}347$; $C_L = {\cdot}289$

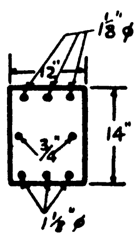
B.M. (Upper Col.) $= \frac{{\cdot}347}{1{\cdot}636} \times 833{,}000 = 177{,}000$ in. lb.

B.M. (Lower Col.) = 147,000 in. lb. $e = 1{\cdot}87"$ $e_1 = {\cdot}138$

Applying Case II: $p_1 = 1{\cdot}5\%$ $Q_1 = 1{\cdot}42$ $c = 690$ lb./in.² Correct

Ditto.

CALCULATION SHEET NO. 16.
EXTERNAL COLUMNS.
(By-laws.)

EXTERNAL COLUMNS D (CONT.).	lb.	
3RD. TO 4TH. FLOOR (Partial Calcⁿ only.)	Loading: from above 4th. Floor = 78,640 4th. Floor Dead Load = 20,500 " " Live " = 90% x 10,600 = 9,550 Wall Panel, Column, etc. = 11,000 Total = 119,690	
$L_B = 16'$ $L_U = L_L = 12'$ $k = 1{\cdot}00$ $f_1 = {\cdot}134$ $D_1 = 13{\cdot}5''$	Trial Section as above 4th Fl. with 4 – 1"φ $K = \frac{119,690}{14 \times 12} = 713$ Ample for axial load Bending Moment: $M_e = 833,000$ in. lb. $I_B = 22,800$ in.⁴ $I_L = I_U = 4940$ in.⁴; $C_L = C_U = {\cdot}289$ B.M. (both Cols.) $= \frac{{\cdot}289}{1{\cdot}578} \times 833,000 = 153,000$ in lb. (N.B. > 147,000) $e = 1{\cdot}28''$ $e_1 = \frac{1{\cdot}28}{14} = 0{\cdot}091$ Case I:– By Calculation:– $p = 1{\cdot}87\%$ $h_o = \frac{10''}{14} = {\cdot}715$ $c_m = \frac{119,690}{12 \times 14}\left[\frac{1}{1+(14 \times 1{\cdot}87)} + \frac{6 \times {\cdot}091}{1+({\cdot}42 \times 1{\cdot}87 \times {\cdot}715^2)}\right]$	$p = \frac{3{\cdot}14 \times 100}{14 \times 12} = 1{\cdot}87\%$ 1 : 2 : 4 MIX 14" x 12" (rib) 4 – 1"φ = 840 lb/in.²
2ND TO 3RD. AND 1ST. TO 2ND.	If maximum stress calculated on rib section is in excess of 950 lb./in.², additional steel to be provided.	SECTION AS 3RD TO 4TH (14" x 12" Rib)
GROUND TO 1ST. FLOOR (Partial Calcⁿ only).	Loading: From above 3rd. Floor = lb. 119,690 Dead Load: 3rd, 2nd, & 1st. Floors = 61,500 Live Load: Ditto = 210% of 10,600 = 22,300 Wall Panels, etc. 1st. to 3rd = 2 x 11,000 = 22,000 Column, Ground to 1st Floor, say 4,100 Total = 229,590	
$L_B = 16'$ $L_U = 12'$ $L_L = 14'$  C_m (permissible) = 1100 lb/in.²	Trial Section: 1 : 1½ : 3 Mix; 14" x 12". $K = \frac{229,590}{14 \times 12} = 1360$; $p = 4{\cdot}0\%$ (for axial load only) $A_c = \frac{4{\cdot}0}{100} \times 14 \times 12 = 6{\cdot}71$ in.² Bending Moment: $M_e = 833,000$ in lb. $I_B = 22,800$ in⁴ $C_U = {\cdot}289$ $I_L = \frac{12 \times 14^3}{12} = 2750$ in⁴: $C_L = \frac{2750 \times 12}{22,800 \times 14} = {\cdot}103$ B.M. (Lower Col.) $= \frac{{\cdot}103}{1{\cdot}392} \times 833,000 = 61,600$ in lb. $e = \frac{61,600}{229,590} = {\cdot}27$ $e_1 = {\cdot}02$ $(< {\cdot}167)$ Apply Case (I):– p (6 – 1⅛"φ) = 3·55%; $h_o = \frac{14 - 3 - 1\frac{1}{8}}{14} = {\cdot}705$ $c_m = 1010$ lb/in.² Slenderness:– $\frac{14 \text{ ft high}}{12'' \text{ least dimension}} = 1{\cdot}17$ ∴ $R_L \not> 1{\cdot}0$	1 : 1½ : 3 Mix. 14" x 12" 6 – 1⅛"φ and 2 – ¾"φ (A_c = 6·84 in.²)

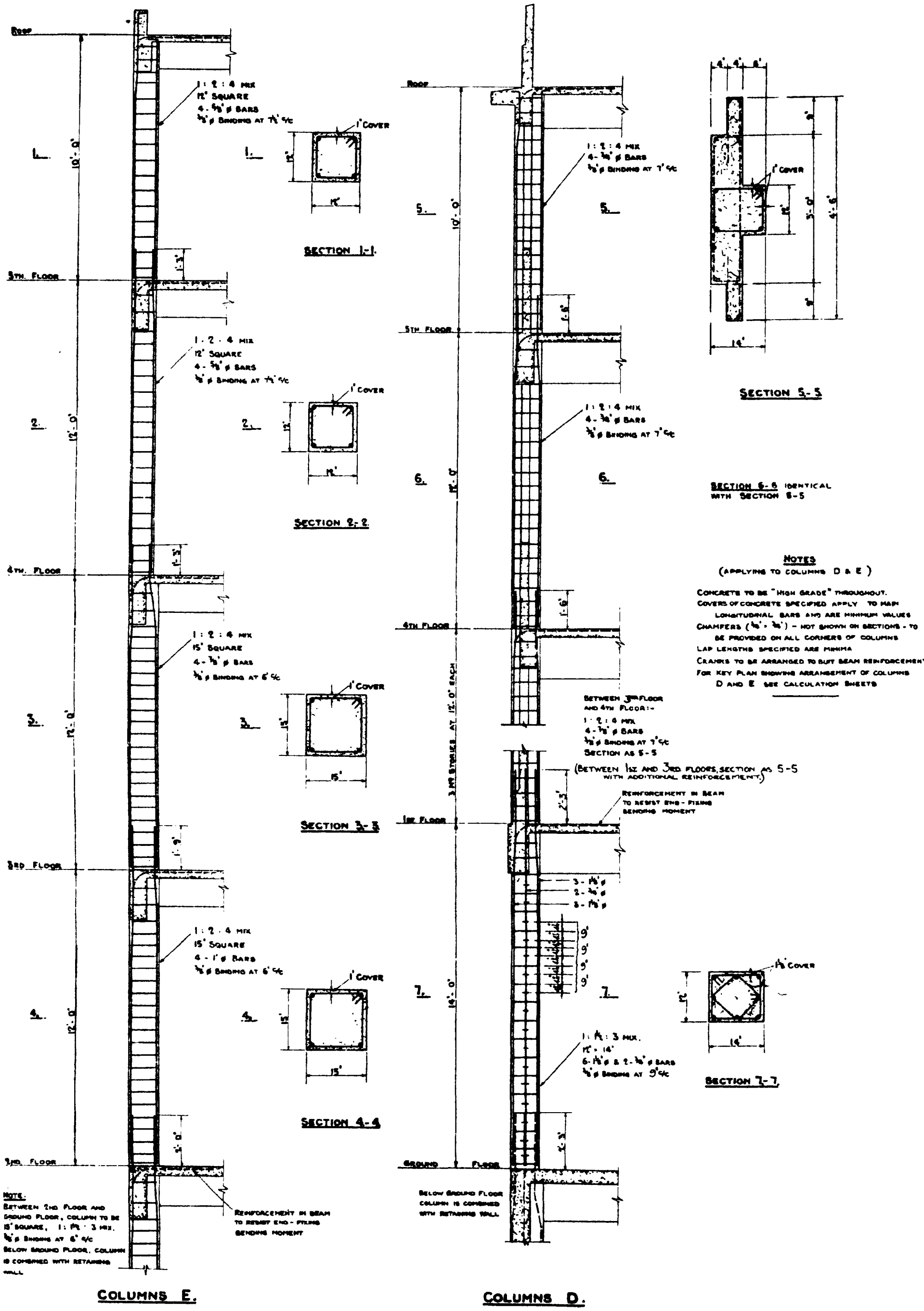
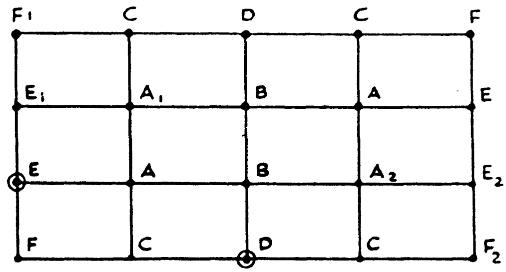


FIG. 15.—DETAILS OF EXTERNAL COLUMNS.
(Code.)

CALCULATION SHEET No. 17.
EXTERNAL COLUMNS.
(Code.)

COLUMN KEY PLAN



HIGH GRADE CONCRETE THROUGHOUT

EXTERNAL COLUMNS E.

5TH Floor To Roof.

Loading:- lb.

Roof Dead Load $= 16 \times 24 \times 90\text{lb} \times \cdot 42 = 14,500$

" Live " $= 16 \times 24 \times 30\text{lb} \times \cdot 53 = 6,100$

Parapet: say $50\text{lb} \times 16 = 800$

Column: $8 \cdot 5 \times 144\text{lb} = 1,225$

Total. 22,625

Trial Section: 12" sq. 4-5/8" ϕ

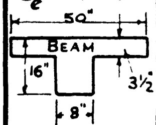
Axial Load only: $K = \dfrac{22,625}{12^2} = 158$ $(p < 0 \cdot 8\%)$

$L_B = 24'$
$L_L = 10'$
$C_e = 1 \cdot 00$

Bending Moment: (per Tables 29 & 30)

$W = 24 \times 900\text{lb (approx)} = 21,600\text{lb.}$

$M_e = 1 \cdot 00 \times 21,600 \times 24 = 518,000$ in lb.



I_B:- $\dfrac{d}{D} = \dfrac{3\frac{1}{2}}{16} = \cdot 219$; $\dfrac{b}{B} = \dfrac{8}{50} = \cdot 16$

$C = \cdot 169$ (per Table 31)

$I_B = \cdot 169 \times 8 \times 16^3 = 5530 \text{ in.}^4$

$I_L = \dfrac{12^4}{12} = 1728 \text{ in.}^4$

$k = 1 \cdot 00$

$C_L = \dfrac{1728 \times 24}{5530 \times 10} = \cdot 75$

B.M. at Col. and Beam Junction

$= \left(\dfrac{\cdot 75}{\cdot 75 + 1}\right) 518,000 = 222,000$ in.lb.
(cf. 262,000 per Sheet No. 3)

$e = \dfrac{222,000}{22,625} = 9 \cdot 8"$ $e_1 = \cdot 817$

Apply Case (ii):-

$Q_1 = K = 157.$ $f_1 = \dfrac{1 \cdot 31}{12} = \cdot 11$

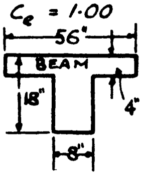
$Q_2 (\text{for } f_1 = \cdot 10) = 157(\cdot 817 + \cdot 4) = 191$

$p < 0 \cdot 8\%$ Trial Section O.K.

1 : 2 : 4 Mix
12" SQUARE
4 - 5/8" ϕ
3/16" ϕ Binding at 7½" c/c

CALCULATION SHEET No. 18.
EXTERNAL COLUMNS.
(Code.)

EXTERNAL COLUMNS E (cont.)

4TH FLOOR TO 5TH FLOOR	Loading: From above 5th. Floor: 22,625 lb.	
	5th. Fl. Dead Load: 16 x 24 x 130lb x ·42 = 21,000	
	" " Live " 16 x 24 x 50lb x ·53 = 10,180	
	Brick Wall · 15 x 8·5 x 90lb. = 11,500	
	Column : 10 x 144 = 1,440	
	Total = 66,745	
	Trial Section : As above 5th. Floor.	
	$K = \frac{66,745}{12^2} = 464$; $p < 0·8\%$ (Axial Load only)	
$L_B = 24'$	Bending Moment: $W = 8 \times 24 \times 170$ lb. (app) = 32,700 lb	
$L_U = 10'$	$M_e = 1·00 \times 32,700 \times 24 = 784,000$ in lb.	
$L_L = 12'$	I_B :- $\frac{d}{D} = \frac{4}{18} = ·222$; $\frac{b}{B} = \frac{8}{56} = ·143$; $C = ·174$	
$C_e = 1·00$	$I_B = ·174 \times 8 \times 18^3 = 8120$ in^4	
	$I_L = I_U = 12^4/12 = 1728$ in.^{4}	
	$C_L = \frac{1728 \times 24}{8120 \times 12} = ·426$ $C_U = \frac{1728 \times 24}{8120 \times 10} = ·511$	
	BM (Upper Col.) $= \frac{·511}{1·937} \times 784,000 = 206,500$ in.lb ($< 222,000$)	
$k = 1·00$	BM (Lower Col.) $= \frac{·426}{1·937} \times 784,000 = 172,000$ in lb.	1 : 2 : 4 MIX 12" SQUARE 4 – 5/8"φ 3/8"φ Binding at 7½" c/c
	$e = \frac{172,000}{66,745} = 2·58''$ $e_1 = ·215$	
	Apply Case (ii):– (Application of Case (i) shows tension).	
$f_1 = ·11$ (as above)	$Q_1 = K = 464$; $Q_2 = 464(·215 + ·4) = 285$	
	$p < ·8\%$ Trial Section O.K.	
3RD FLOOR TO 4TH FLOOR	Loading:– From above 4th Floor: 66,745 lb.	
	4th Fl. Dead Load: = 21,000	
	" " Live " : 90% of 10,180 = 9,150	
	Brick Wall: 15 x 10 x 90lb. = 13,500	
	Column : 10 x 225 = 2,250	
	Total = 112,645	
	Trial Section : 15" x 15" – 4 - 7/8"φ	
	$K = \frac{112,645}{15^2} = 500$. (O.K. for axial load)	
	Bending Moment: I_B, L_B and M_e as for beam at 5th Fl.	
	$C_U = C_L$ for 4th to 5th Floors $= ·426$	

CALCULATION SHEET No. 19.
EXTERNAL COLUMNS.
(Code.)

<u>EXTERNAL COLUMNS E (cont.)</u>

3RD TO 4TH. Floor (CONT.) $L_L = 12'$ $k = 1.00$	$I_L = \frac{15^4}{12} = 4220\ in^4$ $C_L = \frac{4220 \times 24}{8120 \times 12} = 1.04$ B.M. (Upper Col.) $= \frac{.426}{2.47} \times 784{,}000 = 135{,}000$ in.lb. ($\ngtr 172{,}000$) B.M. (Lower Col.) $= \frac{1.04}{2.47} \times 784{,}000 = 330{,}000$ in.lb. $e = \frac{330{,}000}{112{,}645} = 2.93$ $e_1 = .196$ Apply Case (i):– $p = \frac{2.41 \times 100}{15 \times 15} = 1.07\%$ $E = .14$ $h_0 = \frac{15 - 2 - .875}{15} = .81$ $c_m = \frac{112{,}645}{15 \times 15}\left[\frac{1}{1 + .14} \pm \frac{6 \times .196}{1 + (3 \times .14 \times .81^2)}\right]$ $= 900$ lb/in.² (Compⁿ.) and 25 lb/in.² (Tension) Apply Case (ii):– $Q_1 = K = 500$; $f_1 = \frac{1.44}{15} = .096$ Q_2 (for $f_1 = .10$) $= 500\ (.196 + .40) = 298$ $p < 1.0\%$ – Intersection of Q_1 and Q_2 curves beyond top of graph. Trial Section O.K.	1 : 2 : 4 MIX. 15" SQUARE 4 - ⅞" ϕ ⅜" ϕ Binding at 6" c/c
2ND TO 3RD Floor	<u>Loading</u>: From above 3rd. Floor — lb. 112,645 3rd. Floor Dead Load = 21,000 " " Live " = 80% of 10,180 = 8,150 Brick Wall = 13,500 Column. = 2,250 Total: 157,545 Trial Section: 15" × 15" - 4 - 1" ϕ $K = \frac{157{,}545}{15^2} = 700$ O.K. for axial load. <u>Bending Moment</u>: $C_L = C_U = 1.04$ ($= C_L$ for 3rd to 4th) B.M. (Both Cols.) $= \frac{1.04}{3.08} \times 784{,}000 = 264{,}500$ in.lb. ($< 333{,}000$) $e = \frac{264{,}500}{157{,}545} = 1.68''$ $e_1 = .113\ (< .167)$ Apply Case (i): $p = \frac{3.14 \times 100}{15 \times 15} = 1.40\%$ $E = .182$ $h_0 = \frac{15 - 2 - 1.0}{15} = .80$ $C_m = \frac{157{,}545}{15 \times 15}\left[\frac{1}{1 + .182} \pm \frac{6 \times .112}{1 + (3 \times .182 \times .80^2)}\right]$ $= 945$ lb. per. sq. in. Trial Section O.K.	1 : 2 : 4 MIX. 15" SQUARE 4 - 1" ϕ ⅜" ϕ Binding at 6" c/c
1ST TO 2ND. AND GRD. TO 1ST. FLOORS.	Retain 15" Sq. Column throughout, adjusting concrete mix and longitudinal steel as required by load and moment.	

CALCULATION SHEET NO. 20.

EXTERNAL COLUMNS.

(Code.)

EXTERNAL COLUMNS D.

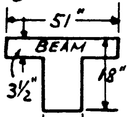
5TH FLOOR TO ROOF

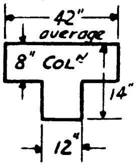
	lb
Loading:— Parapet and Cornice, say	7,000
Roof Dead Load $16 \times 24 \times 90$ lb. $\times \cdot 41$	14,200
" Live " $16 \times 24 \times 30$ lb $\times \cdot 55$	6,340
Wall panel, Column, Column facing, Windows, wall finishes, etc.	9,000
Total	36,540

$L_B = 16'$

$L_L = 10'$

$C_e = 1{\cdot}50$





$k = 1{\cdot}00$

Trial Section (rib only) $14'' \times 12''$ - 4-¾" ϕ

$K = \frac{36{,}540}{12 \times 14} = 217$ O K for axial load

Bending Moment: $W = 24 \times 8 \times 120 = 23{,}040$ lb

$M_e = 1{\cdot}5 \times 23{,}040 \times 16 = 552{,}000$ in lb

I_B:— $\frac{d}{D} = \frac{3\frac{1}{2}}{18} \doteq \cdot 194$; $\frac{b}{B} = \frac{9}{51} = \cdot 176$; $\therefore C = \cdot 163$

$I_B = \cdot 163 \times 9 \times 18^3 = 8560$ in^4

I_L:— $\frac{d}{D} = \frac{8}{14} = \cdot 571$; $\frac{b}{B} = \frac{12}{42} = \cdot 286$; $C = \cdot 15$ approx

$I_L = \cdot 150 \times 12 \times 14^3 = 4940$ in^4

$C_L = \frac{4940 \times 16}{8560 \times 10} = \cdot 922$

BM (Col and beam) $= \frac{\cdot 922}{1{\cdot}922} \times 552{,}000 = 265{,}000$ in lb

(c/f 240,000 per Sheet No 5)

$e = \frac{265{,}000}{36{,}540} = 7{\cdot}25''$ $e_1 = \cdot 517$

Apply Case (vi) :— $f_1 = \frac{1{\cdot}375}{14} = \cdot 098$

$Q_1 = K = 217$; Q_2 (for $f_1 = \cdot 10$) $= 217(\cdot 517 + \cdot 4) = 199$

p required $< \cdot 8\%$ Trial Section O.K.

1 : 2 : 4 Mix

14" x 12" (per section)

4 - ¾" ϕ

⅜" ϕ Binding at 7" c/c

4TH TO 5TH. FLOORS (Partial Calc^n only).

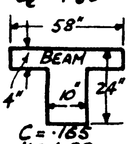
	lb.
Loading:— From above 5th. Floor =	36,540
5th. Floor Dead Load $= 16 \times 24 \times 130$ lb. $\times \cdot 41 =$	20,500
" " Live " $= 16 \times 24 \times 50$ lb. $\times \cdot 55 =$	10,600
Wall Panel, Column, etc.	11,000
Total	78,640

$L_B = 16'$

$L_L = 12'$

$L_U = 10'$

$C_e = 1{\cdot}50$



$C = \cdot 165$

$k = 1{\cdot}00$

Bending Moment: $W = 8 \times 24 \times 180 = 34{,}600$ lb.

$M_e = 1{\cdot}50 \times 34{,}600 \times 16 = 833{,}000$ in.lb.

$I_B = 22{,}800$ in^4; $I_L = I_U = 4940$ in^4; $C_U = \cdot 347$; $C_L = \cdot 289$

B.M. (Upper Col.) $= \frac{\cdot 347}{1{\cdot}636} \times 833{,}000 = 177{,}000$ in. lb.

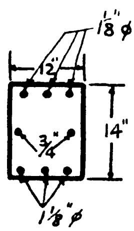
B.M. (Lower Col.) $= 147{,}000$ in.lb. $e = 1{\cdot}87''$ $e_1 = \cdot 133$

Applying Case (i): $p = 1{\cdot}0\%$ $E = \cdot 13$ $h_o = 803$

$c_m = 715$ lb. per sq. in.

DITTO.

CALCULATION SHEET No. 21.
EXTERNAL COLUMNS.
(Code.)

EXTERNAL COLUMNS D (CONT.).	lb.	
3RD. TO 4TH. FLOOR (Partial Calcⁿ only.) $L_B = 16'$ $L_U = L_L = 12'$ $k = 1{\cdot}00$	Loading: from above 4th. Floor = 78,640 4th. Floor Dead Load = 20,500 " " Live " = 90% x 10,600 = 9,550 Wall Panel, Column, etc. = 11,000 Total = 119,690 Trial Section as above 4th Fl. with 4-⅞" ϕ. $K = \frac{119{,}690}{14 \times 12} = 713$ O.K. for axial load Bending Moment: M_e = 833,000 in.lb. I_B = 22,800 in.^{4} $I_L = I_U$ = 4940 in^4; $C_L = C_U = {\cdot}289$ B.M. (both Cols.) $= \frac{{\cdot}289}{1{\cdot}578} \times 833{,}000$ = 153,000 in lb. (N.B. > 147,000) $e = 1{\cdot}28''$ $e_1 = {\cdot}091$ Apply Case (i):- $p = 1{\cdot}43\%$ $E = {\cdot}187$ $h_o = {\cdot}795$ c_m = 890 lb./in.^{2} Trial Section O.K.	1 : 2 : 4 MIX 14" x 12" (rib) 4 - ⅞" ϕ ⅜" ϕ Binding at 7" c/c.
2ND. TO 3RD. AND 1ST. TO 2ND.	If maximum stress calculated on rib section is in excess of 950 lb./in^2, additional steel to be provided.	SECTION AS 3RD TO 4TH (14" x 12" Rib)
GROUND TO 1ST. FLOOR (Partial Calcⁿ only). $L_B = 16'$ $L_U = 12'$ $L_L = 14'$  $m = 12{\cdot}1$ c_m (permissible) = 1100 lb/in^2	Loading: From above 3rd. Floor = 119,690 lb. Dead Load: 3rd, 2nd, & 1st. Floors = 61,500 Live Load: Ditto = 210% of 10,600 = 22,300 Wall Panels, etc. 1st. to 3rd = 2 x 11,000 = 22,000 Column, Ground to 1st Floor, say 4,100 Total = 229,590 Trial Section: 1 : 1½ : 3 Mix; 14" x 12". $K = \frac{229{,}590}{14 \times 12} = 1360$; $p = 4{\cdot}0\%$ (for axial load only) $A_c = \frac{4{\cdot}0}{100} \times 14 \times 12 = 6{\cdot}71$ in.^{2} Bending Moment: M_e = 833,000 in lb. I_B = 22,800 in.^{4} $C_U = {\cdot}289$ $I_L = \frac{12 \times 14^3}{12}$ = 2750 in.^{4}: $C_L = \frac{2750 \times 12}{22{,}800 \times 14} = {\cdot}103$ B.M. (Lower Col.) $= \frac{{\cdot}103}{1{\cdot}392} \times 833{,}000$ = 61,600 in lb. $e = \frac{61{,}600}{229{,}590} = {\cdot}27$ $e_1 = {\cdot}02$ $(< {\cdot}167)$ Apply Case (i):- p (6-1⅛" ϕ) = 3·55%; $E = \frac{3{\cdot}55}{100}(12{\cdot}1 - 1) = {\cdot}394$ $h_o = \frac{14 - 2\frac{1}{4} - 1\frac{1}{8}}{14} = {\cdot}76$. c_m = 1070 lb/in^2 Trial Section O.K. Slenderness :- $\frac{14 \text{ ft high}}{12'' \text{ least dimension}} = 1{\cdot}17$ ∴ $R_L \nless 1{\cdot}0$	1 : 1½ : 3 Mix. 14" x 12" 6 - 1⅛" ϕ and 2 - ¾" ϕ (A_c = 6·84 in.^{2}) ⅜" ϕ Binding at 9" c/c. 

Nos. 15 and 16, column *D*, between the fourth and fifth floors. The bending moment at the head (immediately below the fifth floor) is 147,000 in.-lb. while at the foot (immediately above the fourth floor) the bending moment is 153,000 in.-lb. As the stress with the former moment is low (780 lb. per square inch), reinvestigation is not necessary.

In building designs (without wind moments) it is not usually necessary to investigate the case of minimum direct load (that is, omitting live load) combined with the appropriate bending moment, unless under maximum load a fair amount of tensile stress is developed.

The details of the reinforcement for certain lifts of columns *D* and *E* (see key plan on Calculation Sheet No. 12) are given in *Figs.* 14 and 15. Although both columns are designed as rectangular columns for axial loading and for combined stress it is necessary in the case of column *D* to make some allowance for the effect of the wall facings in calculating the moment of inertia in the same way as the floor slabs are considered in the moment of inertia of the beams.

The remaining external columns *C* and *F* would be designed on similar lines making due allowance in the loading calculations for variations in the elastic reactions from the beams and for the incidence of stairs. The corner columns *F* would be designed for axial load from the two beams supported by them. The increase in stress due to the bending effect of each beam would be considered separately, the maximum unit stress being the sum of the axial load stress and the two increases due to bending.

The shear forces due to moments in external columns are usually so small that the stresses need not be calculated. Consider, for example, the top lift of the external column *E*. According to Calculation Sheet No. 12 the moment at roof level is 222,000 in.-lb., changing to a reverse moment of 206,500 in.-lb. above fifth-floor level. The shear force (= rate of change of moment) is

$$\frac{222{,}000 + 206{,}500}{10 \times 12} = 3{,}554 \text{ lb.}$$

On the whole section of the column this gives a unit shear stress of $\frac{3{,}550}{12 \times 12} = 25$ lb. per square inch, which is negligible.

CHAPTER VII

DESIGN OF BEAM AND SLAB FLOORS

ALTERNATIVE designs for the upper floors of the building will now be considered. The first is a beam-and-slab construction (*Figs.* 1, 16, 17, and 18); the others are rectangular-slab and flat-slab schemes, described in Chapters VIII and IX respectively.

Alternative Superimposed Loads.

In the calculations for floors consideration must be given to the alternative minimum superimposed loads given on *Table* 1. The data on *Tables* 2, 3, and 5 are applicable directly to single spans with any end-fixing conditions, but in a series of continuous spans, where the loading on one span affects the moments in the others, the By-laws do not clearly express whether the alternative load is to be assumed on one span at a time or on two or more spans simultaneously so as to give maximum moments. With the latter interpretation the equivalent minimum unit loadings on *Tables* 2 and 5 would be used directly with the continuous beam moment and shear coefficients (*Tables* 13, 14, 15, and 19). The statement in By-law 4 that the minimum superimposed load should be considered as acting on "an otherwise unloaded floor" seems to indicate that this load may be assumed to act on one span only at a time. The live-load moment coefficients on *Table* 43 have been calculated on this basis, and the figures in the calculation sheets are obtained by designing each section of a beam or slab for the moment due to the normal superimposed load or the minimum alternative superimposed loads, whichever is greater.

The specified superimposed loads are normally sufficient for offices and domestic buildings, but the probable incidence of localised loads such as safes, bookcases, filing cabinets, etc., makes it at times advisable to treat the alternative live load as disposed on one or more spans so as to produce the worst moments. This incidence may be permanent or may occur only when the loads are being moved into position. It is preferable if the magnitude and position of these loads can be ascertained beforehand and allowed for by introducing special beams or otherwise. For the magnitude of normal loads on the floors of buildings reference may be made to the "First Report of the Steel Structures Research Committee" (1931).

Design of Floor Slabs.

Calculation Sheets Nos. 22 to 28 cover the design of the slabs and beams for the upper floors. The methods adopted are similar to those for the roof slab and beams. References T1, T2, etc., applying to *Table* 1, *Table* 2, etc., have been included. The arrangement of the reinforcement is shown in *Figs.* 16, 17 and 18; the first is also a key-plan to the reference numbers for the slab panels and beams.

The principal difference between the calculations for the roof and floor slabs is in the loading. For panels P1 and P3 a dead load of 20 lb. per square foot is included for partitions whose positions are not pre-determined; but in the case

of P2 the weight and position of the brick wall are taken into account and the allowance of 20 lb. per square foot is omitted. The moments due to the alternative superimposed loads on each panel are investigated, the larger moment determining the slab thickness and reinforcement. The minimum alternative superimposed load has been considered on one span only at a time, the coefficients on *Table* 43 being employed. On this basis it will be observed that the minimum alternative live load controls the design of the midspan section of panel P1, while the normal live load of 80 lb. per square foot determines that for the support. If more than one span could be loaded simultaneously with this alternative load, a uniformly distributed superimposed load of 105 lb. per square foot (*Table* 2; span 8 ft.; Class No. 2) would be taken for all sections. This gives a live load moment of $0{\cdot}083 \times 105 \times 8^2 \times 12 = 6{,}720$ in.-lb.; adding the dead load bending moment of 5,370 in.-lb. the total maximum moment is 12,090 in.-lb., compared with 11,418 in.-lb. on the calculation sheet. The calculations for both panels P1 and P2 show that with 8-ft. spans there is little difference between the moments due to the two cases of live load.

Where the normal panels P1 (i.e. P1 A) are supported on the external longitudinal walls of the building, fixity has been assumed owing to the apparent rigidity of the fascia beams and wall panels. With panel P2, however, the edge adjacent to the stair and lift well has been considered freely supported, although this may be conservative owing to the partial continuity with panel P5 and the stairs. Thus panel P2 is treated as the end span of a series of four continuous spans, applying the coefficients (without adjustment) for the distributed load from *Table* 14. The coefficients 0·19 and 0·14 applied to the non-central point load are determined from tables (e.g. the author's "Reinforced Concrete Designer's Handbook," p. 187) for a span free at one support and fixed at the other. The summation of the theoretical moments due to the live load, dead distributed load, and dead point load, for panel P2, indicates that the support moment is so much in excess of the midspan moment that the former can be reduced (within the limit of 15 per cent.) at the expense of the latter to obtain more equable moments at the critical section. As the moment at the support between P2 and P3 is thus reduced it is necessary to increase the midspan moment of panel P3. The slab thickness of $4\frac{1}{2}$ in. required for panel P2 is retained in P3 to provide sufficient depth for the support bending moment.

Only the calculations for P1, P2, and P3 are reproduced; the remainder would be dealt with on similar lines, taking into account variations of span, fixing conditions, and the incidence of special loads.

The calculations on Sheets 22 to 28 apply strictly to design in accordance with the By-laws and Memorandum. Except for the beam design factors and distribution steel, the calculations would apply equally well to design in accordance with the Code. The variations, which incidentally would not affect the ultimate details of the reinforcement, would include the use of the design factors taken from *Table* 9 for 1 : 2 : 4 High-grade concrete.

Design of Floor Beams.

The floor beam calculations, commencing on Sheet No. 23, introduce but few points not considered when designing the roof beams. As the ratio of live to dead load for the secondary beams is $\frac{1}{2}$ the moment coefficients for the com-

bined load can be taken directly from *Table* 14. The only difference in the calculations for secondary beams S1 and S2 and for S3 and S4 is that the latter series is built into the external columns while the former is considered as freely supported on end supports. Beam S5, which is made 9 in. wide to correspond to the thickness of the brick wall below and above, is taken as freely supported at both supports (moment coefficient = 0·125), and the live load of 300 lb. per foot run is determined directly from *Table* 5 (Class No. 2). As shown in the first chapter, the alternative superimposed load is not applicable to the normal secondary and main beams for the upper floor. For the main beams, elastic reactions from the secondary beams have been considered, and the division of the distributed and point loadings has been made more precisely than for the roof beams. From inspection of the relative magnitudes of the various moments on the moment diagram (Calculation Sheet No. 27), it is doubtful whether this refinement is worth while.

The first stage in the construction of the moment diagram for the main

TABLE 43.
MINIMUM LIVE LOAD BENDING MOMENTS.
DUE TO ALTERNATIVE LOADING ON BEAMS AND SLABS.
(By-laws and Code.)

Nº OF SPANS	EQUAL SPANS UNIFORM MOM. OF I; FREELY SUPPORTED ON END SUPPORTS.	SECTION	GENERAL	CLASS Nº 1. SLABS	CLASS Nº 1. BEAMS	CLASSES Nºs 2 TO 6 SLABS	CLASSES Nºs 2 TO 6 BEAMS
1.	A	A	+ 1·50W	+ 840	+ 3360	+ 1260	+ 6720
2.	B C B	B	+ 1·14W	+ 639	+ 2556	+ 958	+ 5110
		C	− 0·75W	− 420	− 1680	− 630	− 3360
3.	D E F E D	D	+ 1·13W	+ 633	+ 2535	+ 949	+ 5070
		E	− 0·80W	− 448	− 1792	− 672	− 3584
		F	+ 0·90W	+ 504	+ 2016	+ 756	+ 4032
4.	G H J K J H G	G	+ 1·13W	+ 633	+ 2535	+ 949	+ 5070
		H	− 0·80W	− 448	− 1792	− 672	− 3584
		J	+ 0·88W	+ 493	+ 1971	+ 739	+ 3942
		K	− 0·65W	− 364	− 1455	− 546	− 2910
FOR OTHER SERIES OF APPROXIMATELY EQUAL SPANS.	MIDSPAN − END SPAN		+ 1·15W	+ 644	+ 2576	+ 966	+ 5152
	" INTERIOR SPAN		+ 0·90W	+ 504	+ 2016	+ 756	+ 4032
	SUPPORT − PENULTIMATE		− 0·80W	− 448	− 1792	− 672	− 3584
	" OTHER INTERIOR SUPPORTS.		− 0·70W	− 392	− 1568	− 588	− 3136

MINIMUM LIVE LOAD BENDING MOMENT (ASSUMING NOT MORE THAN ONE SPAN LOADED AT A TIME WITH MINIMUM SUPERIMPOSED LOAD)

= TABULATED COEFFICIENT × SPAN IN FEET = B.M. IN INCH. LB.

VALUES OF W :− CLASS Nº 1. SLABS = 1/4 TON. BEAMS = 1 TON.
" " 2 TO 6 " = 3/8 TON. " = 2 TONS.

(GARAGE FLOORS IN CLASS Nº 5 EXCLUDED).

FOR CLASSIFICATION OF FLOORS SEE TABLE 1.

beam is to calculate and plot the moments at the critical sections due to the point loads and distributed loads acting separately and assuming no end-fixing adjustment of support moments. The coefficients on *Table* 13 are used. From consideration of the maximum free moment the diagrams for each loading can be completed. The second step is the calculation and plotting of the ordinates for the support moments due to the end-fixing moment, applying *Table* 16. The magnitude of the end-fixing moment has already been established in the external column calculation. The diagrams for the distributed loading, point loads, and end-fixing can now be combined to give the unadjusted envelopes of positive and negative moments. The peak moment at the interior support is then reduced to 85 per cent. of its maximum value and the amount of this reduction is added to the maximum positive moments in the adjacent spans. The envelope of the positive bending moment is then adjusted in such a way that the positions of the points of contraflexure are unaltered (compare curve C, *Fig.* 3.). The resulting diagrams, indicated by the heavy line, give the moments for which each section should be designed.

The reinforcement in the secondary and main beams is illustrated in *Figs.* 17 and 18. The stopping-off points of the bars and the anchorage lengths are determined as described for roof beams. The additional construction on the moment diagram on Calculation Sheet No. 27 indicates the adoption of the diagram for this purpose in connection with the main beams.

The shear calculations require little comment. For the secondary beams S3 and S4 the calculated values of V apply to the centre of the support ; the value of V corresponding to the stirrups provided (= 990) is generally less than the calculated value and will be satisfactory a few inches from the support. The stirrups in the main beam are nominal only.

The shear resistance for beam S5 (*Fig.* 17) is provided by bent-up bars in combination with stirrups. The anchorage length available at the end of the 1-in. bars *b* adjacent to the outer wall is 9 in. from the beginning of the hook to the point of bending down plus 7 in. down the slope to the neutral axis, giving a total of 16 in. By proportion from *Table* 24 this allows a stress of only $\frac{16}{38} \times 18{,}000 = 7{,}600$ lb. per square inch. The shear taken on these two bars is, by proportion from *Table* 23, $2 \times \frac{7{,}600}{18{,}000} \times 10{,}000 = 8{,}400$ lb. if they are bent up at 45 deg. and arranged as a single system. The shear left to be taken on stirrups is $14{,}850 - 8{,}400 = 6{,}450$ lb. From *Table* 22, $\frac{5}{16}$-in. stirrups at 5-in. centres take $12\frac{1}{2} \times 554 = 6{,}900$ lb. The shear resistance calculations for the inner support of beam S5 are given on Calculation Sheet No. 26. In this case the bent-up bars *b* can be embedded sufficiently far in the 4-in. slab to develop the stress required to resist the total shear. This stress equals $\frac{14{,}850}{2 \times 10{,}000} \times 18{,}000$, say, 14,000 lb. per square inch. In accordance with *Table* 24, at this stress the length of bar required without a hook is 44 in. ; this length is provided by 37 in. along the straight plus 7 in. down the inclined portion to the neutral axis. The point of bending up, 1 ft. 6 in. from the face of the support (which is well within the limiting distance from the support to the point where the bars are required in the bottom to resist bending), is determined from the value $D = 1{\cdot}41d$ given on *Table* 23.

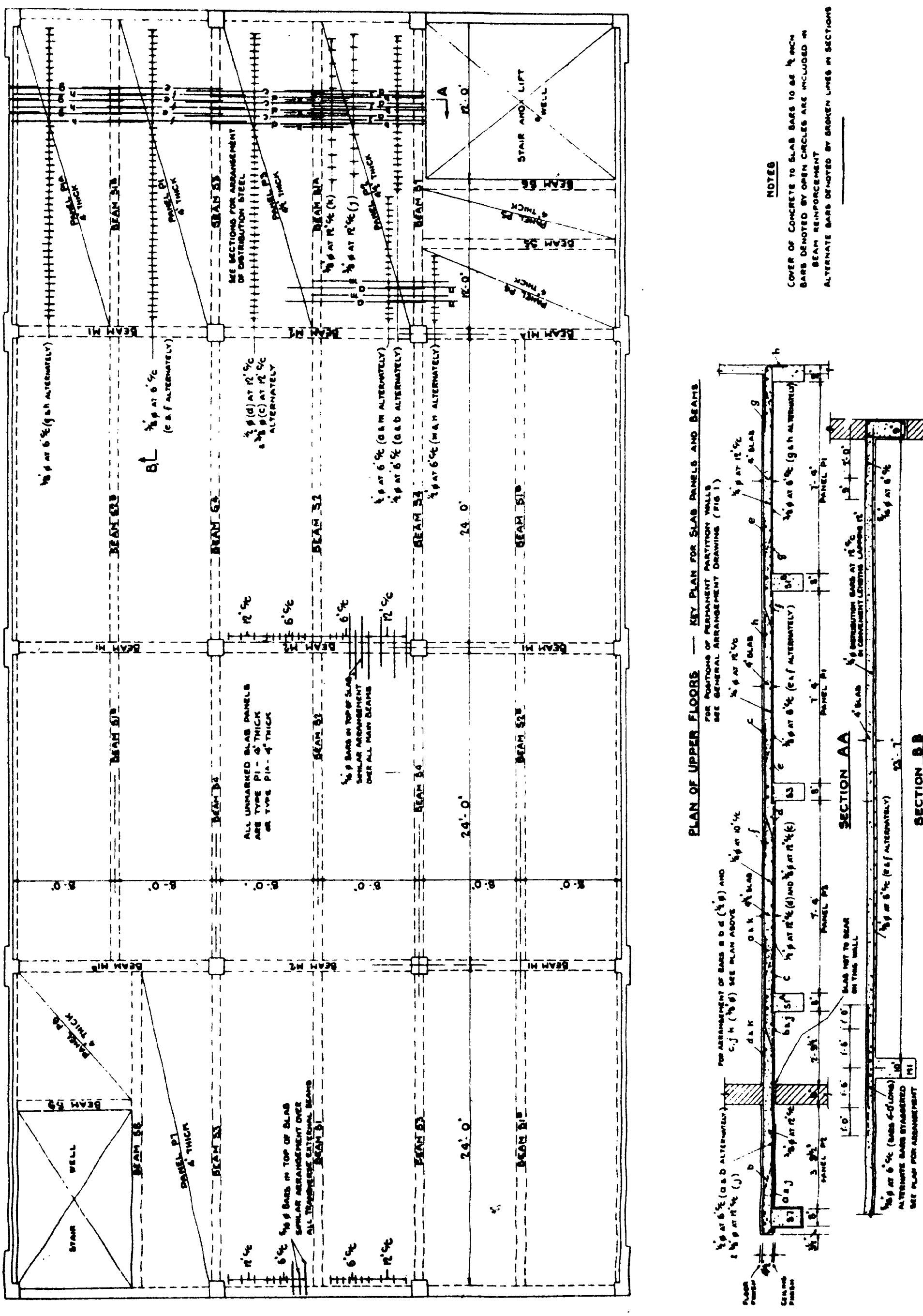


FIG. 16.—DETAILS OF FLOOR SLAB.

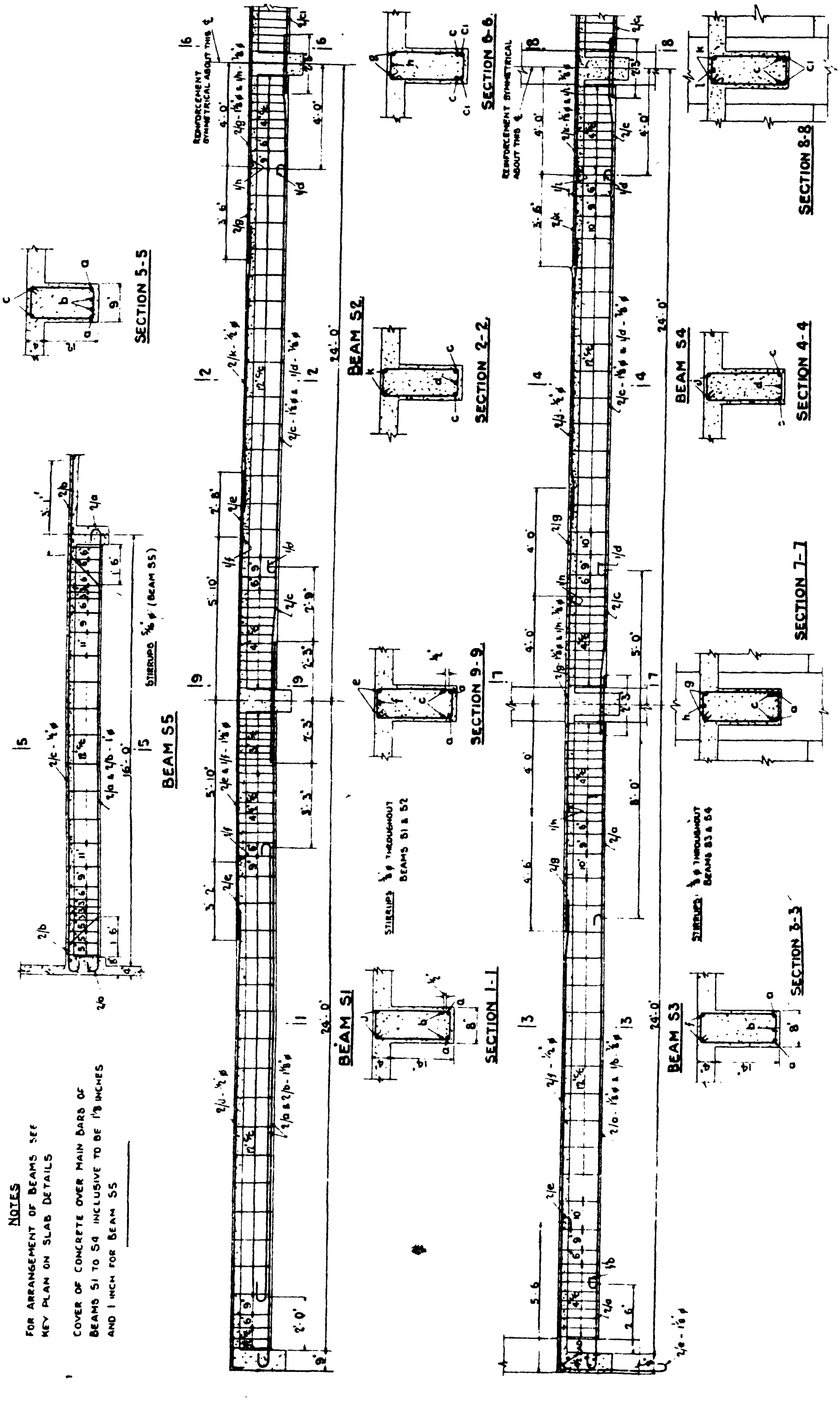


FIG. 37.—DETAILS OF FLOOR SECONDARY BEAMS.

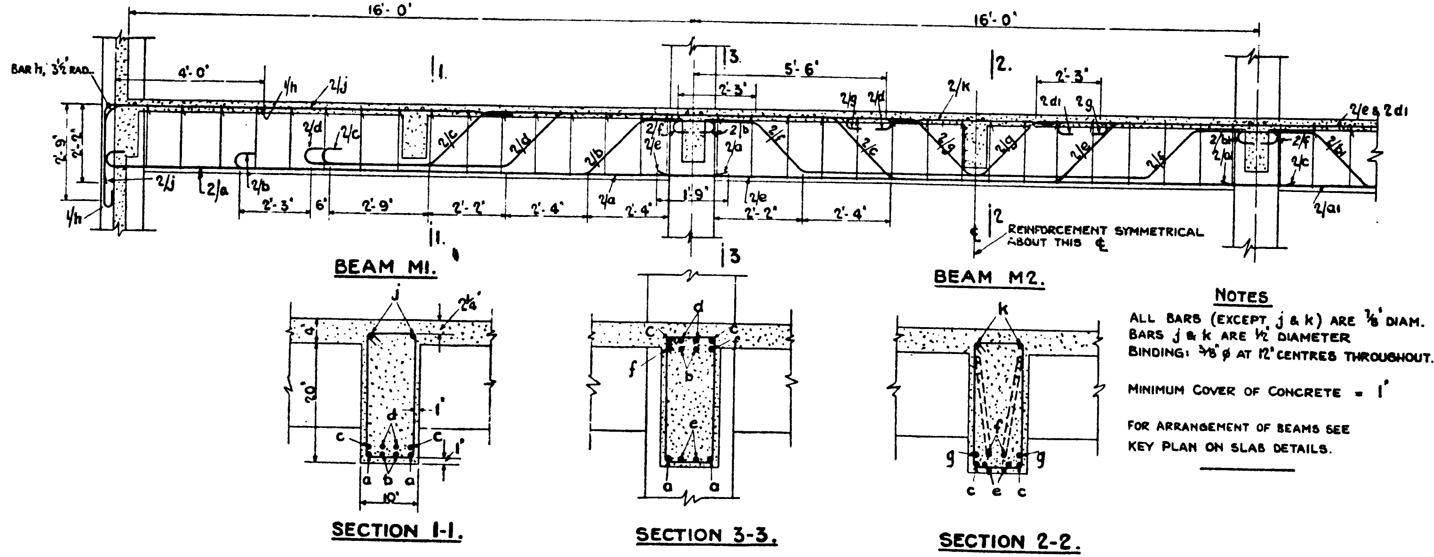


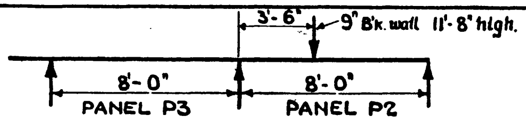
FIG. 18.—DETAILS OF FLOOR MAIN BEAMS.

CALCULATION SHEET NO. 22.

UPPER FLOORS.

BEAM-AND-SLAB CONSTRUCTION.

(By-laws.)

FLOOR SLAB. PANEL P1.	Dead Load:– Floor Finish, say. 10 lb/ft²		
	Ceiling Finish 6 "		
	4" R.C. Slab 48 "		
	Partitions (min.) 20 "		
	Total Dead Load = 84 "		
	B.M. midspan and support due to normal live load = ·083 × 80 × 8² × 12 = 5120 in. lb.	T1 T2 T15	
Effective Span = 8·0 ft. (End spans = 8·33 ft.)	B.M. at midspan due to minimum alternative live load = 756 × 8 = 6048 in. lb.	T43	
	B.M. (dead load) = 0·083 × 84 × 8² × 12 = 5370 " "		
	Max. Midspan B.M = 11,418 " "		
	B.M. at support due to minimum alternative live load = 588 × 8 = 4704 in. lb. (≯ 5120)	T43	
1:2:4 Mix III A Quality A.	Max. B.M. at Support: Dead – 5370 in. lb.		
	Live = 5120 " "		
	10,490 " "		
t = 18,000 :– Q = 17	Effective depth req'd = $\sqrt{\frac{11,418}{179 \times 12}} = 2 \cdot 3$ "	T8	4" SLAB
$a_1 = \cdot 85$	$A_T = \frac{11,418}{18,000 \times \cdot 85 \times 3 \cdot 31} = 0 \cdot 226$ in.²		3/8" φ at 6" c/c
d (provided) = 3·31"	Distribution Steel:	T17	1/4" φ at 12" c/c
	Flange Steel (over Main Beams):	T18	5/16" φ at 6" c/c
FLOOR SLAB. PANEL P2			
	Dead Load = Finishes (16) + 4½" Slab (54) = 70 lb/ft²		
	Normal Live Load = 80 "	T2	
	Total Load = 150 "		
$R = \frac{80}{70} = 1 \cdot 15$	Load from 9" B'k Wall = 90 lb. × 11·67 ft. = 1050 lb.		
	Support B.M. with normal Live Load:		
	= (·112 × 150 × 8² × 12) + (·19 × 1050 × 8 × 12) = 32,100 in. lb.	T14	
	Midspan B.M. with normal Live Load:		
	= (·089 × 150 × 8² × 12) + (·14 × 1050 × 8 × 12) = 24,400 " "	T14	
(≮ 85% of 32,100)	Equalising Support & Midspan B.Ms. = $\frac{32,100 + 24,400}{2}$ = 28,250 in. lb.		

CALCULATION SHEET No. 23.
UPPER FLOORS.
BEAM-AND-SLAB CONSTRUCTION.
(By-laws.)

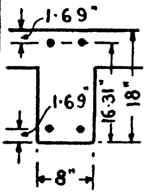
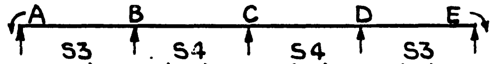
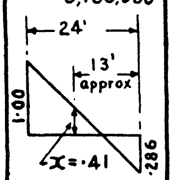
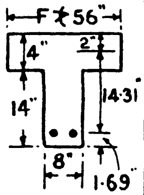
FLOOR SLAB PANEL P2 (cont^{c}.)	Alternative Support B.M. due to minimum live load:		
	Dead Load $= \cdot 107 \times 70 \times 8^2 \times 12 =$ 5,750 in. lb.	T14	
	Min^{m} Live Load $= 672 \times 8$ = 5,376 " "	T43	
	B'k Wall $= \cdot 19 \times 1050 \times 8 \times 12$ = 19,200 " "		
	Total = 30,326 " "		
	Alternative Midspan B.M. due to minimum live load:		
	Dead Load $= \cdot 077 \times 70 \times 8^2 \times 12 =$ 4,140 in. lb		
	Min^{m} Live Load $= 949 \times 8$ = 7,592 " "		
	B'k Wall $= \cdot 14 \times 1050 \times 8 \times 12$ = 14,150 " "		
	Total = 25,882 " "		
	Equalising Alternative Support and Midspan B.M's:-		
	$= \frac{30,326 + 25,882}{2} =$ 28,104 in lbs. (< 28,250)		$4\frac{1}{2}''$ SLAB
d (provided) = 3·75"	$d = \sqrt{\frac{28,250}{179 \times 12}} = 3 \cdot 62''$	T8	$\frac{1}{2}''\phi$ at $6''$ c/c, $\frac{3}{8}''\phi$ at $12''$ c/c
	$A_T = \frac{28,250}{18,000 \times 3 \cdot 75 \times \cdot 85} = \cdot 49$ ins.^{2}		
	Distribution Steel: $\not<$ 0·05 ins.^{2}		$\frac{3}{8}''\phi$ at $12''$ c/c provided
FLOOR SLAB PANEL P3 (Partial calc^{n}.)	Midspan B.M. as Panel P1 = 11,418 in. lb.		
	Add Reduction of Support B.M. for Panel P2		
	$= 32,100 - 28,250$ = 3,850 " "		$4\frac{1}{2}''$ SLAB
	Total = 15,268 " "		$\frac{1}{2}''\phi$ at $12''$ c/c
d = 3·75"	$A_T = \frac{15.268}{18,000 \times 3 \cdot 75 \times \cdot 85} = \cdot 27$ ins.^{2}		$\frac{3}{8}''\phi$ at $12''$ c/c
	Distribution Steel:- $\not<$ 0·03 ins.^{2}		$\frac{1}{4}''\phi$ at $10''$ c/c provided
SECONDARY BEAMS S1, S2, S3 & S4.	Note:- Following Calculations apply principally to Beams S3 and S4. They can be adapted to Beam S1 and S2 by omitting effect of end-fixing at columns.		
	Loading:- Dead - Slab etc. $= 8 \times 84$ = 672 lb per ft.		
	Beam Rib $= 8 \times 14$ = 112 " " "		
$R = \frac{400}{784} = \frac{1}{2}$	Total Dead = 784 " " "		
	Live Load $= 8 \times 50$ = 400 " " "	T5	
	Total = 1184 " " "		

CALCULATION SHEET NO. 24.

UPPER FLOORS.

BEAM-AND-SLAB CONSTRUCTION.

(By-laws.)

SECONDARY BEAMS S1, S2, S3 & S4 (cont^t). Span = 24' (S1 = 24·4') d = 16·31"  Approx. Lever arm = $\frac{1}{2}(13·9 + 14·62)$ = 14·3"	 Supports A & E:- B.M = End-Fixing B.M = sum of Column B.Ms. per Calc^n Sheets 10, 11 & 12:- At 5^th Floor = 206,500 + 172,000 = 378,500 in. lb. At 4^th Floor = 135,000 + 333,000 = 468,100 " " At 3^rd, 2^nd & 1^st F = 264,500 + 264,500 = 529,000 " " Design for approx. B.M = 500,000 in. lb. $C_c = 179 \times 8 \times 16·31^2 = 381,000$ " " $C_s = 119,000$ " " $n = ·44 \times 16·31 = 7·2"$ $a_c = ·85 \times 16·31 = 13·9"$ $a_s = 14·62"$ $c_s = \frac{7·2 - 1·69}{7·2} \times 950 \times 14 = 10,200\ lb./in.^2$ $A_c = \frac{119,000}{10,200 \times 14·62} = 0·80\ ins.^2$ $A_T = \frac{500,000}{18,000 \times 14·3} = 1·86\ ins.^2$	T8	S3 & S4:- 2-1⅛"φ provided. 2-1⅛"φ (S1 & S2:- $A_c = A_T = 0$)
$12wL^2$ = 8,180,000  	Midspan AB & DE:- B.M. = ·101 × 8,180,000 = 826,000 in. lb. Deduct 0·41 × 500,000 = 205,000 " " Net B.M. = 621,000 " " $F = \frac{2 \times 621,000}{4 \times 950 \times 14·31} = 22·8"$ approx. (≯ 56") $A_T = \frac{621,000}{18,000 \times 14·31} = 2·41\ ins^2$	T14 T16 T18	S3 & S4:- 2-1⅛"φ & 1-⅞"φ (S1 & S2:- 4-1⅛"φ two layers)
	Midspan BC & CD:- B.M = ·067 × 8,180,000 = 548,000 in lb. $A_T = \frac{548,000}{18,000 \times 14·31} = 2·26\ ins^2$		S1, S2, S3 & S4 2-1⅛"φ & 1-⅞"φ
	Supports B & D:- B.M. = ·094 × 8,180,000 = 769,000 in. lb. Deduct ·286 × 500,000 = 143,000 " " 626,000 " " C_c = 381,000 " " C_s = 245,000 " "	T14 T16	S3 & S4:- A_c = 2-1⅛"φ A_T = 2-1⅛"φ & 1-⅞"φ

CALCULATION SHEET No. 25.
UPPER FLOORS.
BEAM-AND-SLAB CONSTRUCTION.
(By-laws.)

SECONDARY BEAMS S1, S2, S3 & S4 (cont.)	$A_c = \dfrac{245,000}{10,200 \times 14.62} = 1.62 \text{ ins}^2$		(S1 & S2:- $A_c = 4-1\frac{1}{8}''\phi$
	$A_T = \dfrac{626,000}{18,000 \times 14.3} = 2.43 \text{ ins}^2$		$A_T = 3-1\frac{1}{8}''\phi$)
	Support C :-		
	B.M. = 0·071 × 8,180,000 = 581,000 in. lb	T14	
	Add ·142 × 500,000 = 71,000 " "	T16	S1, S2, S3 & S4:-
	652,000 " "		
	C_c = 381,000 " "		
	C_s = 271,000 " "		$A_c = 2-1\frac{1}{8}''\phi$
	$A_c = \dfrac{271,000}{10,200 \times 14.62} = 1.81 \text{ in.}^2$		$A_T = 2-1\frac{1}{8}''\phi$ & $1-\frac{7}{8}''\phi$
	$A_T = \dfrac{652,000}{18,000 \times 14.3} = 2.53 \text{ in.}^2$		
	Shear:- Total Load per span = 24 × 1184 = 28,400 lb.		
Due to End-Fixing $\dfrac{M}{L} = \dfrac{500,000}{24}$ = 20,800 lb	Max. Shear Force without Reinforcement = 95 × 14·3 × 8 = 10,900 lb.	T21	
	Spans AB and DE:-		
	At A & E: S = ·41 × 28,400 = 11,650 lb.	T19	S3 & S4:-
	Add ·107 × 20,800 = 2,250 "	T20	At every support.
	13,900 "		$\frac{3}{8}''\phi$ Stirrups
	$V = \dfrac{13,900}{14.3} = 970$		at 4" c/c
	At B & D:- S = ·61 × 28,400 = 17,300 lb.	T19	(T 20)
	Deduct ·107 × 20,800 = 2,250 "	T20	
	15,050 "		
	V = 1050		(S1 & S2:-
	Spans BC and CD:-		$\frac{3}{8}''\phi$ Stirrups
	At B and D:- S = ·56 × 28,400 = 15,900 lb.	T19	End Spans:-
	Deduct ·036 × 20,800 = 750 "	T20	At A & E: $4\frac{1}{2}''$ c/c
	15,150 "		At B & D:
	V = 1060		3" c/c
	At C:- S = ·50 × 28,400 = 14,200 lb.	T19	Interior Spans:-
	Add ·036 × 20,800 = 750 "	T20	At B, C & D
	14,950 "		4" c/c)
	V = 1050		

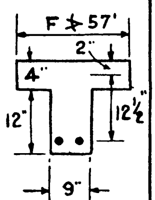
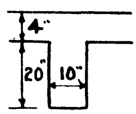
(Shear forces for Beams S1 & S2 are equal to values of S without additions and deductions for end fixing.)

CALCULATION SHEET No. 26.

UPPER FLOORS.

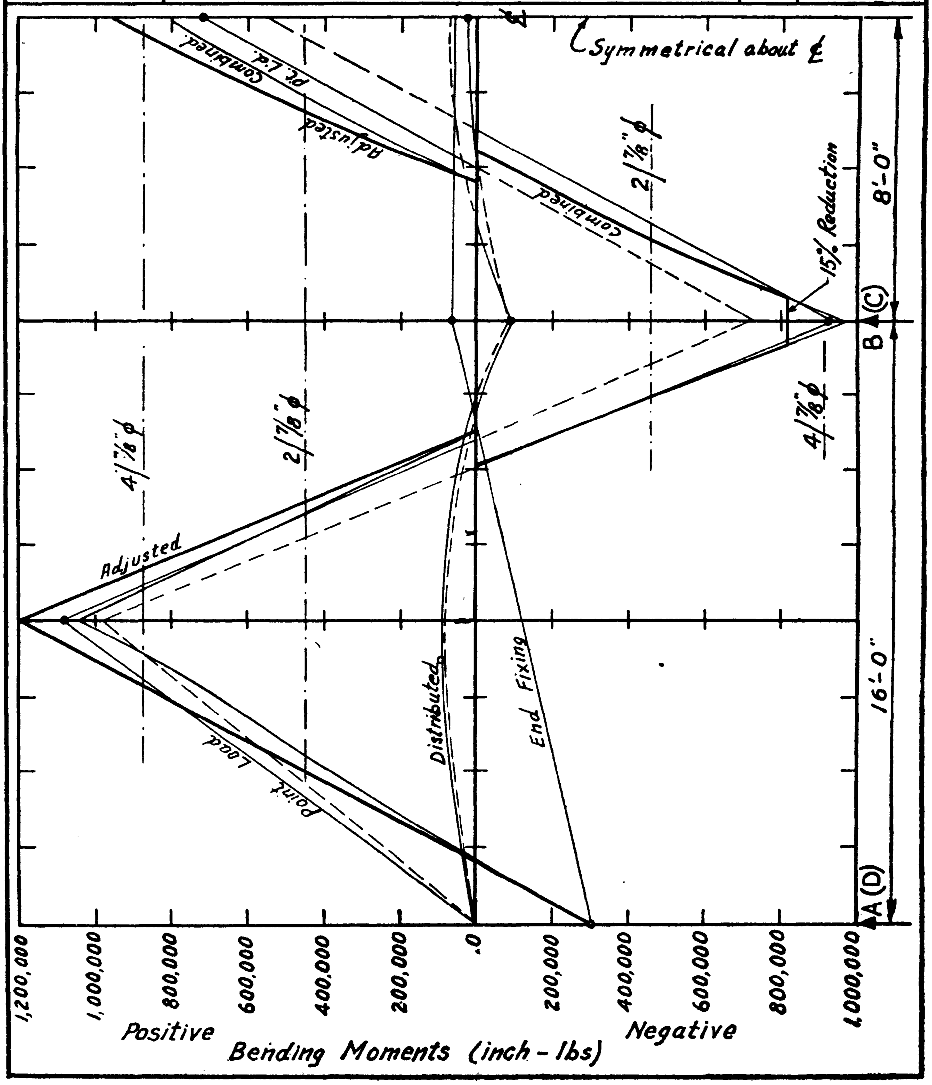
BEAM-AND-SLAB CONSTRUCTION.

(By-laws.)

SECONDARY	Span 16'-0" Freely Supported at Ends.		
BEAM	Width of Floor Slab supported = 6 ft.		
S5	Loading - Live Load = 6 × 50 lb = 300 lb/ft.	T5	
	Slab = 84 lb × 6 = 504 "		
	Beam Rib = 12 × 9 = 108 "		
	9" B'K. Wall = 90 lb × 10·5 = 945 "		
	Total 1857 "		
	Max. Positive B. M. = $0{\cdot}125 \times 1857 \times 16^2 \times 12$ = 712,000 in.lb.		
	$F = \dfrac{2 \times 712{,}000}{4 \times 950 \times 12{\cdot}5} = 30''$ approx ($\not> 57''$)		
	$A_T = \dfrac{712{,}000}{18{,}000 \times 12{\cdot}5} = 3{\cdot}16$ ins.^{2}		4-1" ϕ in bottom
	Shear :- S = 1857 × 8 = 14,850 lbs.		
	$s = \dfrac{14{,}850}{9 \times 12{\cdot}5} = 132$ lb/in.^{2}		
	At inner support: Bent-up Bars at 14,000 lbs/in.^{2} take 15,500 lbs.	T23	2-1" ϕ at 45°
	Shear at 1'-10" from support centre: $= 1857\ (\frac{16}{2} - 1{\cdot}83) = 11{,}450$ lbs.	T22	5/16" ϕ at 3" c/c
MAIN BEAMS M1 & M2	Loading :- Reaction from Secondary Beams. = 1184 × 23·16' × 1·1 av. = 30,200 lb.	T19	
Span = 16'	Distributed Load : Beam Rib = 10 × 20 = 200 lb/ft.		
	Slab, finishes, partitions, etc. = $\frac{10''}{12}$ × 134 = 112 "		
	Total = 312 "		
R (for Point Load) = ½	Total Distributed Load = 16 × 312 = 4980 lb.		
	Data for Construction of B.M. Diagram:-		
	Point Loads:- (30,200 × 16 × 12 = 5,790,000)		
R (for dist^d Load) = 1/6	(Free B.M. = 0·25 × 579,000 = 1,449,600 in.lb)		
	Midspan AB & CD = 0·188 × 5,790,000 = 1,090,000 " "	T13	
	Supports B & C = 0·159 × 5,790,000 = 920,000 " "		
	Midspan B.C = 0·125 × 5,790,000 = 725,000 " "		
	End Fixing Moment :-		
	At A & D (per average of sum of Column Moments on Calc^n Sheets N^{os} 13 & 14) = − 300,000 in.lb.		
	At B & C = ·200 × 300,000 = + 60,000 in.lb.	T16	

CALCULATION SHEET NO. 27.
UPPER FLOORS.
BEAM-AND-SLAB CONSTRUCTION.
(By-laws.)

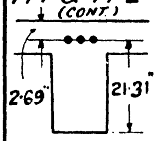
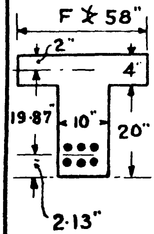
MAIN BEAMS M1 & M2 (cont.)	Distributed Loads:- (4980×16×12 = 964,000) (Free B.M. = ·125 × 964,000 = 120,500 in.lb) Midspan AB & CD = ·082 × 964,000 = 79,000 " Support B & C = ·101 × 964,000 = 97,400 " Midspan BC = ·030 × 964,000 = 28,900 "	T13	



UPPER FLOORS.

BEAM-AND-SLAB CONSTRUCTION.

(By-laws.)

MAIN BEAMS M1 & M2 (CONT.)	<u>Moment Resistance Calculations.</u> (B.M.s taken from Diagram on Sheet № 20)		
	<u>Supports A & D</u> :- B.M. = 300,000 in. lb.		
	$C_c = 179 \times 10 \times 21{\cdot}25^2 = 810{,}000$ in. lb.		$A_c = 0$,
	$A_T = \dfrac{300{,}000}{18{,}000 \times {\cdot}85 \times 21{\cdot}31} = 0{\cdot}92$ in.^{2}	A_T =	2 - ½″ ϕ & 1 - ⅞″ ϕ
	<u>Midspan AB & CD</u>:- B.M. = 1,200,000 in. lb.		
	$F = \dfrac{2 \times 1{,}200{,}000}{4 \times 950 \times 19{\cdot}87} = 32''$ approx.	T18	
	$A_T = \dfrac{1{,}200{,}000}{18{,}000 \times 19{\cdot}87} = 3{\cdot}35$ in.^{2}		6 - ⅞″ ϕ (Effective)
	<u>Midspan BC</u> :- B.M. = 960,000 in. lb.		
	$A_T = \dfrac{960{,}000}{18{,}000 \times 19{\cdot}87} = 2{\cdot}69$ in.^{2}		6 - ⅞″ ϕ (Effective)
	<u>Supports B & C</u> :- B.M. = 815,000 in. lb.		
	C_c = 810,000 ″		
	C_s = 5,000 ″		A_c - nominal
	$A_T = \dfrac{815{,}000}{18{,}000 \times 21{\cdot}31 \times {\cdot}85} = 2{\cdot}48$ in.^{2}		4 - ⅞″ ϕ (Effective)
$a = {\cdot}85 \times 21{\cdot}31 = 18{\cdot}0''$	<u>Shear Calculations:</u> Max. Shear Force without reinforcement = 95 × 18 × 10 = 17,000 lb.		
	<u>Spans AB & CD</u>:-		
	At A & D : S (Point Load) = ·38 × 30,200 = 11,450 lb.	T19	
$\dfrac{M}{L} = \dfrac{300{,}000}{16} = 18{,}700$	(Dist. Load) = ·41 × 4,980 = 2,040 ″	T19	
	(End Fixing) = ·100 × 18,700 = 1,870 ″	T20	Nominal Stirrups
	15,360 ″		
	At B & C : S (Point Load) = ·66 × 30,200 = 19,900 lb.	T19	
	(Dist$^{\underline{t}}$ L'd) = ·60 × 4,980 = 2,990 ″	T19	
	22,890 ″		
	Deduct (End Fixing) = ·100 × 18,700 = 1,870 ″	T20	
	21,020 ″		
	<u>Span BC</u>:-		
	At B & C : S (Point L'd) = ·56 × 30,200 = 16,900 lb.	T19	
	(Dist. L'd) = ·51 × 4,980 = 2,540 ″	T19	2 - ⅞″ ϕ at 45° Single
	19,440 ″		
	<u>Shear Resistance</u> provided throughout Beam (except in end spans): Bent-up bars = 2 × 7,650 = 15,300 lb	T23	⅜″ ϕ Stirrups at 12″ c/c
	Stirrups = 330 × 180 = 5,950 ″	T22	
	21,250 ″		

CHAPTER VIII

SLABS SPANNING IN TWO DIRECTIONS

Moment Reduction Coefficients for Rectangular Panels.

THE clauses in the Memorandum relating to rectangular panels of solid slabs uniformly loaded and spanning in two directions contain tabulated reduction coefficients for

Case (*a*) Slabs fixed at or continuous over four sides, and

Case (*b*) Slabs simply supported on four sides.

The same coefficients are recommended in the Code, but in it the Case (*a*) coefficients are also applicable to slabs simply supported on four sides if the corners are prevented from lifting and adequate provision is made to resist torsion at the corners of the slab, and where such provision does not obtain, the Case (*b*) coefficients apply.

In view of the failure of the Code to specify what amount and arrangement of torsion reinforcement would be considered satisfactory, the German regulations might be taken as a guide since they define what this steel shall be. These regulations require each corner to be reinforced with a series of bottom bars normal to the diagonals of the panel and a series of top bars parallel to the diagonals. A permissible alternative to the bottom series is a mesh of bars parallel to the two adjacent edges of the panel, and it may be suggested that a similar alternative could be allowed for the top series. Each series of bars should provide an area per foot width (normal or parallel to the diagonal) equal to that provided at the middle of the short span, and the extent of each series (or their alternatives) is given in *Fig.* 19. In accordance with the suggested provision (*Fig.* 19), the effective area of reinforcement provided is not less than the minimum prescribed in the German regulations for series parallel and normal to the diagonals. These suggestions can usually be easily complied with in practical details as in scheme *B* (*Fig.* 22). The danger to avoid is the provision of a large number of layers of bars in either the top or bottom of the slab ; if the latter is thin the effectiveness of most of the reinforcement is lost if it is arranged in numerous layers.

Subject to the foregoing comments the Code conforms with the Memorandum in regard to slabs spanning in two directions, including the assessment of positive and negative bending moments in continuous spans.

The Case (*b*) coefficients agree with those obtained from the Grashof and Rankine formula (except for large ratios of length to breadth), while Case (*a*) coefficients are similar to those obtained mathematically by Pigeaud's theory when Poisson's ratio is taken as zero. For slabs fixed or continuous on four sides the midspan moment can be taken as 80 per cent. of the "free-span" moment calculated in accordance with Case (*a*) coefficients, while at the support

the moment must be taken as equal (but of opposite sign) to this " free-span " positive moment. The reduction coefficients for Cases (*a*) and (*b*) and the resulting moment coefficients are given on *Table* 44.

Presumably it is left to the designer to estimate the moments when the slabs are continuous along one, two, or three sides and freely supported along the remainder. Considering a single span, the theoretical maximum negative moment

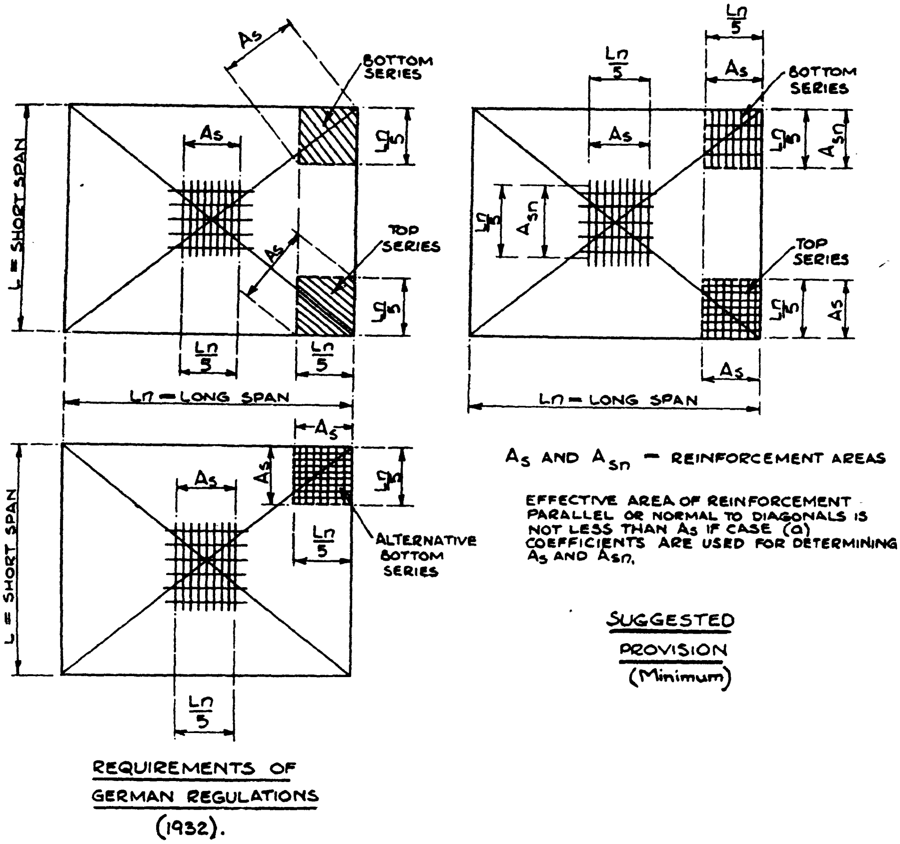


FIG. 19.—CORNER REINFORCEMENT FOR SLABS SPANNING IN TWO DIRECTIONS.

at the support occurs when there is complete fixity at one end and free support at the other ; this moment is $-\frac{wL^2}{8}$ and is numerically equal to the positive moment at the middle of a span freely supported at both ends. This value has, however, been appropriated for the supports of two-way slabs with spans fixed at both ends. Where Case (*a*) coefficients apply it should therefore be satisfactory to take the moment at all supports where fixity or continuity exists as equal to the appropriate positive midspan moment with free supports.

Again where Case (*a*) applies it would be rational to take the midspan positive moment at 90 per cent. of the corresponding support moment when the span is

TABLE 44.

SLABS SPANNING IN TWO DIRECTIONS.

RATIO OF SPANS	CASE (b).				CASE (a).							
	REDUCTION COEFFICIENTS.		FREE SUPPORT ON FOUR SIDES.		REDUCTION COEFFICIENTS.		FREE SUPPORT ON FOUR SIDES.		FIXED OR CONTINUOUS ON FOUR SIDES.			
			MIDSPAN B.M. COEFFICIENT.				MIDSPAN B.M. COEFFICIENT.		MIDSPAN B.M. COEFFICIENT.		SUPPORT B.M. COEFFICIENT.	
	SHORT SPAN.	LONG SPAN.	SHORT SPAN.	LONG SPAN.	SHORT SPAN.	LONG SPAN.	SHORT SPAN.	LONG SPAN.	SHORT SPAN.	LONG SPAN.	SHORT SPAN.	LONG SPAN.
1·0	0·500	0·500	·063	·063	0·295	0·295	·037	·037	·030	·030	·037	·037
1·1	0·594	0·406	·074	·051	0·358	0·237	·045	·030	·036	·024	·045	·030
1·2	0·675	0·325	·084	·041	0·419	0·191	·052	·024	·042	·019	·052	·024
1·3	0·741	0·259	·093	·032	0·477	0·154	·060	·019	·048	·015	·060	·019
1·4	0·794	0·206	·099	·026	0·532	0·127	·067	·016	·053	·013	·067	·016
1·5	0·835	0·165	·104	·021	0·581	0·107	·073	·013	·058	·011	·073	·013
1·6	(0·867)	(0·133)	·108	·017	(0·625)	(0·090)	·078	·011	·063	·009	·078	·011
1·7	(0·895)	(0·105)	·112	·013	(0·663)	(0·078)	·083	·010	·066	·008	·083	·010
1·75	0·904	0·096	·113	·012	0·681	0·071	·085	·009	·068	·007	·085	·009
1·8	(0·915)	(0·085)	·114	·011	(0·697)	(0·068)	·087	·009	·070	·007	·087	·009
1·9	(0·930)	(0·070)	·116	·009	(0·730)	(0·058)	·091	·007	·073	·006	·091	·007
2·0	0·941	0·059	·118	·007	0·757	0·051	·095	·006	·076	·005	·095	·006
2·5	0·975	0·032	·122	·004	0·869	0·032	·109	·004	·087	·003	·109	·004
3·0	0·988	0·022	·124	·003	0·940	0·022	·118	·003	·094	·002	·118	·003

NOTES. REDUCTION COEFFICIENTS IN BRACKETS ARE INTERPOLATED.

$B.M. = (B.M.\ COEFF.) \times (LOAD\ PER\ SQ.\ FT) \times (SPAN)^2.$

continuous over one support and freely supported on the other, since 80 per cent. is allowed for conditions of continuity over both supports.

The foregoing considerations cover the three cases (1) free support on four sides, (2) continuity along four edges, and (3) free support on two adjacent sides and continuity along the other two edges. For these the ratio $\frac{\text{long span}}{\text{short span}}$ determines the appropriate reduction coefficients and consequently the moment coefficients, as given on *Table* 44. Case (*b*) coefficients would be applied to (1) and Case (*a*) coefficients to (2), but with free support on two adjacent sides it would probably be necessary and reasonable to adopt Case (*b*) coefficients combined with a bending moment factor of, say, $\frac{1}{10}$ for both midspan and support moments.

Other sequences of supporting conditions are (1) continuity along one edge only with free support on the remaining three sides; (2) continuity along two opposite edges and free support on the other two opposite sides; and (3) continuity along three edges, with free support on the remaining side. These can be considered by one of several methods such as the development of "equal deflection" expressions similar to the basis of the Grashof and Rankine formula or the use of the approximate "equivalent span" method.

The approximate "equivalent span" method is proposed here, as it is simple and easily memorised and is sufficiently accurate having regard to the uncertainty of the more precise mathematical treatments. This conforms to the preference expressed in the Memorandum for methods that are simple and sufficiently approximate. By this method the span ratio of any panel is expressed as that for an equivalent panel freely supported on four sides, each actual span being reduced to an "equivalent freely-supported span" by applying the following reduction factors:

Condition of Ends.	*Equivalent span.*
Both ends continuous	$\frac{2}{3}\times$ actual span
One end continuous, one end free	$\frac{4}{5}\times$ actual span
Both ends free	actual span

The coefficients applicable to a span ratio equal to $\frac{\text{longer equivalent span}}{\text{shorter equivalent span}}$ are then employed.

For conditions of continuity along three edges with free support along one edge, it seems reasonable to assume that Case (*a*) coefficients would be acceptable with a support-moment factor of $\frac{1}{8}$ and a midspan-moment factor of 90 per cent. of $\frac{1}{8}$ for the free-fixed span. The reduction coefficients would be those for a ratio of

$$\frac{\frac{4}{5}\text{ of “free-fixed” span}}{\frac{2}{3}\text{ of “fixed both ends” span}}$$

or the reciprocal of this expression, whichever is greater than unity.

For conditions of continuity along one edge only and free support along three edges, since the major portion of the panel is freely supported it seems reasonable to assume that Case (*b*) reduction coefficients should be adopted and combined with a support-moment factor of $\frac{1}{10}$ and midspan-moment factors of

$\frac{1}{10}$ for the "fixed-free" span and $\frac{1}{8}$ for the span free at both ends. The reduction coefficients would be based on a span ratio of

$$\frac{\text{Span freely supported at both ends}}{\frac{4}{5} \text{ of "free-fixed" span}}$$

or the reciprocal of this ratio, whichever is greater than unity.

The remaining case of continuity along two opposite edges and free support on the other two edges seems to be a border-line case between Cases (*a*) and (*b*). If treated as Case (*a*) the reduction coefficients would be those applicable to a span-ratio of

$$\frac{\text{Span freely supported at both ends}}{\frac{2}{3} \text{ of "fixed both ends" span}}$$

or the reciprocal of this ratio, whichever is greater than unity. The support moment factor in this case would be $\frac{1}{8}$ and the midspan-moment factors $\frac{1}{8}$ for the "free both ends" span and 80 per cent. of $\frac{1}{8}$ for the "fixed both ends" span. If treated as Case (*b*) the same equivalent span ratio would apply for determining the reduction coefficients, but a support moment factor of $\frac{1}{12}$ and midspan moment factors of $\frac{1}{8}$ for the span free at each end and $\frac{1}{12}$ for the span fixed at both ends would be reasonable. Either method gives similar bending moments as will be seen from the following results applicable to a square panel of span *L* ft.

Bending Moment	Considered as Case (*a*)	Considered as Case (*b*)
Support	0·073 wL^2 ft.-lb.	0·070 wL^2 ft.-lb.
Midspan of fixed both ends span	0·058 wL^2 ,,	0·070 wL^2 ,,
Midspan of free both ends span	0·013 wL^2 ,,	0·021 wL^2 ,,

Design of Floor Slab Panels.

The manner in which the preceding proposals are adopted and the use of *Table* 44 are demonstrated on Calculation Sheets Nos. 29, 30, and 31, which apply to the designs illustrated in *Figs.* 20, 21, and 22. A design for the upper floors with slabs spanning in two directions can be based on the following arrangements without altering the column spacing shown on the general plan of the building (*Fig.* 1), (i) panels 16 ft. square, (ii) panels 12 ft. by 16 ft., and (iii) panels 24 ft. by 16 ft. The third has been chosen. Compared with the other two schemes this design would require a slightly thicker slab but much less beam construction.

Panels P1 (*Figs.* 20 and 21) are continuous along all four edges and the appropriate coefficients for the moments at midspan and support for both the short and the long span can be taken directly from *Table* 44. When the effective depth is the same at the support as at midspan (for instance, on the short span of panel P1, Calculation Sheet No. 29) the steel area at midspan can be assessed directly by taking 80 per cent. of the area of the reinforcement at the support.

Along both the longitudinal edges and the inner transverse edge of panels P2 continuity exists, but at the outer transverse edge conditions of free support are assumed. Thus the short span is continuous at both ends and the long span is

continuous at one end and free at the other. The equivalent span ratio is therefore

$$\frac{\frac{4}{5} \times \text{actual long span } (= 24 \text{ ft.})}{\frac{2}{3} \times \text{actual short span } (= 16 \text{ ft.})} = 1{\cdot}8.$$

The moment coefficients for the supports of both the long and short spans are taken from *Table* 44 for a span ratio of 1·8. The midspan moment for the short span will be 80 per cent. of the corresponding support moment, while that for the long span will be 90 per cent. of the long-span support moment, these percentages being in accordance with the preceding discussion.

Panel P3 offers features of interest. The supporting conditions are similar to those for panel P2, except that on the longitudinal edge against the stair well there is continuity on part of the length and free support over the remainder as shown on Calculation Sheet No. 30. The " equivalent span " of the short span is therefore intermediate between two-thirds and four-fifths (say three-quarters) of the actual span. The 9-in. brick wall (see *Fig.* 1) carried by this panel introduces further complications that are not covered by the regulations, since the latter apply to uniformly distributed loading only. A reasonable way of dealing with this load would be to assume that the proportion taken by each span is the same as if the load were uniformly distributed. The reduction coefficients for Case (*a*) for a span ratio of 1·6 are 0·625 and 0·090 (*Table* 44); the allocation of the load between the short and long spans is given in the calculations. On the short span the load from the wall acts as a point load and the coefficients 0·10 and 0·035 apply to this type of load on spans fixed at one end and partially continuous at the other. On the long span the wall load can be approximately converted into an equivalent uniformly distributed load, assuming, say, only 3 ft. width of slab assists in carrying the additional load. The moments obtained by this approximation are not less than those obtained by applying Pigeaud's method for loads not fully covering the panel.

Details for panels $P1^A$ and $P2^A$ are given in *Figs.* 20 and 21, but the calculations are omitted. The latter would be prepared on the assumption that the outer edges of the panels are freely supported, the inner edges being continuous. Panel $P2^A$ adjacent to panel P3 has been made similar to other panels $P2^A$, but theoretically the resistance moment at the support common to this panel and panel P3 should be the same in the former panel as in the latter. This would require a $6\frac{1}{2}$-in. slab for panel $P2^A$, but in providing a 6-in. slab consideration has been given to the fact that it is reasonable to assume that the small difference in support moment can be absorbed by the beam at this support.

The remaining panels (P4, P5, etc.) adjacent to the stair wells are not conveniently designed as rectangular panels; their design can be identical with those previously prepared for a slab-and-beam construction.

The determination of the superimposed and dead loadings and the required sections for the floor slab are similar to those already given for the roof and floor slabs. The arrangement of the reinforcement (*Figs.* 20 and 21) conforms to the By-laws as regards spacing, cover, and similar requirements. With high span ratios the bars in the direction of the long span correspond to the distribution bars in a slab spanning in one direction. Distribution bars must not be less than 10 per cent. of the principal steel (that is, across the short span), and adoption of the

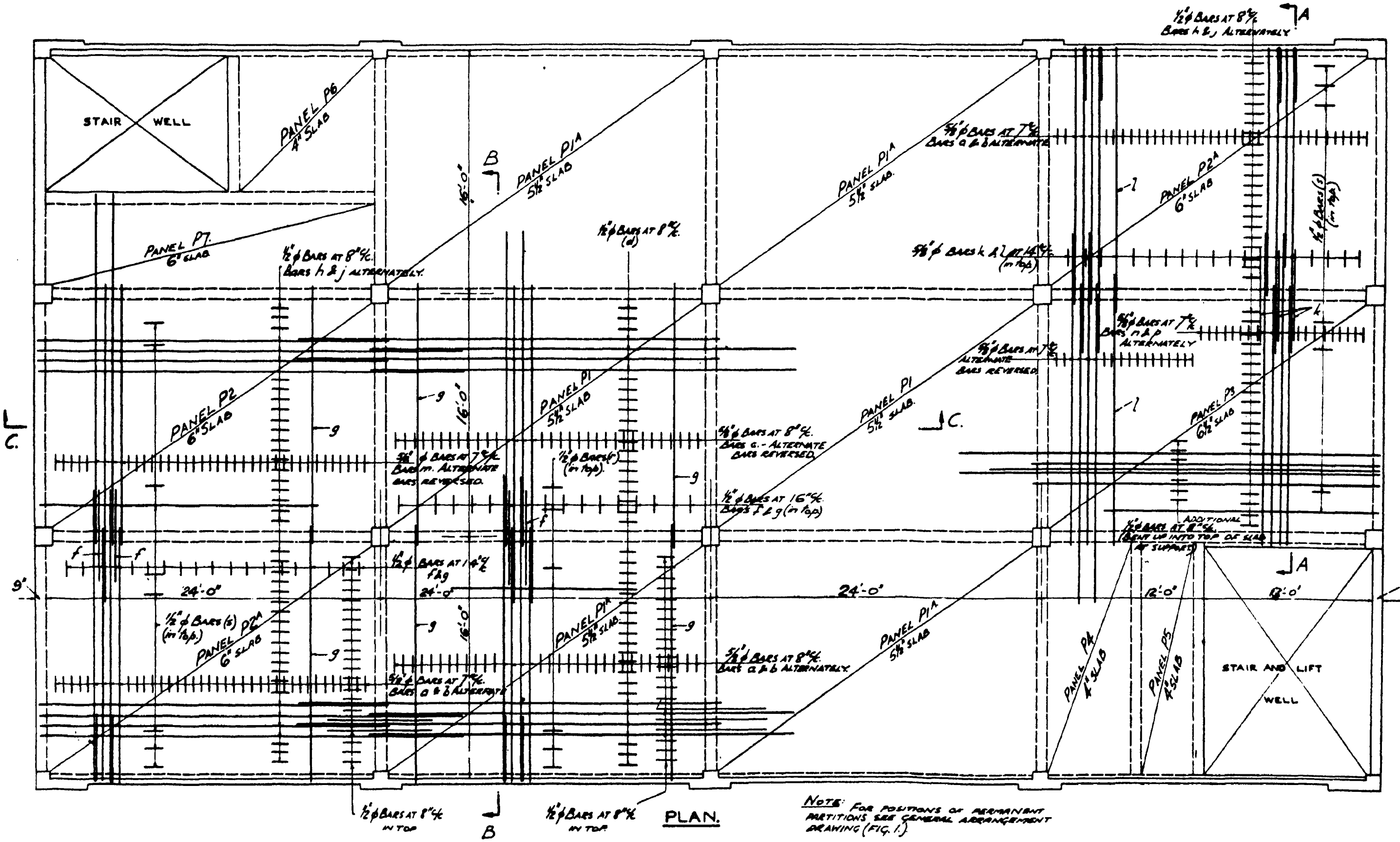


FIG. 20.—PLAN OF SLAB SPANNING IN TWO DIRECTIONS.

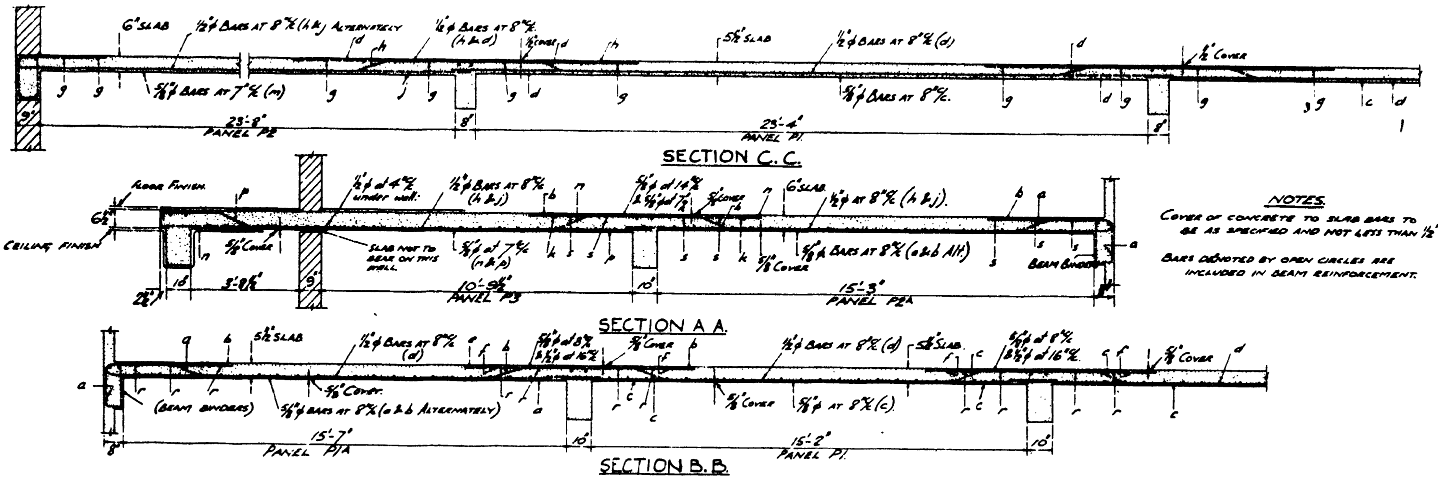


FIG. 21.—SECTIONS THROUGH SLAB SPANNING IN TWO DIRECTIONS.

Calculation Sheet No. 29.

UPPER FLOORS.

Slabs Spanning in Two Directions.

(By-laws.)

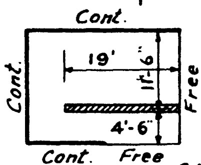
	(See Fig. 20 for Key Plan of Panels.)	
PANELS P1	Loading: Live Load (Class No 2) = 80 lb/ft²	
	Floor Finish = 10 " "	
Span Ratio $= \frac{24'}{16'} = 1{\cdot}5$	5½" R.C. Slab = 66 " "	
	Partitions (min.) = 20 " "	
	Ceiling Finish = 6 " "	
(Continuous on Four Sides)	Total = 182 " "	
	Short Span:- (= 16'-0")	
	Support B.M. $= {\cdot}073 \times 182 \times 16^2 \times 12$	5½" SLAB
	$= 40{,}800$ in. lb	
$d = 4{\cdot}56''$	Eff. depth required $\sqrt{\frac{40{,}800}{179 \times 12}} = 4{\cdot}35''$	⅝" ϕ at 8" c/c
	$A_T = \frac{40{,}800}{18{,}000 \times {\cdot}85 \times 4{\cdot}56} = {\cdot}58$ ins²	& ½" ϕ at 16" c/c
	Midspan A_T = 80% of ·58 = ·46 ins²	⅝" ϕ at 8" c/c
	Long Span:- (= 24'-0")	
	Support B.M. $= {\cdot}013 \times 182 \times 24^2 \times 12$	
	$= 16{,}380$ in. lbs	
$d = 4{\cdot}75''$	$A_T = \frac{16{,}380}{18{,}000 \times {\cdot}85 \times 4{\cdot}75} = {\cdot}225$ ins²	½" ϕ at 8" c/c
	Midspan B.M. $= {\cdot}011 \times 182 \times 24^2 \times 12$	
	$= 13{,}850$ in lbs.	
$d = 4{\cdot}0''$	$A_T = \frac{13{,}850}{18{,}000 \times {\cdot}85 \times 4{\cdot}0} = {\cdot}226$ ins²	½" ϕ at 8" c/c
PANELS P2	Loading as P1 but with 6" slab = 188 lb/ft²	
	Short Span:- (16'-0")	
Equivalent Span Ratio	Support B.M. $= {\cdot}087 \times 188 \times 16^2 \times 12$	
	$= 51\,000$ in lb	6" SLAB
$= \frac{{\cdot}8 \times 24}{{\cdot}67 \times 16}$	d Reqd $= \sqrt{\frac{51{,}000}{179 \times 12}} = 5{\cdot}0$	⅝" ϕ at 7" c/c
$= 1{\cdot}8$ $d = 5{\cdot}06''$	$A_T = \frac{51{,}000}{18{,}000 \times {\cdot}85 \times 5{\cdot}06} = {\cdot}655$ ins²	& ½" ϕ at 14" c/c
	Midspan A_T = 80% of ·655 = ·524 ins²	⅝" ϕ at 7" c/c
	(continued on Sheet 30)	

CALCULATION SHEET No. 30.

UPPER FLOORS.

SLABS SPANNING IN TWO DIRECTIONS.

(By-laws.)

PANELS P2 (cont.)	Long Span:- (24'-0")	
	Support B.M. = $\cdot 009 \times 188 \times 24^2 \times 12$ = 11,700 in. lb.	
$d = 5{\cdot}25''$	$A_T = \dfrac{11{,}700}{18{,}000 \times \cdot 85 \times 5{\cdot}25} = \cdot 145$ ins.^{2}	½" ϕ at 8" c/c. provided.
	Midspan B.M. = 90% of 11,700 = 10,500 in. lb.	
$d = 4{\cdot}5''$	$A_T = \dfrac{10{,}500}{18{,}000 \times \cdot 85 \times 4{\cdot}5} = \cdot 151$ ins.^{2}	½" ϕ at 8" c/c.
PANEL P3.	Loading:- As P1 but with 6½" slab = 194 lb/ft.^{2}	
Equivalent span ratio $= \dfrac{\cdot 8 \times 24}{\cdot 75 \times 16} = 1{\cdot}6$	Additional Load (9" B'k Wall, 11'-6" high)	
	= 90 lb. × 11·5 × 19 ft. = 19,665 lb.	
	Allocate as point load on short span:	
	$\left(\dfrac{\cdot 625}{\cdot 625 + \cdot 090}\right) 19{,}665$ = 17,200 lbs. (on say 20 ft.)	
	= 860 lbs. per ft. width.	
	Allocate as uniform load on long span:	
	19,665 − 17,200 = 2,450 lbs.	
	Assume spread on area 20 ft. × 3 ft.	
	equivalent uniform load = 35 lb. per ft.^{2} on whole span.	
	Short Span (= 16 ft.):-	
	Support B.M. = $\cdot 078 \times 194 \times 16^2 \times 12$ = 46,600 in. lb.	
	+ $0.10 \times 860 \times 16 \times 12$ = 16,500 " "	
	63,100 " "	
	d Req'd $= \sqrt{\dfrac{63{,}100}{179 \times 12}} = 5{\cdot}40''$	6½" SLAB
$d = 5{\cdot}56$	$A_T = \dfrac{63{,}100}{18{,}000 \times \cdot 85 \times 5{\cdot}56} = \cdot 74$ ins.^{2}	⅝" ϕ at 7" c/c. ⅝" ϕ at 14" c/c.
	Midspan B.M. = 85% of 46,600 = 39,600 in. lb.	
	+ $0{\cdot}035 \times 860 \times 16 \times 12$ = 5,800 " "	
	45,400 " "	
	$A_T = \dfrac{45{,}400}{18{,}000 \times \cdot 85 \times 5{\cdot}56} = \cdot 53$ ins.^{2}	⅝" ϕ at 7" c/c.
	Long Span (= 24 ft.)	
	Support B.M. = $\cdot 011 \times 194 \times 24^2 \times 12$ = 14,720 in. lb.	Long Span Support and Midspan:-
	+ $\cdot 100 \times 35 \times 24^2 \times 12$ = 24,200 " "	
	38,920 " "	
$d = 5{\cdot}75''$	$A_T = \dfrac{38{,}920}{18{,}000 \times \cdot 85 \times 5{\cdot}75} = \cdot 44$ ins.^{2}	½" ϕ at 4" c/c under wall.
	Midspan B.M. = 90% of 14,720 = 13,250 in. lb.	½" ϕ at 8" c/c Elsewhere.
	+ 24,200 " "	
	37,450 " "	
$d = 5{\cdot}0''$	$A_T = \dfrac{37{,}450}{18{,}000 \times \cdot 85 \times 5{\cdot}0} = \cdot 485$ ins.^{2}	

Memorandum reduction coefficients ensures that the steel in the long span is at least equal to one-fifth of that in the short span. Taking the extreme case, span ratio = 3·0, the reduction coefficients are: short span 0·988, long span 0·022 [Case (*b*)]. The ratio of the long-span moment to the short-span moment is $\frac{0{\cdot}022 \times 3^2}{0{\cdot}988}$, which is 20 per cent. Owing to the different lever arms for the two layers of steel, the long-span steel slightly exceeds 20 per cent. of the short-span steel.

The amount of reinforcement in the slab over the beams along the short sides of the panel should be checked against the requirements for flange steel. In accordance with *Table* 18, a 5½-in. slab, for instance, requires 0·198 sq. in. Since ½-in. bars at 8-in. centres (= 0·295 sq. in.) have been provided, there is an ample amount.

Design in accordance with the Code would not differ in essentials from the calculations on Sheets 29 and 30 and the details in *Figs.* 20 and 21; a small variation would occur when calculating the effective depth and area of reinforcement owing to the different design factors consequent upon using the modular ratio appropriate to 1 : 2 : 4 High-grade concrete.

Freely-supported Rectangular Slabs.

In ordinary building work these slabs do not frequently occur, and in the present structure the only instance would probably be the roof over the lift motor room or a similar structure above the general roof level. The motor room is assumed to be a brick structure covered by a concrete roof slab of which the design is introduced here to complete the study of slabs spanning in two directions.

In *Fig.* 22 are given the details and on Calculation Sheet No. 31 the computations for Scheme A for which the moment reduction factors for Case (*b*) are used in accordance with the requirements of the Memorandum.

Scheme B (*Fig.* 22) is an illustration of design in accordance with the Code requirements for the adoption of Case (*a*) coefficients. The edges of the slab are anchored by the 9-in. brick parapet wall and reinforcement is added to resist the torsion in the corners of the panel. In the calculations for this scheme, the Code design factors are incorporated.

Loading on Beams.

Details of the beams supporting the slabs spanning in two directions are omitted as they present only one feature that has not been dealt with already. This exception concerns the amount of the total panel load carried by each beam. There are several ways of considering this aspect of design upon which no recommendation is expressed in the Memorandum or Code, and among the methods in current practice are (i) direct proportion to the moment-reduction coefficients; (ii) direct proportion to the reduction coefficients in accordance with the Grashof-Rankine formula irrespective of what coefficients are used for the moment calculation; (iii) "trapezium" method (*Fig.* 23); and (iv) "parabola" method (*Fig.* 23).

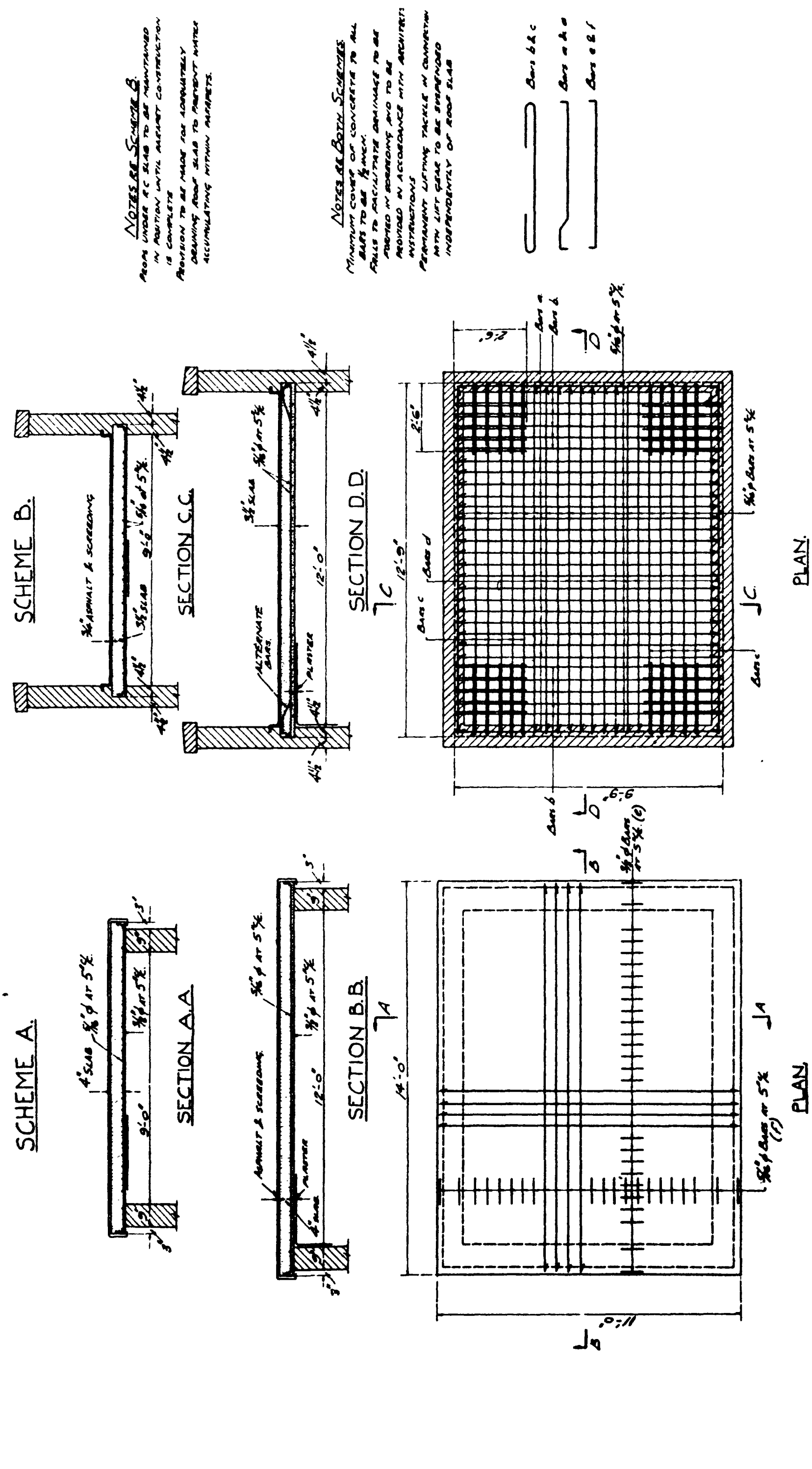


FIG. 22.—DETAILS OF MOTOR ROOM ROOF SLAB.

CALCULATION SHEET No. 31.
ROOF OVER MOTOR ROOM.
(By-laws and Code.)

Span Ratio: $\frac{12'-9''}{9'-9''} = 1{\cdot}3$ (Case b)	**SCHEME A. (BY LAWS)** WITHOUT PARAPETS. Loading:- $\frac{3}{4}''$ Asphalt = 8 lbs/ft² ; Screeding, say = 14 " " ; Ceiling Finish = 6 " " ; 4" Slab = 48 " " ; Live Load = 50 " " ; total 126 " "	
$d = 3{\cdot}31''$	Short Span:- $B.M. = {\cdot}093 \times 126 \times 9{\cdot}75^2 \times 12 = 13{,}300$ in. lbs. $d = \sqrt{\frac{13{,}300}{179 \times 12}} = 2{\cdot}6''$; $A_T = \frac{13{,}300}{18{,}000 \times {\cdot}85 \times 3{\cdot}31} = {\cdot}266$ ins.²	4" SLAB. $\frac{3}{8}''\phi$ at 5" c/c
$d = 2{\cdot}97''$	Long Span.- $B.M. = {\cdot}032 \times 126 \times 12{\cdot}75^2 \times 12 = 7{,}890$ in. lb. $A_T = \frac{7{,}890}{18{,}000 \times {\cdot}85 \times 2{\cdot}97} = {\cdot}175$ ins²	$\frac{5}{16}''\phi$ at 5" c/c
Span Ratio: $\frac{12'-4\frac{1}{2}''}{9'-4\frac{1}{2}''} = 1{\cdot}32$ (Case a)	**SCHEME B (CODE)** WITH PARAPETS. Loading as for Scheme A but with $3\frac{1}{2}''$ Slab = 120 lbs/ft²	
$d = 2{\cdot}84''$	Short Span:- $B.M. = 0{\cdot}60 \times 120 \times 9{\cdot}375^2 \times 12 = 7{,}600$ in. lb ; $d = \sqrt{\frac{7{,}600}{174 \times 12}} = 1{\cdot}91''$; $A_T = \frac{7{,}600}{18{,}000 \times {\cdot}858 \times 2{\cdot}84} = {\cdot}170$ ins²	$3\frac{1}{2}''$ SLAB. $\frac{5}{16}''\phi$ at 5" c/c
$d = 2{\cdot}53''$	Long Span:- $B.M. = {\cdot}019 \times 120 \times 12{\cdot}375^2 \times 12 = 4{,}190$ in. lbs. $A_T = \frac{4{,}190}{18{,}000 \times {\cdot}858 \times 2{\cdot}53} = {\cdot}107$ ins.²	$\frac{5}{16}''\phi$ at 5" c/c (Minimum)

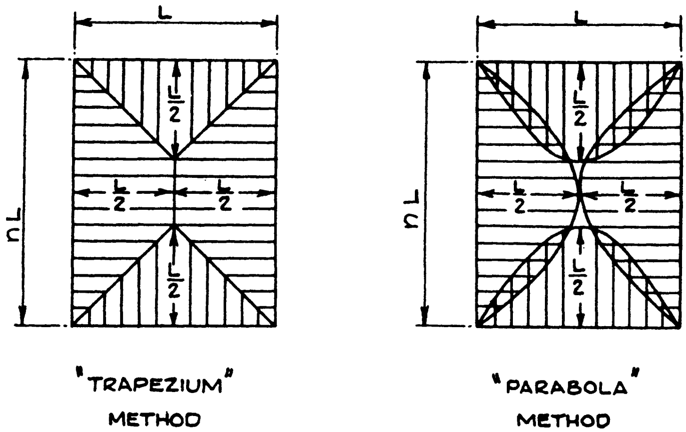


FIG. 23.—DISTRIBUTION OF LOAD TO BEAMS SUPPORTING SLABS SPANNING IN TWO DIRECTIONS.

The following formulæ can be derived for each of these when L = short span; nL = long span; n = span ratio; w = total unit dead and live load on slab; W_A = total load from each panel on each beam along long sides; and W_B = total load from each panel on each beam along short sides.

(i) Based on moment-reduction coefficients [Case (*a*) or (*b*)].

$$\left.\begin{array}{l}\alpha = \text{short-span reduction coefficient}\\ \beta = \text{long-span reduction coefficient}\end{array}\right\}\text{from } \textit{Table } 44.$$

$$W_A = \alpha\frac{nwL^2}{2(\alpha+\beta)};\quad W_B = \beta\frac{nwL^2}{2(\alpha+\beta)}.$$

(ii) Based on Grashof-Rankine formula ($\alpha + \beta$ = unity).

$$W_A = \alpha\frac{nwL^2}{2};\quad W_B = \beta\frac{nwL^2}{2}.$$

(iii) " Trapezium " method.

$$W_A = (n - \tfrac{1}{2})\frac{wL^2}{2};\quad W_B = \frac{wL^2}{4}.$$

(iv) " Parabola " method. (The method illustrated in *Fig.* 23 represents the mean of extreme methods.)

$$W_A = \frac{nwL^2}{3};\quad W_B = \frac{wL^2}{3}.$$

For various values of the span ratio n the percentage of the total panel load carried by each beam is shown in the table on the opposite page.

The " trapezium " method is the one most generally advocated in this country, while the Grashof-Rankine method is adopted in Germany and a more conservative form of the " parabola " method figures in Australian regulations.

Span Ratio.	By Moment Reduction Coefficient				Grashof-Rankine		"Trapezium"		"Parabola"	
	Case (*a*)		Case (*b*)							
n.	W_A	W_B	W_A	W_B	W_A	W_B	W_A	W_B	W_A	W_B
1·0	25	25	25	25	25	25	25	25	$33\frac{1}{3}$	$33\frac{1}{3}$
1·3	38	12	37	13	37	13	31	19	$33\frac{1}{3}$	26
1·5	42	8	42	8	42	8	33	17	$33\frac{1}{3}$	22
2·0	47	3	47	3	47	3	37	13	$33\frac{1}{3}$	17
3·0	49	1	49	1	50	—	42	8	—	—

Applying the "trapezium" method to the beams supporting panel P1, we have $L = 16$ ft. and $n = 1{\cdot}5$.

Dead load (Calculation Sheet No. 29) = 102 lb. per square foot
Live load (from *Table* 5) = 50

Total = 152

Total load on beams along the long edges of panel

$$= \text{load from two panels} = 2W_A$$

$$= 2\,(1{\cdot}5 - 0{\cdot}5)\,\frac{152 \times 16^2}{2} = 38{,}900 \text{ lb.}$$

Total load on beams along the short edges of panel

$$= \text{load from two panels} = 2W_B$$

$$= 2 \times \frac{152 \times 16^2}{4} = 19{,}450 \text{ lb.}$$

These loads, which exclude the dead load of the beam rib, will not be uniformly distributed along the beam. In the case of the beams along the short edges the distribution of the load is triangular; the loading diagram is a trapezium in the case of the beams along the long edge. With square slabs ($n = 1{\cdot}0$) the loading diagram for all four beams is triangular. In all cases the loading is symmetrically placed on the beam. (*Note:* Bending moment coefficients for beams carrying triangular loadings are given on *Tables* 7 and 9 in the author's "Reinforced Concrete Designers' Handbook." The corresponding coefficients for beams subject to "trapezium" load distribution are intermediate between those for triangular and uniformly distributed loading.)

It would probably be preferable in designs under the By-laws to base the beam loadings on the actual moment-reduction factors used. Thus in the above example the coefficients [Case (*a*)] are 0·581 and 0·107. The total load on the beam along each long edge of the panel

$$= \frac{2 \times 0{\cdot}581 \times 1{\cdot}5 \times 152 \times 16^2}{2(0{\cdot}581 + 0{\cdot}107)} = 49{,}500 \text{ lb.}$$

The total load on the beam along each short edge of the panel

$$= \frac{2 \times 0{\cdot}107 \times 1{\cdot}5 \times 152 \times 16^2}{2(0{\cdot}581 + 0{\cdot}107)} = 9{,}100 \text{ lb.}$$

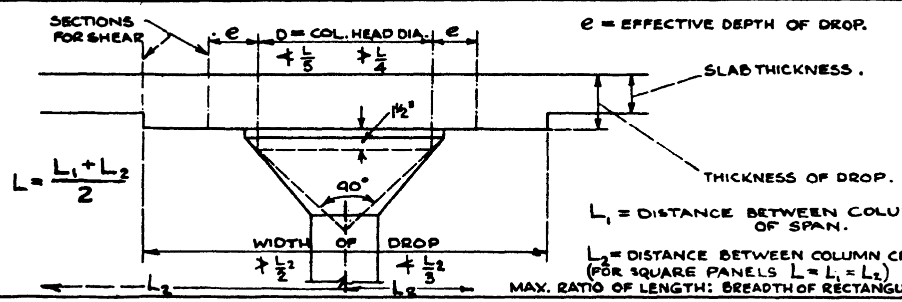
The shape of the loading diagrams can be considered as triangles and trapeziums as explained above.

TABLE 45.

BENDING MOMENTS FOR FLAT SLABS.

(By-laws and Memorandum.)

–	TYPE OF SLAB	WIDTH OF COLUMN HEAD (D)	WIDTH OF DROP (NORMAL TO L_1)	COLUMN STRIP			MIDDLE STRIP			B.M. FORMULA.
				WIDTH OF STRIP	NEG. B.M.	POS.VE B.M.	WIDTH OF STRIP	NEG. B.M.	POS.VE B.M.	
COEFFICIENTS FOR TOTAL BENDING MOMENT (f)	WITHOUT DROPS.	$\not< \frac{L}{5}$ $\not> \frac{L}{4}$	NIL	$\frac{L_2}{2}$	·042	·022	$\frac{L_2}{2}$	·018	·018	TOTAL B.M. $= f\omega L_2(L_1-\frac{2}{3}D)^2$
	WITH DROPS.	$\not< \frac{L}{5}$ $\not> \frac{L}{4}$	$\not< \frac{L_2}{3}$ $\not> \frac{L_2}{2}$	$\frac{L_2}{2}$ ($\frac{L_2}{3}$	·046 ·041	·022 ·017	$\frac{L_2}{2}$ $\frac{2L_2}{3}$	·016 ·021	·016 ·021)	
SQUARE PANEL: COEFFICIENTS. FOR BENDING MOMENT PER UNIT WIDTH (f_1)	WITHOUT DROPS.	$\frac{L}{5}$	NIL	$\frac{L}{2}$	·063	·033	$\frac{L}{2}$	·027	·027	SQUARE PANELS: B.M. PER UNIT WIDTH $= f_1 \omega L^2$ ω = TOTAL DEAD PLUS LIVE LOAD PER UNIT AREA.
		$\frac{L}{4}$	NIL	$\frac{L}{2}$	·058	·031	$\frac{L}{2}$	·025	·025	
	WITH DROPS.	$\frac{L}{4}$	$\frac{L_2}{2}$	$\frac{L}{2}$	·065	·031	$\frac{L}{2}$	·022	·022	
		$\frac{L}{5}$	$\frac{L_2}{2}$	$\frac{L}{2}$	·068	·033	$\frac{L}{2}$	·024	·024	
		$\frac{L}{4}$	$\frac{L_2}{3}$	$\frac{L}{3}$	·085	·035	$\frac{2L}{3}$	·022	·022	
		$\frac{L}{5}$	$\frac{L_2}{3}$	$\frac{L}{3}$	·092	·038	$\frac{2L}{3}$	·024	·024	



e = EFFECTIVE DEPTH OF DROP.

L_1 = DISTANCE BETWEEN COLUMN CENTRES IN DIRECTION OF SPAN.

L_2 = DISTANCE BETWEEN COLUMN CENTRES NORMAL TO L_1 (FOR SQUARE PANELS $L = L_1 = L_2$)

MAX. RATIO OF LENGTH: BREADTH OF RECTANGULAR PANELS = 1·33.

for both sections are given on the calculation sheets. When the panels are not square some adjustment should be made for the shear not being uniformly distributed throughout the perimeter of the critical sections, but in the present example the shear stresses are so low that adjustment is unnecessary. The permissible shear stresses on the critical sections are the same as for ordinary beams and slabs (see *Table* 21).

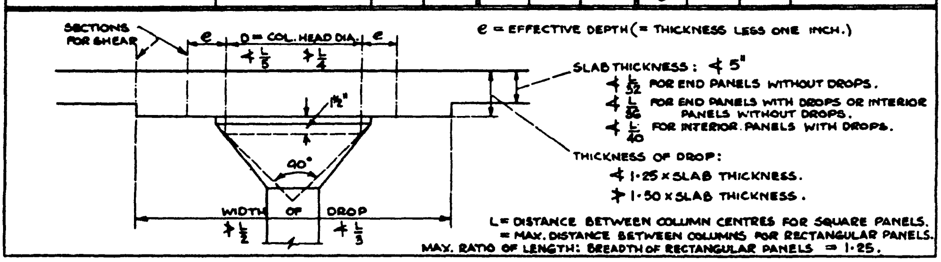
In view of the fact that there are a number of differences between the Memorandum and the Code, *Table* 46 has been prepared to give information comparable to that given for the Memorandum on *Table* 45. On the former the Code's recommendations relating to the limiting thicknesses of slabs and drops are indicated.

TABLE 46.

BENDING MOMENTS FOR FLAT SLABS.

(Code.)

	TYPE OF SLAB	WIDTH OF COLUMN HEAD (D)	WIDTH OF DROP	COLUMN STRIP: WIDTH OF STRIP	COLUMN STRIP: NEG. B.M.	COLUMN STRIP: POS.VE B.M.	MIDDLE STRIP: WIDTH OF STRIP	MIDDLE STRIP: NEG. B.M.	MIDDLE STRIP: POS.VE B.M.	B.M. FORMULA.
COEFFICIENTS FOR TOTAL BENDING MOMENT (f)	WITHOUT DROPS.	$\nless \frac{L}{5}$ $\ngtr \frac{L}{4}$	NIL	$\frac{L}{2}$	·042	·022	$\frac{L}{2}$	·018	·018	TOTAL B.M. $= f\omega L(L-\frac{2}{3}D)^2$
	WITH DROPS.	$\nless \frac{L}{5}$ $\ngtr \frac{L}{4}$	$\nless \frac{L}{3}$ $\ngtr \frac{L}{2}$	$\frac{L}{2}$	·046	·022	$\frac{L}{2}$	·016	·016	
COEFFICIENTS FOR BENDING MOMENT PER UNIT WIDTH (f_1)	WITHOUT DROPS.	$\frac{L}{5}$	NIL	$\frac{L}{2}$	·063	·033	$\frac{L}{2}$	·027	·027	B.M. PER UNIT WIDTH $= f_1 \omega L^2$.
		$\frac{L}{4}$	NIL	$\frac{L}{2}$	·058	·031	$\frac{L}{2}$	·025	·025	
	WITH DROPS.	$\frac{L}{4}$	$\frac{L}{2}$	$\frac{L}{2}$	·065	·031	$\frac{L}{2}$	·022	·022	
		$\frac{L}{5}$	$\frac{L}{2}$	$\frac{L}{2}$	·068	·033	$\frac{L}{2}$	·024	·024	
		$\frac{L}{4}$	$\frac{L}{3}$	$\frac{L}{3}$	·085	·035	$\frac{2L}{3}$	·022	·022	ω = TOTAL DEAD PLUS LIVE LOAD PER UNIT AREA.
		$\frac{L}{5}$	$\frac{L}{3}$	$\frac{L}{3}$	·092	·038	$\frac{2L}{3}$	·024	·024	



Calculation of Bending Moments.

For the purpose of assessing the bending moments on flat slabs, each panel is divided in each direction into column strips and a middle strip. The width of the column strip must be taken as half the width of the panel except where drops are provided, when it can be taken as equal to the width of the drop. The width of the middle strip must be taken as the width of the panel less the width of the column strip. The total bending moments specified apply to column and middle strips each equal to half the panel width. When the width of the column strip is taken as the width of the drop and is less than half the panel

width the total bending moments on the middle strip should be increased in proportion to its increased width, and the moments on the column strip can then be decreased by an amount such that there is no reduction in the total moments taken by the column and middle strips together.

Coefficients (f) for the total positive and negative moments for the column and middle strips of panels, with and without drops, are given on *Table* 45. These coefficients should be substituted in the expression

$$\text{B.M.} = fwL_2(L_1 - \tfrac{2}{3}D)^2$$

where L_1 = the distance between column centres in the direction of span,
L_2 = width between column centres normal to the direction of span,
w = combined dead and live load, and
D = effective diameter of column head.

To conform to the formulæ in the Code the coefficients given on *Table* 46 should be substituted in the expression

$$\text{B.M.} = fwL(L - \tfrac{2}{3}D)^2$$

where L = the span of the panel between centre line of columns ; in the case of panels which are not square the larger of the two dimensions is taken in accordance with the Code.

The foregoing expression gives the total moment to be taken on the width of each column and middle strip if the width of each strip equals half the span as in the case of panels without drops. In panels with drops, if the column strip is taken as equal to the width of the drop and the middle strip is increased in width accordingly, the moments on the middle strip must be increased in proportion to its increased width, while the moment to be taken by the column strip may be decreased by an amount such that there is no reduction in either the total negative or positive moments taken by the combined strips.

For square panels, the coefficients for the moment per foot width of column or middle strip resulting from the adoption of extreme cases of column head diameter and width of drop are also given on *Table* 45. The coefficients (f_1) should be substituted in the expression B.M. per foot width $= f_1wL^2$ where L = span of a square panel. Coefficients for intermediate cases of drop width and column head can be closely interpolated from the tabulated values.

The coefficients f_1 tabulated on *Table* 46, which apply principally to square panels for designs in accordance with the Code, can be substituted in the expression

$$\text{B.M.} = f_1wL^2$$

to determine directly the moment per foot width across the short span of rectangular panels, and by multiplying by the ratio $\dfrac{\text{long span}}{\text{short span}}$, the long-span moments can be derived. This is illustrated in the alternative methods of finding the moments for panels P1 (Calculation Sheet No. 37). The first method given illustrates the adoption of coefficients f and the formulæ tabulated in the Code.

The positive moments in strips normal to the discontinuous edge in end spans must be 25 per cent. in excess of those in interior spans, and the negative bending moment provided for at and normal to the discontinuous edge must be 90 per cent. in the column strip, and 60 per cent. in the middle strip, of the moments for interior spans. The slab thickness in end panels must not be less

than that in interior panels, but the use of compression steel in end panels is permitted when the positive moments are such that otherwise a thicker slab than for the interior panels would be required. For such panels the amount of positive moment tension steel may exceed 1 per cent.

Where end spans are shorter than interior spans, the Memorandum and the Code state that the moments " may be suitably modified," but no indication is given of how the modification shall be made. A reasonable method of dealing with end spans differing in length from interior spans is to compute the moments for an interior span of the same dimensions as the given end span and to increase the positive moments so derived by 25 per cent. One advantage in making end spans shorter than interior spans is that it enables the same slab thickness to be maintained throughout without the complication of introducing compression steel.

The calculations on Sheets 32 to 36 inclusive and the details in *Fig.* 25 relate to design in accordance with the Memorandum.

An alternative design in accordance with the Code recommendations for flat slabs is given in *Fig.* 26 and Calculation Sheets 37 to 41 inclusive, the latter allowing for the design factors appropriate to the modular ratio specified in the Code.

The following comments apply equally to the Code or Memorandum designs unless otherwise stated.

The calculations for panels P2 (Sheet No. 35) are based on those for panel P1. The short-span positive moments are found by increasing those in panel P1 by 25 per cent., since in this direction the panel is an end panel. The negative moments for the short span are identical with those for panel P1, as are also the negative and positive moments along the long span, except that in the half column strip adjacent to the external beam only a quarter of the total steel for the column strip need be provided ; that is, the area of the reinforcement per foot width need only be half the value calculated for panel P1. The positive moment in the column strip does not exceed the concrete resistance moment of the slab ; if there had been a deficiency this could be made up by providing compression steel, thus maintaining a constant slab thickness.

Negative moment steel must be provided adjacent to and normal to the non-continuous edges. The amount of this steel for the column strip should be at least 90 per cent., and at least 60 per cent. for the middle strip, of the corresponding negative moment steel for interior panels. This negative reinforcement and the positive reinforcement normal to the discontinuous edges must extend to within three inches of the edge of the panel.

The details for panel P3 are not reproduced but the calculation is prepared as follows. The bending moments are computed for an interior panel 18 ft. by 16 ft. with column strips 8 ft. wide in both directions. The middle strip is 10 ft. wide across the short span and 8 ft. wide across the long span. The positive and negative moments across the short span are the same as for an internal panel, except that in the half column strip adjacent to the edge beam the unit area of the reinforcement can be half that provided in the interior column strip. The negative moments at the interior support of the long span would also be the same as for an interior panel, but the negative moments in the column and middle strips at the outer support of this span must be not less than 90 and 60 per cent. respectively

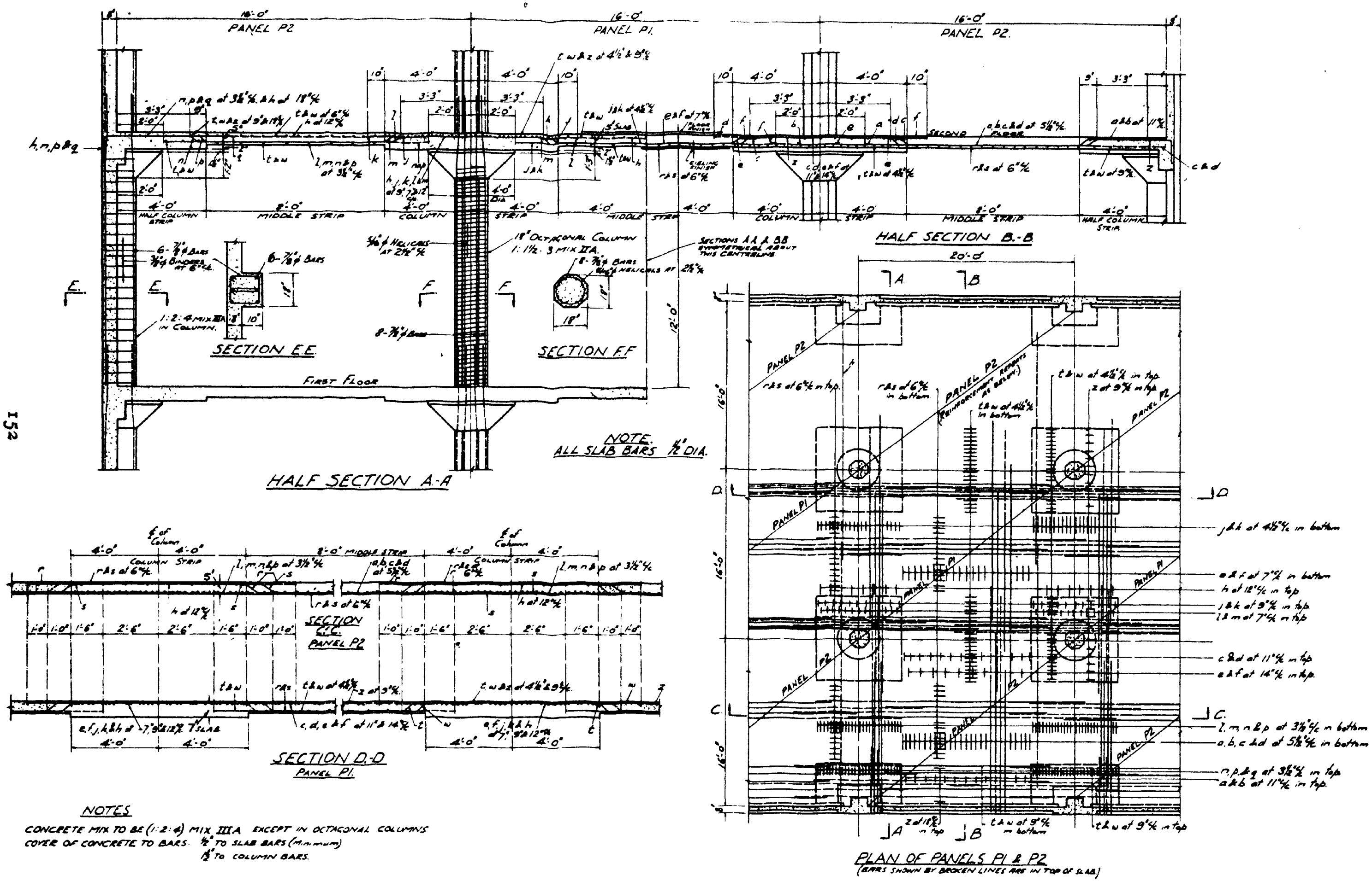


FIG. 25.—DETAILS OF FLAT-SLAB CONSTRUCTION.
(By-laws and Memorandum.)

CALCULATION SHEET NO. 32.
UPPER FLOORS.
FLAT-SLAB CONSTRUCTION.
(By-laws and Memorandum.)

PANEL P1.

Interior Panel
20′ × 16′
D = 48″ = L/5
L = 18 ft.
width of drop = 8 ft. = 20/2½
and = 16/2

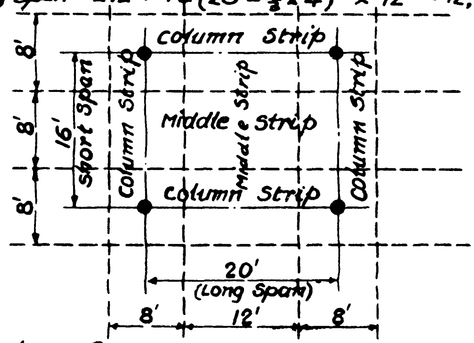
Loading: — Floor Finish = 10 lb./ft.²
Ceiling Finish = 6 " "
Concrete, say, = 96 " "
Partitions = 20 " "
Total Dead = 132 " "
Super. = 80 " "
Total = 212 " "

B.M. Calculation:-

$(\text{Span} \times \text{load})\ \text{Factor} = wL_2\left(L_1 - \frac{2}{3}D\right)^2 \times 12$

Short Span $= 212 \times 20\left(16 - \frac{2}{3} \times 4\right)^2 \times 12 = 9{,}000{,}000$

Long Span $= 212 \times 16\left(20 - \frac{2}{3} \times 4\right)^2 \times 12 = 12{,}000{,}000$



Long Span:-

Middle Strip: Width of strip = 8′ $\left(= \frac{\text{short span}}{2}\right)$

Total Pos. and Neg. B.Ms. (f = 0·016)

= ·016 × 12,000,000 = 192,000 in. lb.

= 192,000/8 = ±24,000 in. lb per ft. width

Column Strip: Width of Strip = 8′ $\left(= \frac{\text{short span}}{2}\right)$

Total Pos B.M. (f = ·022)

= ·022 × 12,000,000 = 264,000 in. lb.

= 264,000/8 = +33,000 in lb per ft. width

CALCULATION SHEET No. 33.
UPPER FLOORS.
FLAT-SLAB CONSTRUCTION.
(By-laws and Memorandum.)

PANEL P1 (cont.)

B.M. Calculation (cont):

Total Neg. B.M. ($f_1 = \cdot 046$)

$= \cdot 046 \times 12{,}000{,}000 = 552{,}000$ in lb.

$= 552{,}000/8 = -69{,}000$ in lb per ft width

Short Span:–

Middle strip: Width of strip $= 12' = \dfrac{\text{long span}}{1^2/_3}$

i.e. middle strip is 20 per cent wider than half span

Total Pos. and Neg. B.M. on width equal to half span ($f = \cdot 016$)

$= \cdot 016 \times 9{,}000{,}000 = 144{,}000$ in lb

Add 20 per cent $= 28{,}800$ " "

Total $172{,}800$ " "

$= \dfrac{172{,}800}{12} = \pm 14{,}400$ in lb per ft

Column Strip: Width of Strip $= 8$ ft. ($<$ half span)

Total Pos. B.M. (for width of strip = half span) $f = \cdot 022$

$= \cdot 022 \times 9{,}000{,}000 = 198{,}000$ in lb

Deduct addition to mid strip $= 28{,}800$ " "

$169{,}200$ " "

$= 169{,}200/8 = +21{,}200$ in lb per ft

Total Neg. B.M. (for width = half span) $f = \cdot 046$

$= \cdot 046 \times 9{,}000{,}000 = 414{,}000$ in lb

Deduct addition to mid strip $= 28{,}800$ " "

$385{,}200$ " "

$= 385{,}200/8 = -48{,}100$ in lb per ft

(For calculation of slab thickness and area of reinforcement see Calculation Sheet No 34.)

CALCULATION SHEET NO. 34.
UPPER FLOORS.
FLAT-SLAB CONSTRUCTION.
(By-laws and Memorandum.)

PANEL P1. (cont.)	Slab Thickness:- Normal effective depth required (per +ve. B.M., Column Strip, Long Span): $= d = \sqrt{\dfrac{33,000}{179 \times 12}} = 3{\cdot}9''$	TOTAL THICKNESS OF SLAB $= 5''$
	Shear around edge of drop:- Perimeter of drop $= 4 \times 8' \times 12 = 384$ ins. Total Area of panel $= 20 \times 16 = 320\ ft^2$ Area of drop $= 8 \times 8 = 64$ " Net area $= 256$ " Average shear stress $= \dfrac{256 \times 212}{384 \times 4 \times {\cdot}85} = 44\ lb/in^2$	
	Thickness of Drop:- Effective depth req'd (per -ve. B.M., Column strip, Long span) $= d = \sqrt{\dfrac{69,000}{179 \times 12}} = 5{\cdot}7''$	TOTAL THICKNESS OF DROP $= 7''$
	Shear at critical section at column head: Perimeter $= \pi(48'' + 2 \times 6'') = 188$ ins Total Area of panel $= 320\ ft^2$ Area of Col. Head section (5'-2" dia) $= 21$ " Net area 299 Average shear stress $= \dfrac{299 \times 212}{188 \times 6 \times {\cdot}85} = 66\ lbs/in^2$	
	Reinforcement. (Two-way system; long span steel nearest faces of slab)	Reinforcement Provided:—
	Long span:	
$d = 4{\cdot}25''$	Middle Strip +ve & -ve $\Big\}$ $A_T = \dfrac{24,000}{18,000 \times {\cdot}85 \times 4{\cdot}25} = {\cdot}368\ in^2$	$\frac{1}{2}''\phi$ at $6''$ c/c
$d = 4{\cdot}25''$	Col. Strip +ve. $= \dfrac{33,000}{18,000 \times {\cdot}85 \times 4{\cdot}25} = {\cdot}501$	$\frac{1}{2}''\phi$ at $4\frac{1}{2}''$ c/c
$d = 6{\cdot}25''$	-ve. $= \dfrac{69,000}{18,000 \times {\cdot}85 \times 6{\cdot}25} = {\cdot}722$	$\frac{1}{2}''\phi$ at $4\frac{1}{2}''$ c/c & $9''$ c/c
	Short span:	
$d = 3{\cdot}75''$	Middle Strip +ve. $= \dfrac{14,400}{18,000 \times {\cdot}85 \times 3{\cdot}75} = {\cdot}25$	$\frac{1}{2}''\phi$ at $7''$ c/c
$d = 4{\cdot}25''$	-ve. $= \dfrac{14,400}{18,000 \times {\cdot}85 \times 4{\cdot}25} = {\cdot}222$	$\frac{1}{2}''\phi$ at $11''$ & $14''$ c/c
$d = 3{\cdot}75''$	Col. Strip +ve $= \dfrac{21,200}{18,000 \times {\cdot}85 \times 3{\cdot}75} = {\cdot}37$	$\frac{1}{2}''\phi$ at $4\frac{1}{2}''$ c/c
$d = 5{\cdot}75''$	-ve $= \dfrac{48,100}{18,000 \times {\cdot}85 \times 5{\cdot}75} = {\cdot}55$	$\frac{1}{2}''\phi$ at $9''$, $7''$ & $12''$ c/c

CALCULATION SHEET No. 35.

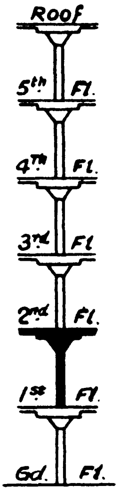
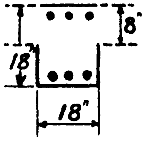
UPPER FLOORS.

FLAT-SLAB CONSTRUCTION.

(By-laws and Memorandum.)

PANEL P2.	Loading:- As for Panel P1	
	Thickness of Slab and Drop: As Panel P1	SLAB = 5" DROP = 7"
End Panel	Bending Moments and Reinforcement:-	Reinforcement Provided:-
Dimensions as Panel P1	Long Span:-	
Short Span non-continuous	Middle Strip +ve and -ve B.Ms as Panel P1	(+ve) ½" φ at 6" c/c (-ve) ½" φ at 6" c/c
	Column Strip along interior edge: +ve and -ve BMs as for Panel P1	(+ve) ½" φ at 4½" c/c (-ve) ½" φ at 4½" & 9" c/c
	Column strip along outer edge:	
	+ve BM: A_T = 50% of int^r strip $= \cdot 501/2 = \cdot 251$ ins^2/ft.	½" φ at 9"
	-ve B.M.: A_T = 50% of int^r strip $= \cdot 722/2 = \cdot 361$ ins^2/ft	½" φ at 9" & 18" c/c
	Short Span:-	
	Middle Strip +ve B.M.; $A_T = 1{\cdot}25 \times$ Panel P1 $= 1{\cdot}25 \times \cdot 25 = \cdot 32$ ins^2	½" φ at 5½" c/c
	Middle Strip -ve B.M. at interior support = as for Panel P1	½" φ at 14" & 11" c/c
	-ve B.M. at outer support = 60% of interior support $= \cdot 6 \times \cdot 222 = \cdot 143$ ins^2	½" φ at 11" c/c
	Column Strip +ve B.M. = $1{\cdot}25 \times$ Panel P1 $= 1{\cdot}25 \times 21{,}200 = 26{,}500$ in.lb (Check concrete resistance: $= 179 \times 12 \times 3{\cdot}75^2 = 30{,}300$)	
	$A_T = 1{\cdot}25 \times 0{\cdot}37 = \cdot 47$ ins^2	½" φ at 3½" c/c
	Column Strip:- -ve B.M. at interior support = as Panel P1	½" φ at 9", 7" & 12"
	-ve B.M. at outer support = 90% of interior support $A_T = 0{\cdot}9 \times \cdot 81 = \cdot 73$ ins^2	½" φ at 3½" & 12" c/c
	Shear: As for Panel P1.	

CALCULATION SHEET No. 36.
COLUMNS SUPPORTING FLAT SLABS.
(By-laws and Memorandum.)

<table>
<tr>
<td>INTERNAL COLUMNS

</td>
<td>BETWEEN 1ST & 2ND FLOOR
Loading:- lb.
Roof Live and Dead = $20 \times 16 \times 120 \text{ lb/ft}^2$ = 38,400
2nd to 5th Floors dead = $4 \times 20 \times 16 \times 132$ = 169,000
Do. Live (per Table 7) = $3.4 \times 20 \times 16 \times 50$ = 54,400
Total = 261,800
Column weight: 1st to Roof = 15,000
Total = 276,800
Total Bending Moment on Upper and Lower Cols.
= 50% of Column strip neg. B.M. (without adjustment for decreased width of strip.)
$= \cdot 5 \times \cdot 046\, wL_2(L_1 - \tfrac{2}{3}D)^2 \times 12$
Max.^{m} $= \cdot 5 \times \cdot 046 \times 212 \times 16(20 - \tfrac{2}{3} \times 4)^2 \times 12 = 282{,}000$ in. lb.
For columns of constant concrete section between ground and 3rd floors. B.M. on each
$= \tfrac{1}{2} \times 282{,}000 = 141{,}000$ in. lbs.
$e = 141{,}000/276{,}800 = 0{\cdot}52''$
Combined Stress:-
Area: Concrete = $\cdot 828 \times 18^2$ = 269 ins^2
Steel = $4{\cdot}81 \times 14$ = 68 "
337 "
M. of I: Concrete = $\cdot 055 \times 18^4$ = 5770 ins^4
Steel, say, $2{\cdot}5 \times 7{\cdot}5^2 \times 14$ = 1960 "
7730 "
Min. Section Modulus $= \dfrac{7730}{\cdot 54 \times 18}$ = 795 ins^3
Max. Stress $= \dfrac{276{,}800}{337} + \dfrac{141{,}000}{795} = 1000$ lbs/ins^2
(> 1100 per Table 8)</td>
<td>18" OCTAGONAL
1 : 1½ : 3
MIX II A
QUALITY A
8 - ⅞" ϕ Bars
5/16" ϕ HELICALS AT 2½" C/C</td>
</tr>
<tr>
<td>EXTERNAL COLUMNS
(SIDE WALLS)


Max.^{m} $L_2 = 16'$
$L_1 = 20'$</td>
<td>BETWEEN 1ST & 2ND FLOOR lb.
Approx. floor and roof loads $= \dfrac{261{,}800}{2}$ = 130,900
Panel walls and columns, say 53,000
Total Load = 183,900
Total B.M. = 90% of interior column strip neg. B.M.
$= \cdot 9 \times \cdot 046\, wL_2(L_1 - \tfrac{2}{3}D)^2 \times 12 = 510{,}000$ in. lb.
B.M. on each column $= 510{,}000/2 = 255{,}000$ in. lb.
$e = 1{\cdot}4''$ $e_1 = \cdot 078$. $p = 1{\cdot}11\%$.
Apply Case I (See Chapter VI): $Q_1 = 1{\cdot}2$
Max. stress $= \dfrac{183{,}900}{18 \times 18} \times 1{\cdot}2 = 680$ lb/in^2
(> 950 per Table 8)</td>
<td>18" SQUARE
(Rib Section)
1 : 2 : 4
MIX III A
QUALITY A
6 - ⅞" ϕ Bars</td>
</tr>
</table>

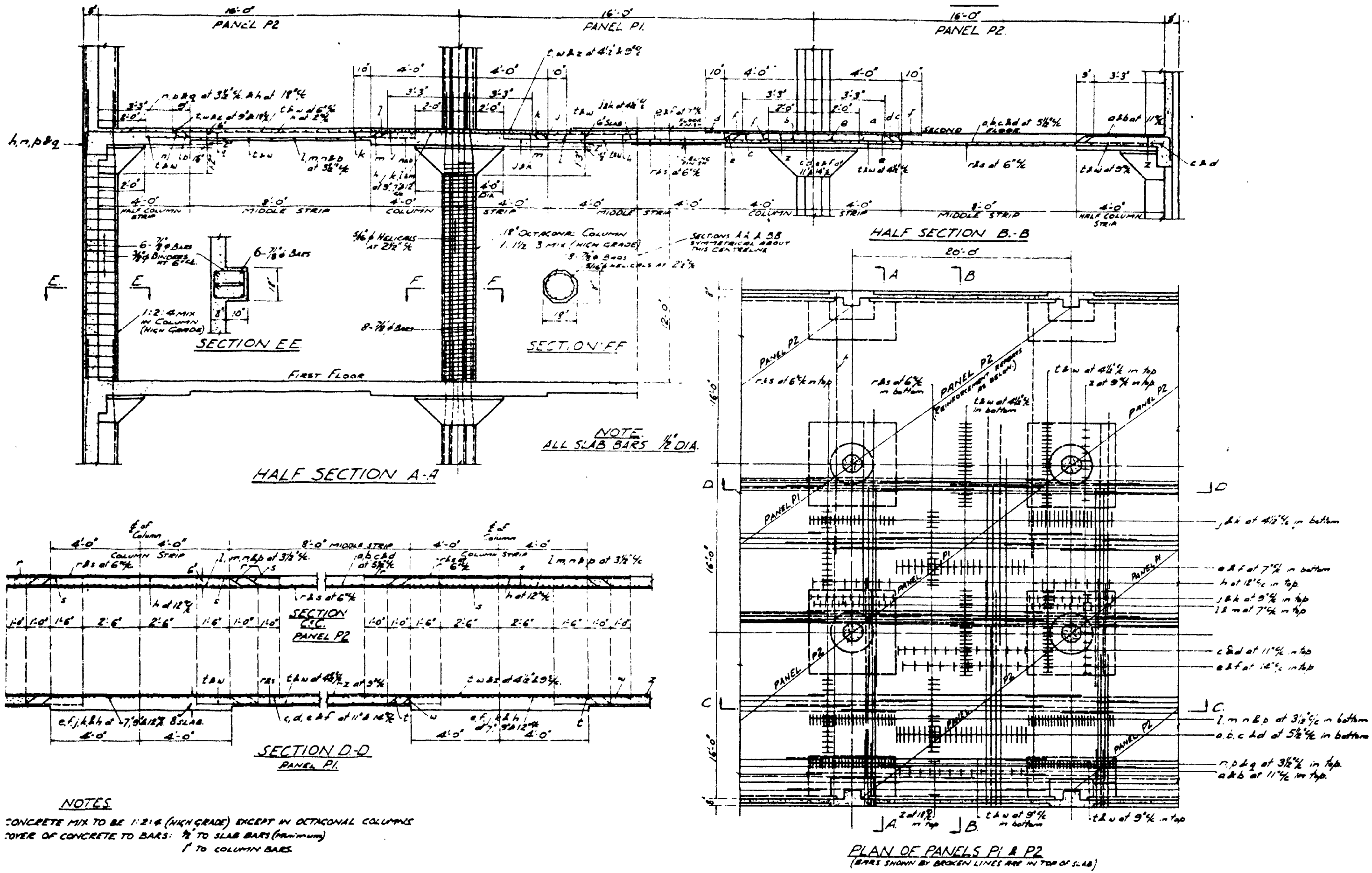


FIG. 26.—DETAILS OF FLAT-SLAB CONSTRUCTION.
(Code.)

CALCULATION SHEET NO. 37.

UPPER FLOORS.

FLAT-SLAB CONSTRUCTION.

(Code.)

PANEL P1.

Interior Panel

20′ × 16′

$D = 48'' = \frac{L}{5}$

$L = 20$ ft

width of drop $= 8$ ft $= \frac{L}{2\frac{1}{2}}$

(See Fig. 24 for Key Plan of Panels)

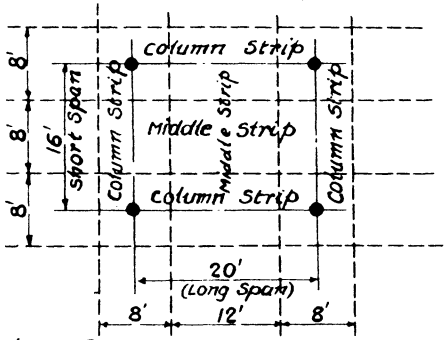
Loading: — Floor Finish = 10 lb./ft^2

Floor Finish	=	10 lb./ft^2
Ceiling Finish	=	6 " "
Concrete, say,	=	96 " "
Partitions	=	20 " "
Total Dead	=	132 " "
Super.	=	80 " "
Total	=	212 " "

B.M. Calculation:-

(Span × load) Factor $= wL(L - \frac{2}{3}D)^2 \times 12$

$= 212 \times 20(20 - \frac{2}{3} \times 4)^2 \times 12 = 15,300,000$



Long Span:-

Middle Strip: Width of strip $= 8'\left(= \frac{\text{short span}}{2}\right)$

Total Pos. and Neg. BMs. ($f = 0.016$)

$= \cdot 016 \times 15,300,000 = 245,000$ in. lb

$= 245,000/8 = \pm 30,630$ in lb per ft. width

(Alternative: $f_1 = \cdot 024$;

$B.M. = \cdot 024 \times 212 \times 20^2 \times 12 \times \frac{20}{16}$

$= 30,630$ in. lb.)

Column Strip: Width of Strip $= 8'\left(= \frac{\text{short span}}{2}\right)$

Total Pos BM ($f = \cdot 022$)

$= \cdot 022 \times 15,300,000 = 337,000$ in. lb.

$= 337,000/8 = +42,130$ in lb per ft width

(Alt^{ve}: $f_1 = \cdot 033$;

$B.M. = \cdot 033 \times 212 \times 20^2 \times 12 \times \frac{20}{16} = 42,130$ in. lb)

CALCULATION SHEET No. 38.

UPPER FLOORS.

FLAT-SLAB CONSTRUCTION.

(Code.)

PANEL P1 (cont.)

B.M. Calculation (cont):

Total Neg B.M ($f_1 = \cdot 046$)

$= \cdot 046 \times 15{,}300{,}000 = 704{,}000$ in lb.

$= \frac{704{,}000}{8} = -88{,}000$ in. lb per ft width

(Altve. $f_1 = \cdot 068$

B.M. $= \cdot 068 \times 212 \times 20^2 \times 12 \times \frac{20}{16} = 86{,}500$ in lb)

Short Span:-

Middle strip: Width of Strip $= 12' = \frac{\text{long span}}{1\frac{2}{3}}$

i.e middle strip is 20 per cent wider than half span

Total Pos. and Neg B.M on width equal to half span ($f = \cdot 016$)

$= \cdot 016 \times 15{,}300{,}000 \quad = 245{,}000$ in lb

Add 20 per cent $\quad = 49{,}000$ " "

Total $\quad 294{,}000$ " "

$= \frac{294{,}000}{12} = \pm 24{,}500$ in. lb per ft.

(Altve: f_1 (for width of drop $= \frac{L}{2\frac{1}{2}}$ & $D = \frac{L}{5}$) $= \cdot 024$

B.M $= \cdot 024 \times 212 \times 20^2 \times 12 = 24{,}500$ in lb)

Column Strip: Width of Strip = 8 ft. (< half span)

Total Pos. B.M. (for width of strip = half span) $f = \cdot 022$

$= \cdot 022 \times 15{,}300{,}000 = 337{,}000$ in. lb.

Deduct addition to mid strip $= 49{,}000$ " "

$288{,}000$ " "

$= \frac{288{,}000}{8} = +36{,}000$ in. lb per ft

(Altve: f_1 (for drop $= \frac{L}{2\frac{1}{2}}$ & $D = \frac{L}{5}$) $= \cdot 035$

B.M $= \cdot 035 \times 212 \times 20^2 \times 12 = 36{,}000$ in. lb)

Total Neg. B.M (for width = half span) $f = \cdot 046$

$= \cdot 046 \times 15{,}300{,}000 = 704{,}000$ in lb

Deduct addition to mid strip $= 49{,}000$ " "

$655{,}000$ " "

$= \frac{655{,}000}{8} = -81{,}880$ in. lb per ft

(Altve:- f_1 = mean of $\cdot 092$ and $\cdot 068 = \cdot 080$

B.M. $= \cdot 080 \times 212 \times 20^2 \times 12 = 81{,}400$ in lb)

(For calculation of slab thickness and area of reinforcement see Calculation Sheet No 39.)

Calculation Sheet No. 39.
UPPER FLOORS.
Flat-Slab Construction.
(Code.)

PANEL P1 (cont.)	Slab Thickness:- Normal effective depth required (per +ve. B.M., Column Strip, Long Span): $= d = \sqrt{\dfrac{42,130}{174 \times 12}} = 4.5''$ (Check $\not< 5''$ and $\not< \dfrac{span}{40} = \dfrac{20}{40} = 6''$)	TOTAL THICKNESS OF SLAB $= 6''$
$d - 6'' - 1'' = 5''$	Shear around edge of drop:- Perimeter of drop $= 4 \times 8' \times 12 = 384$ ins Total Area of panel $= 20 \times 16 = 320$ ft² Area of drop $= 8 \times 8 = 64$ " Net area $= 256$ " Average shear stress $= \dfrac{256 \times 212}{384 \times 5 \times \cdot 858} = 33$ lb/in²	
	Thickness of Drop:- Effective depth req'd (per − ve. B.M., Column strip, Long span) $= d = \sqrt{\dfrac{88\,000}{174 \times 12}} = 6 \cdot 5''$ (Check: $\not< 1 \cdot 25 \times 6'' = 7\tfrac{1}{2}''$, $\not> 1 \cdot 50 \times 6'' = 9''$)	TOTAL THICKNESS OF DROP $= 8''$
$d = 8'' - 1'' = 7''$	Shear at critical section at column head: Perimeter $= \pi(48'' + 2 \times 7'') = 194$ ins Total Area of panel $= 320$ ft² Area of Col Head section (5'-2" dia) $= 21$ " Net area 299 Average shear stress $= \dfrac{299 \times 212}{194 \times 7 \times \cdot 858} = 55$ lbs/in²	
	Reinforcement: (Two-way system; long span steel nearest faces of slab)	
	Long span:	
$d = 5 \cdot 25''$	Middle Strip +ve & −ve $A_T = \dfrac{30,630}{18,000 \times \cdot 858 \times 5 \cdot 25} = \cdot 379$ in²	$\tfrac{1}{2}''\phi$ at $6''$ c/c
$d = 5 \cdot 25''$	Col Strip +ve $= \dfrac{42,130}{18,000 \times \cdot 858 \times 5 \cdot 25} = \cdot 520$	$\tfrac{1}{2}''\phi$ at $4\tfrac{1}{2}''$ c/c
$d = 7 \cdot 25''$	−ve $= \dfrac{88,000}{18,000 \times \cdot 858 \times 7 \cdot 25} = \cdot 785$	$\tfrac{1}{2}''\phi$ at $4\tfrac{1}{2}''$ c/c & $9''$ c/c
	Short span:	
$d = 4 \cdot 75''$	Middle Strip +ve $= \dfrac{24,500}{18,000 \times \cdot 858 \times 4 \cdot 75} = \cdot 335$	$\tfrac{1}{2}''\phi$ at $7''$ c/c
$d = 5 \cdot 25''$	−ve $= \dfrac{24,500}{18,000 \times \cdot 858 \times 5 \cdot 25} = \cdot 303$	$\tfrac{1}{2}''\phi$ at $11''$ & $14''$ c/c
$d = 4 \cdot 75''$	Col Strip +ve $= \dfrac{36,000}{18,000 \times \cdot 858 \times 4 \cdot 75} = \cdot 490$	$\tfrac{1}{2}''\phi$ at $4\tfrac{1}{2}''$ c/c
$d = 6 \cdot 75''$	−ve $= \dfrac{81,880}{18,000 \times \cdot 858 \times 6 \cdot 75} = \cdot 786$	$\tfrac{1}{2}''\phi$ at $9''$, $7''$ & $12''$ c/c

CALCULATION SHEET No. 40.

UPPER FLOORS.

FLAT-SLAB CONSTRUCTION.

(Code.)

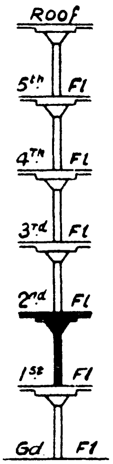
PANEL P2.	Loading:- As for Panel P1.	
	Thickness of Slab and Drop: As Panel P1	SLAB = 6"
End Panel	(Slab thickness $\not<$ $\frac{16 ft}{36}$ = $5\frac{1}{3}$ ins)	DROP = 8"
Dimensions as Panel P1	Bending Moments and Reinforcement:-	
Short Span non-continuous	Long Span:-	
	Middle Strip +ve and -ve B.Ms as Panel P1	(+ve) $\frac{1}{2}$" ϕ at 6" c/c (-ve) $\frac{1}{2}$" ϕ at 6" c/c
	Column Strip along interior edge: +ve and -ve BMs as for Panel P1	(+ve) $\frac{1}{2}$" ϕ at $4\frac{1}{2}$" c/c (-ve) $\frac{1}{2}$" ϕ at $4\frac{1}{2}$" & 9" c/c
	Column Strip along outer edge:	
	+ve BM: A_T = 50% of intr. strip = $\cdot 520/2$ = $\cdot 260$ ins²/ft.	$\frac{1}{2}$" ϕ at 9"
	-ve BM: A_T = 50% of intr. strip = $\cdot 785/2$ = $\cdot 393$ ins²/ft	$\frac{1}{2}$" ϕ at 9" & 18" c/c
	Short Span:-	
	Middle Strip +ve B.M; $A_T = 1\cdot 25 \times$ Panel P1 = $1\cdot 25 \times \cdot 333 = \cdot 417$ ins².	$\frac{1}{2}$" ϕ at $5\frac{1}{2}$" c/c
	Middle Strip -ve BM at interior support = as for Panel P1	$\frac{1}{2}$" ϕ at 14" & 11" c/c
	-ve B.M at outer support = 60% of interior support = $\cdot 6 \times \cdot 303$ = $\cdot 181$ ins²	$\frac{1}{2}$" ϕ at 11" c/c (provided)
	Column Strip +ve B.M = $1\cdot 25 \times$ Panel P1 = $1\cdot 25 \times 36{,}000$ = 45,000 in lb	
	(Check concrete resistance: = $174 \times 12 \times 4\cdot 75^2 = 47{,}000$)	
	$A_T = 1\cdot 25 \times 0\cdot 490 = \cdot 613$ ins²	$\frac{1}{2}$" ϕ at $3\frac{1}{2}$" c/c
	Column Strip:- -ve. B.M at interior support = as Panel P1	$\frac{1}{2}$" ϕ at 9", 7" & 12"
	-ve B.M. at outer support = 90% of interior support $A_T = 0\cdot 9 \times \cdot 786$ = $\cdot 707$ ins²	$\frac{1}{2}$" ϕ at $3\frac{1}{2}$" & 12" c/c
	Shear. as for Panel P1	

CALCULATION SHEET No. 41.

COLUMNS SUPPORTING FLAT SLABS.

(Code.)

INTERNAL COLUMNS



BETWEEN 1ST & 2ND FLOOR

Loading:- lb

Roof Live and Dead = $20 \times 16 \times 120$ lb/ft^2 = 38,400

2nd to 5th Floors dead = $4 \times 20 \times 16 \times 132$ = 169,000

Do Live (per Table 7) = $3{\cdot}4 \times 20 \times 16 \times 50$ = 54 400

Total = 261,800

Column weight. 1st to Roof = 15 000

Total = 276,800

Total Bending Moment on Upper and Lower Cols

= 50% of Column strip neg B·M (without adjustment for decreased width of strip.)

$= \cdot 5 \times \cdot 046\, wL(L - \tfrac{2}{3}D)^2 \times 12$

$= \cdot 5 \times \cdot 046 \times 212 \times 20(20 - \tfrac{2}{3} \times 4)^2 \times 12 = 351{,}900$ in. lb

For columns of constant concrete section between ground and 3rd floors, B.M. on each

$= \tfrac{1}{2} \times 351{,}900 = 175{,}950$ in lbs

$e = {}^{175{,}950}/_{276{,}800} = 0{\cdot}64''$

Combined Stress:-

Area: Concrete = $\cdot 828 \times 18^2$ = 269 ins^2

Steel = $4{\cdot}81 \times 11{\cdot}1$ = 53 "

322 "

M. of I: Concrete = $\cdot 055 \times 18^4$ = 5770 ins^4

Steel, say, $2{\cdot}5 \times 7{\cdot}5^2 \times 11{\cdot}1$ = 1560 "

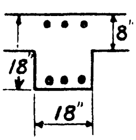
7330 "

Min. Section Modulus $= \dfrac{7330}{\cdot 54 \times 18} = 750$ ins^3

Max. Stress $= \dfrac{276{,}800}{322} + \dfrac{175{,}950}{750} = 1095$ lbs/ins^2 (c/1100 per Table 8)

18" OCTAGONAL
1 : 1½ : 3 MIX
HIGH GRADE
8 - ⅞" ϕ Bars
5/16" ϕ HELICALS AT 2½" c/c
(Safe Direct Load = 279,700 lb.)

EXTERNAL COLUMNS



BETWEEN 1ST & 2ND FLOOR

lb

Approx. floor and roof loads $= \dfrac{261{,}800}{2} = 130{,}900$

Panel walls and columns, say 53,000

Total Load = 183,900

Total B.M. = 90% of interior column strip neg. B.M.

$= \cdot 9 \times \cdot 046\, wL(L - \tfrac{2}{3}D)^2 \times 12 = 632{,}000$ in. lb.

B.M. on each column $= {}^{632{,}000}/_{2} = 316{,}000$ in lb

$e = 1{\cdot}72''.\quad e_1 = \cdot 096.\quad p = 1{\cdot}11\%.\quad h_o = \cdot 84$

$E = 0{\cdot}01 \times 1{\cdot}11(14{\cdot}03 - 1) = \cdot 145$

Max. stress $= \dfrac{183{,}900}{18 \times 18}\left(\dfrac{1}{1{\cdot}145} + \dfrac{6 \times \cdot 096}{1 + 3 \times \cdot 145 \times \cdot 84^2}\right) = 746$ lb/in^2

18" SQUARE
(Rib Section)
1 : 2 : 4 MIX
HIGH GRADE
6 - ⅞" ϕ Bars

of the inner support moments. The positive moments in both the column and middle strips across the long span are 25 per cent. in excess of the corresponding interior panel moments.

The calculations for panel P4 are based on those for panel P3, making allowances in the short-span positive and negative moments for lack of continuity along one of the longitudinal edges.

Checking the areas of reinforcement for panel P1 (Calculation Sheet No. 34), in the slab the maximum area of reinforcement required = 0·501 sq. in. per foot width, which is not greater than 1 per cent. of 12 × 4·25 = 0·51 sq. in. Similarly, in the drops the maximum area of 0·722 sq. in. does not exceed 1 per cent. of 12 × 6·25 = 0·75 sq. in. This limitation on the percentage of steel does not apply to end spans. Therefore, to resist the increased positive moment in the column strip of the short span of panel P2 (Calculation Sheet No. 40), 0·613 sq. in. can be provided, although this is in excess of 1 per cent. of 12 × (6 — 1) = 0.60 sq. in.

Arrangement of Reinforcement.

Two-way or four-way arrangements of the reinforcement in flat slabs is permissible. The former, consisting of two series of bars mutually at right angles and parallel to the sides of the panels, is the simpler. The four-way system has two diagonal bands of reinforcement in addition, and leads to four layers of steel over each column with consequent congestion of the bars and decrease in the lever arm. Such a system is only suitable for thick slabs to support heavy loads on large spans.

For the present design two-way reinforcement has been selected. With this arrangement each strip must be reinforced over its full width. Forty per cent. of the positive reinforcement must be continuous in the lower part of the slab and extend to within a distance of $\frac{L}{8}$ from the line joining the column centres. The full amount of negative reinforcement must be provided in the top for a distance measured from the line joining the column centres of not less than $\frac{L}{5}$, and the full area of positive reinforcement must be provided for a distance of $\frac{L}{4}$ on either side of the centre of the panel. The negative steel must extend in the top of the slab into adjacent panels for an average distance beyond the column centre line not less than $\frac{L}{4}$ and in no case less than $\frac{L}{5}$. In these ratios L represents the mean of the two spans of rectangular panels.

The minimum percentage of reinforcement in any strip is 0·3 per cent., and the maximum for interior panels 1·0 per cent. of the area represented by the product of the width of the strip and the effective depth. End anchorage must be provided at non-continuous edges of end panels.

The reinforcement detailed in *Fig.* 25 has been arranged to comply with the appropriate recommendations.

The Code recommends that 50 per cent. of the positive reinforcement should be continuous in the bottom of the slab and that the remaining 50 per cent. should be bent up to assist in providing negative moment reinforcement. Subject

to this variation, the Code requirements comply with the Memorandum except that L represents the longer span of rectangular panels. The reinforcement detailed on *Fig.* 26 complies with the recommendations of the Code.

Beams, Openings, and Partitions.

The beams at the edges of panels P2, P3, and P4 must be designed to carry any direct load together with a portion of the panel loads. In the case of approximately square panels each beam would carry at least one-quarter of the total panel load in addition to any direct load. No recommendation in the Memorandum applies to beams supporting rectangular panels, such as those now being designed, but it might be reasonable to apply a modification of the trapezium method of distribution as discussed in the consideration of slabs spanning in two directions.

The Memorandum allows openings of limited size to be provided in flat slab floors, but such openings should not encroach upon a column head or drop. Openings must be trimmed on all sides with beams to carry the loads to the columns, unless in the case of openings in an area common to two middle strips the greatest dimension parallel to the panel centre line does not exceed $\frac{2L}{5}$. Openings in an area common to two column strips do not require to be framed with beams if the aggregate length or width of the openings does not exceed 10 per cent. of the width of the column strip. Similarly openings without beams may be formed in an area common to one column strip and one middle strip if the aggregate lengths or widths do not exceed 25 per cent. of the width of the strip.

Sufficient resistance must be provided in the reduced section of the slab on either side of the openings to take the total specified negative and positive moments. Since the stair openings in the present building exceed the permissible sizes, the panels in which they occur are framed bv beams as indicated in *Fig.* 24.

The Code requires beams to be provided to carry partitions and walls when the weight exceeds 5 per cent of the sum of the dead and live loads on any panel. The weight of the 9-in. brick partition wall, 18 ft. long and 11 ft. 6 in. high, adjacent to the stair well is 90 lb. $\times$ 11·5 $\times$ 18 = 18,600 lb. The total panel load is 18 $\times$ 16 $\times$ 212 lb. = 61,000 lb. Since the weight of the wall is considerably in excess of 5 per cent. of the total panel load, it must be carried on a beam. The slab panels thus formed would be designed as simple one-way or two-way slabs.

Columns supporting Flat Slabs.

The limiting size of column heads has already been described and given on *Table* 45. Exterior wall columns must be provided with such portion of the enlarged head as will lie within the adjoining walls, and the Memorandum should be consulted for details of heads that are not completely circular or similar. The details in *Fig.* 25 show the permissible arrangement for the present scheme.

Both internal and external columns supporting flat slabs must be designed to take bending. The value of the moment is taken as 50 per cent. of the negative moment in the column strip for internal columns and 90 per cent. for external

columns. The total moment is divided between the upper and lower columns at any floor level in proportion to their relative stiffnesses. The calculations on Sheet No. 36 and the appropriate details in *Fig.* 25 indicate the method of designing typical interior and exterior columns supporting flat slabs. The column sections are designed for the modular ratio and stresses specified in the By-laws and explained in Chapter V.

The calculations on Sheet No. 41 give the corresponding design for columns in accordance with the Code.

CHAPTER X

GROUND FLOOR AND STAIRS

Ground Floor—Garage Portion.

It was established in Chapter I that the slab of the ground floor (garage portion) should be designed for superimposed loads in excess of the minimum of 150 lb. per square foot specified in By-law 4 for this type of floor (Class No. 5). The curves (*Table* 3) for a 1-ton wheel load were drawn for a simply-supported span and a span fixed at both ends, whereas in the example (*Fig.* 27) the slab is continuous over a number of 8-ft. spans and is between these extremes. If the midspan and support sections are designed for the same moment the mean of the tabulated equivalent uniformly distributed loads at midspan and support of a fixed-end span would be a reasonable compromise. For a 1-ton load the

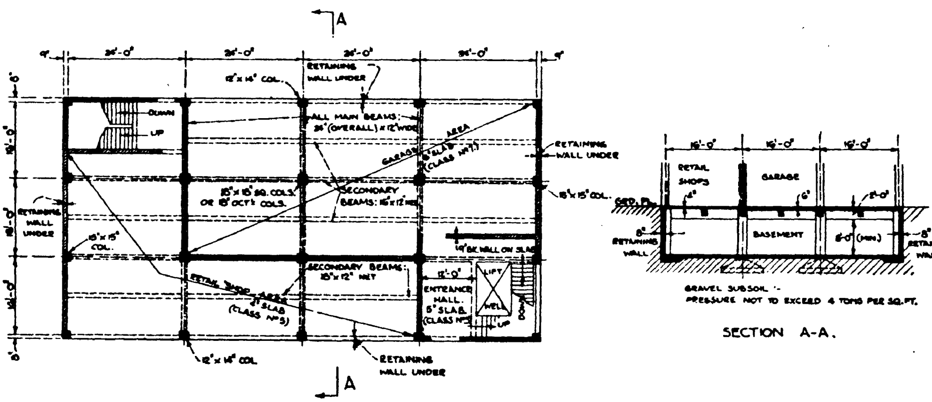


Fig. 27.—Ground Floor.

equivalent loads for an 8-ft. span are 162 lb. per square foot for the support and 242 lb. per square foot for midspan, the mean being 202 lb. per square foot. As the maximum wheel load plus 50 per cent. for a motor coach is 3·56 tons (*Table* 4), the equivalent superimposed uniformly distributed load is $3{\cdot}56 \times 202 = 720$ lb. per square foot. This assumes that the wheel load is spread over an area 2 ft. 6 in. square and that in any width of 2 ft. 6 in. only one wheel load can occur on any one span. If more or less than a 2-ft. 6-in. distributive width is permissible the load of 720 lb. per square foot must be adjusted pro rata. An inspection of the possible position of wheels on an 8-ft. span shows that the most severe bending moments occur when one central wheel load is in position. A further inspection

of the possible arrangement of vehicles on the ground floor as a whole indicates that the sequences of loading necessary to produce maximum moments (*Fig.* 2) can be realised. Thus, allowing for dead load and finishes, the maximum bending moment is $\frac{800 \times 8^2 \times 12}{12} = 51{,}200$ in.-lb. With Quality A, Mix IIIA, (1 : 2 : 4) concrete and a stress of 18,000 lb. per square inch on the reinforcement the resistance moment factor (*Table* 8) is 179, and the required effective depth of slab is $\sqrt{\frac{51{,}200}{179 \times 12}} = 5{\cdot}0$ in. A 6-in. slab is provided.

The loading on the beams supporting the garage floor requires careful consideration to ensure that the worst possible combination of wheel loads is taken. For a single wheel placed at the middle of any span of secondary beams *Table* 6 can be used, and it will be seen that a 3·56-ton wheel load would not give a greater equivalent uniformly distributed load than that from an 8-ft. panel of slab loaded at the minimum rate of 120 lb. per square foot (= 960 lb. per foot run = 23,000 lb. on a span of 24 ft.). Any beam may, however, support simultaneously a series of wheel loads, and the incidence depicted in *Fig.* 28(*a*) represents a possible maximum. The total load of this arrangement = 6 × 3·56 × 2,240 = 48,000 lb. Assuming a reduction factor of, say, 0·95 (*Table* 6) to allow for part of the load being taken on the adjacent beams, and including a factor of 1·75 to account for the non-uniform distribution of the loading shown in *Fig.* 28(*a*), the equivalent total uniform load = 0·95 × 1·75 × 48,000 = 80,000 lb., which considerably exceeds the 23,000 lb. due to the normal superimposed uniformly distributed load. The factor 1·75 can be assessed by comparing the free moment due to the critical loading or by inspecting the relation between the coefficients for point loads and uniform loads given on *Tables* 13, 14, and 15. The foregoing calculations are based on the wheel load being spread on an area 2 ft. 6 in. square and should be modified if any other area is stipulated.

On a 24-ft. span the total dead load is about 20,000 lb. and gives a total load of 80,000 + 20,000 = 100,000 lb. Allowing for the full 15 per cent. reduction of support moments, the maximum moments are found by employing coefficients estimated from *Table* 14 (four equal spans, uniform load, $R = 4$). The maximum positive bending moment (at A) is 0·114 × 100,000 × 24 × 12 = 3,280,000 in.-lb. The maximum negative bending moment (at B) is

$$0{\cdot}098 \times 100{,}000 \times 24 \times 12 = 2{,}820{,}000 \text{ in.-lb.}$$

If a beam section 12 in. wide and 16 in. deep below the slab is adopted, similar calculations for the upper floor beams show that seven 1½-in. bars are required at the middle of the end spans. At the penultimate supports, seven 1½-in. bars are necessary both in tension and compression.

The loading in *Fig.* 28(*a*) would also give practically the maximum shear forces (this can be checked by taking alternative arrangements of the wheels); that is, the maximum shear force (using a factor of 0·65 to allow for elastic reaction—estimated from *Table* 19) = 0·65 [(0·95 × 48,000) + 20,000] = 43,500 lb. On the 16-in. by 12-in. net beam section, the unit shear stress would be less than the maximum allowable (380 lb. per square inch by *Table* 21), but requires the provision of shear reinforcement.

The alternative superimposed loadings for the main beams are:

(1) The minimum load of 120 lb. per square foot (*Table* 1) = 120 × 24 × 8 = 23,000 lb. total, acting as a central point load;

(2) The whole floor covered by the heaviest vehicles likely to use the garage; say, 8 tons each plus 50 per cent. impact on, say, 200 sq. ft. each, which gives 135 lb. per square foot = 26,000 lb. at the centre.

(3) The wheels concentrated as adversely as possible over the beam. This arrangement of loading is shown in *Fig.* 28(*b*), and the total load would be approximately 4 × 3·56 × 2,240 = 31,900 lb. Although this load is less concentrated

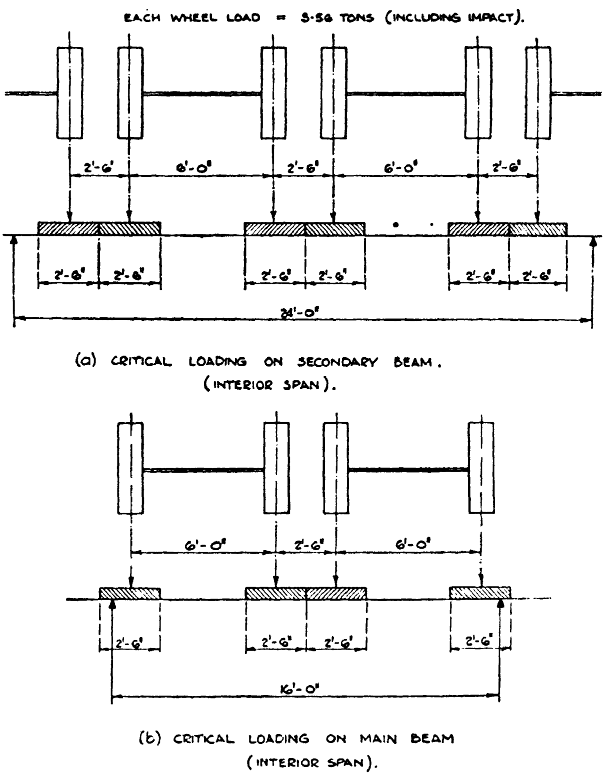


FIG. 28.—LOADING ON GARAGE FLOOR BEAMS.

at one point on the beam than the load of 26,000 lb. acting through the secondary beams, this latter loading will control the bending resistance of the beam, while the shear resistance will depend on the 31,900-lb load. A beam section 12 in. wide projecting 18 in. below the 6-in. slab would be suitable, giving a beam not exceeding the maximum of 2 ft. overall constructional depth specified in *Fig.* 27.

For the design of the garage portion of the ground floor in accordance with the Code the following amendments to the foregoing calculations are required.

With High-grade 1 : 2 : 4 concrete and a stress of 18,000 lb. per square inch in the reinforcement, the resistance moment factor is 174 (*Table* 9) and therefore

an effective depth of 5 in. for the slab will still be required. Since the minimum live load on garage floor beams of this class is 200 lb. per square foot instead of 120 lb. per square foot permitted by the By-laws, the minimum total live load on a 24-ft. span of secondary beam would be $200 \times 8 \times 24 = 38{,}400$ lb. The equivalent loading from the worst wheel arrangement will however considerably exceed this figure; hence the design bending moments for the secondary beams will be identical with the design in accordance with the By-laws. In accordance with the Code, the corresponding superimposed loadings for the main beams are:

(1) The minimum load of 200 lb. per square foot $= 200 \times 24 \times 8 = 38{,}400$ lb. total, acting as a central point load;

(2) The whole floor covered by the heaviest vehicles likely to use the garage; say, 8 tons each plus 50 per cent. impact on, say, 200 sq. ft. each, which is less than 200 lb. per square foot; and

(3) With the wheels concentrated as adversely as possible over the beam as shown in *Fig.* 28(*b*), the total load would be approximately $4 \times 3{\cdot}56 \times 2{,}240 = 31{,}900$ lb. As well as being less in total value, this load is less concentrated at one point on the beam than the minimum superimposed load of 38,400 lb. acting through the secondary beams. In this case, then, the minimum superimposed load controls the design of the main beams both for bending and shear.

Entrance Hall.

In accordance with *Fig.* 27 the entrance hall is principally a slab panel 12 ft. by 16 ft. that can be considered as continuous on three sides and freely supported along the longer edge adjacent to the lift and stair well. It is therefore convenient to design this panel as a slab spanning in two directions as described in Chapter VIII. The equivalent ratio of sides $= \dfrac{\frac{2}{3} \times 16}{0{\cdot}8 \times 12} = 1{\cdot}1$, for which Case (*a*) moment coefficients (*Table* 44) are 0·045 for the short span and 0·030 for the long span.

The loading for this type of floor falls under Class No. 3 (*Tables* 1 and 3), that is, entrance floor of offices, the minimum superimposed load for which is 80 lb. per square foot. The smaller 12-ft. span controls the alternative superimposed load, but reference to *Table* 2 indicates that for this span the load of 80 lb. per square foot is applicable. Allowing a 5-in. slab and 22 lb. per square foot for floor and ceiling finishes, the total load is 162 lb. per square foot. On this panel of the entrance hall there are no partitions, the latter being carried on the beams.

The negative moment at the support for the short span is

$$0{\cdot}045 \times 162 \times 12^2 \times 12 = 12{,}650 \text{ in.-lb.},$$

and the midspan positive moment is 90 per cent. of $12{,}650 = 11{,}385$ in.-lb.

The negative moment at the support for the long span is

$$0{\cdot}030 \times 162 \times 16^2 \times 12 = 14{,}950 \text{ in.-lb.},$$

while the midspan positive moment is 80 per cent. of $14{,}950 = 11{,}960$ in.-lb. A 5-in. slab is ample for these moments.

Ground Floor—Retail Shop Portion.

The remaining ground floor areas are allocated to retail shops (Class No. 5, *Table* 1), the minimum superimposed load on which is 80 lb. per square foot.

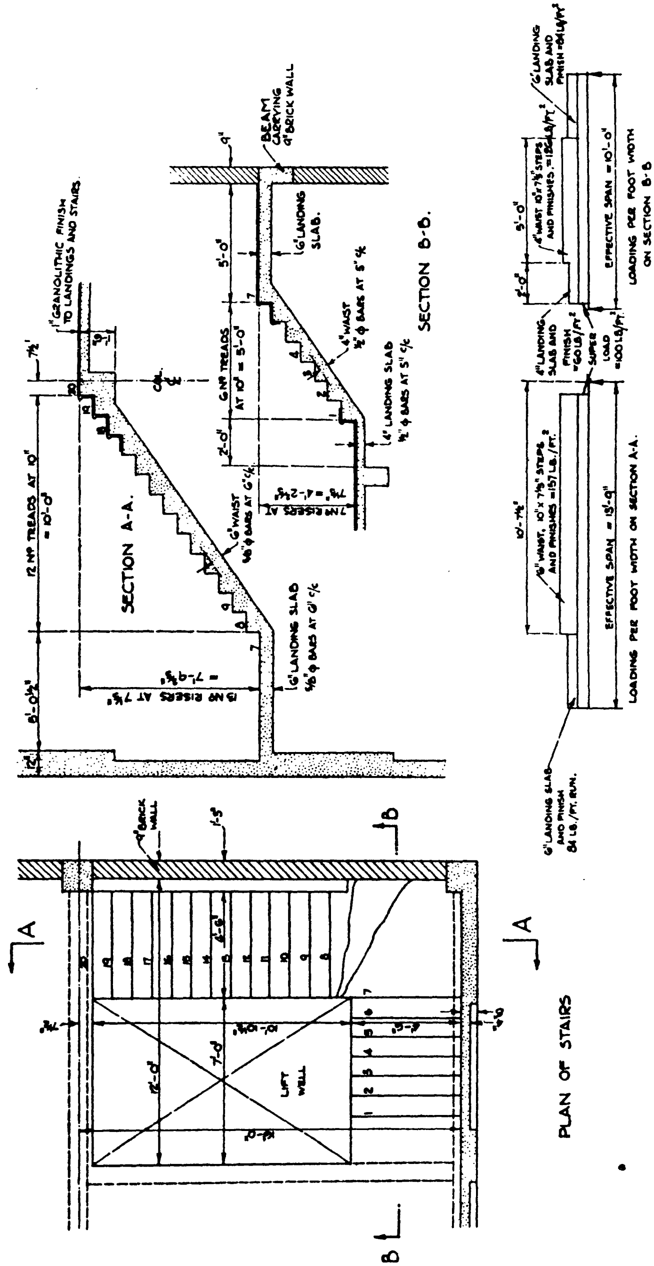


FIG. 29.—DETAIL OF STAIRS.

A slab span of 8 ft., however, is controlled by the alternative superimposed load which (*Table* 2) is equivalent to 105 lb. per square foot. The compulsory minimum additional dead load of 20 lb. per square foot for partitions is only applicable to office floors, but this allowance will be included in the present calculation together with 75 lb. per square foot for the weight of the slab and finishes, giving a total load of 200 lb. per square foot. The corresponding moment is

$$0{\cdot}083 \times 200 \times 8^2 \times 12 = 12{,}800 \text{ in.-lb.}$$

If the alternative consideration, in accordance with the assumptions upon which *Table* 43 is based, is taken, the maximum live load moment is $756 \times 8 = 6{,}048$ in.-lb. The dead load moment is $0{\cdot}083 \times 95 \times 8^2 \times 12 = 6{,}080$ in.-lb., and the total moment is $6{,}048 + 6{,}080 = 12{,}128$ in.-lb. for which a 4-in. slab suffices.

Staircase.

By confining attention to the principal staircase adjacent to the lift well, the points in the By-laws affecting staircase design can be adequately covered. Staircases and landing loads are included in Class No. 4 (*Table* 1), for such cases as that under consideration, the superimposed load for which is 100 lb. per square foot. The critical span below which the alternative superimposed loading controls is 8 ft. 3 in. (*Table* 2). If, instead of providing stringer beams, these stairs are designed to span from top to bottom of each flight, the effective spans (taken to the centre of the common landing) are approximately 13 ft. 9 in. and 10 ft. (*Fig.* 29) and the minimum load of 100 lb. per square foot applies. Making allowances for the dead loads and the weight of the granolithic finish on the landings, treads, and risers of the stairs, the loads on each flight are those shown in *Fig.* 29. The longer flight will be supported at the top on the beam trimming the lift and stair well, and at the bottom (at landing level) on the external wall which here, to act as a support, should be made thicker than the minimum of 4-in. specified elsewhere. The shorter flight can be supported at the bottom on the beam trimming the lift well and on the wall beam at landing level. An appreciable, but indefinite, amount of the load will be carried directly by the external wall and by the beam carrying the brick wall.

The principal moments in both flights can be calculated by taking a coefficient of $\frac{1}{10}$ (owing to the indefinite continuity at each support), and by adopting the stresses of 950 lb. per square inch and 18,000 lb. per square inch appropriate to 1 : 2 : 4 Mix IIIA concrete and ordinary rolled mild steel bars the sections and reinforcement in the stairs and landings as indicated in *Fig.* 29 can be determined, the detailed calculations being similar to those already given for slabs.

CHAPTER XI

FOUNDATIONS

THE principal types of building foundations are:

(1) Separate column footings, constructed either in reinforced concrete or in plain concrete.

(2) A reinforced concrete strip footing or raft foundation for two or more columns when the permissible ground pressure is low or when one or more of the columns is close to the boundary of the site.

(3) A piled foundation when good ground occurs at some depth below the surface.

These three primary classes allow of considerable variation to suit different conditions. A number of typical problems is dealt with in this chapter, including the design of a typical foundation for an internal column of the building illustrated in *Fig.* 1.

These designs are prepared in accordance with the requirements of the By-laws and Memorandum where clauses relevant to such designs exist. If similar designs are required in accordance with the Code the principal variations would be the numerical values of the design factors for bending, but there would be practically no difference in the concrete sections or amount of reinforcement.

In the design of foundations the Memorandum requires that eccentricity of loading and horizontal or other forces that may affect the distribution of ground pressure and cause uplift, sliding, or overturning should be taken into account. These various requirements are conformed to where applicable to the design of the foundations and retaining walls.

Plain Concrete Footing for Column.

A conventional type of column foundation for small loads and high ground pressures consists of a small reinforced concrete footing bearing on a block of plain concrete as illustrated in *Fig.* 30. Consider the foundation for a reinforced concrete column 16 in. square reinforced with four $\frac{7}{8}$-in. bars and carrying an axial load of 100 tons, the concrete being 1 : 2 : 4 Quality A, Mix IIIA. The upper reinforced concrete footing will be made from the same class of concrete and is designed to spread the column load on to the lower plain concrete footing, the latter in turn distributing the load on the ground. It is assumed that the ground is capable of taking a safe pressure of 4 tons per square foot.

The lower footing will be of plain concrete of Mix IV (1 : 6), that is, $7\frac{1}{2}$ cu. ft. of combined fine and coarse aggregate to 1 cwt. of Portland cement (see *Table* 8 and By-law 14). According to By-law 35, the safe bearing pressure on such a

concrete is 20 tons per square foot. Thus the minimum area of the reinforced footing will be $\frac{100 \text{ tons}}{20 \text{ tons per sq. ft.}} = 5$ sq. ft., say 2 ft. 6 in. square. The depth of this footing should be sufficient to spread the load at 45 deg. from the

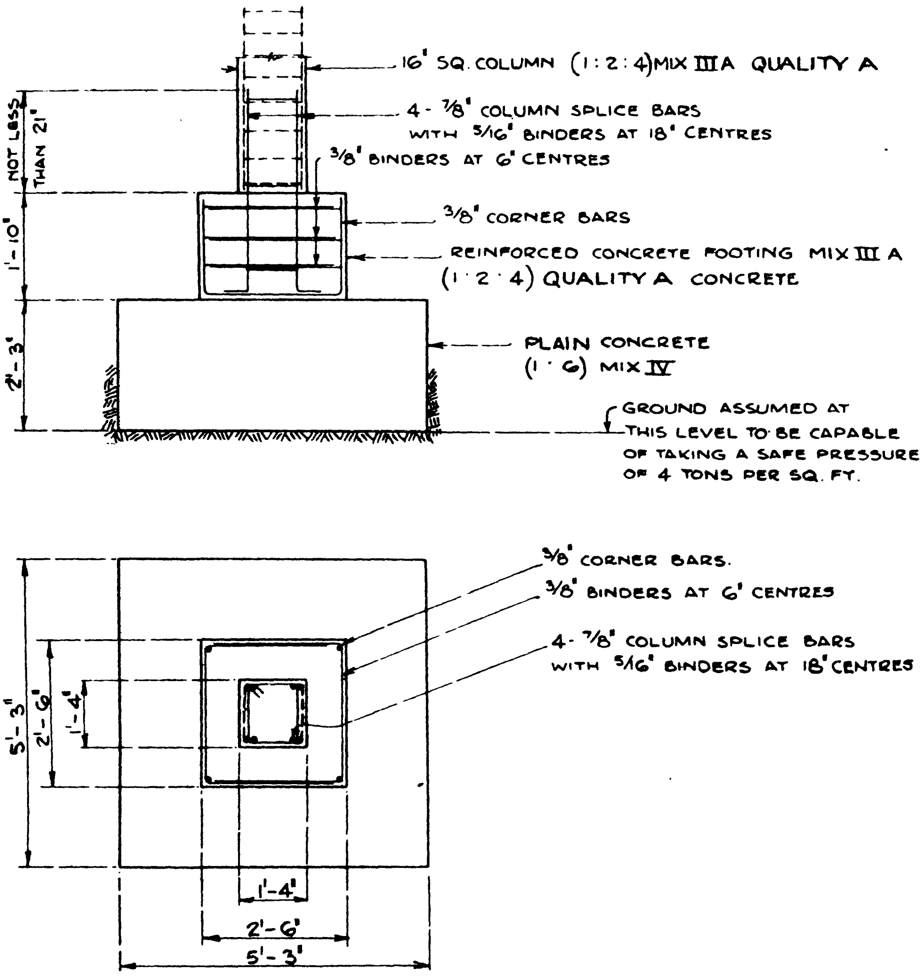


FIG. 30.—PLAIN CONCRETE FOOTING.

16-in. column to the 2-ft. 6-in. bearing plane, that is, the depth should not be less than $\frac{30 \text{ in.} - 16 \text{ in.}}{2} = 7$ in. The depth will also be controlled by the permissible punching shear stress which in accordance with By-law 99 must not exceed twice the ordinary shear stress. The amount of load producing punching shear around the perimeter of the column is

$$\frac{2 \cdot 5^2 - 1 \cdot 33^2}{2 \cdot 5^2} \times 100 \times 2{,}240 = 160{,}000 \text{ lb.}$$

The permissible punching shear stress (see *Table* 21) for the concrete mix used is 190 lb. per square inch. Thus the minimum shear area required is $\frac{160,000}{190} =$ 842 sq. in. The perimeter of the column being $4 \times 16 = 64$ in., the depth of the footing must not be less than $\frac{842}{64} = 13{\cdot}1$ in.

It is also necessary to make the footing deep enough to ensure that the $\frac{7}{8}$-in. diameter column splice bars shall be embedded for a minimum length of 24 diameters, which equals 21 in. Thus by making the upper footing 2 ft. 6 in. square and 1 ft. 10 in. deep all the foregoing requirements are complied with. The footing need only be nominally reinforced as shown in *Fig.* 30.

To transmit the column load to the ground at a pressure not exceeding 4 tons per square foot the minimum area of the mass concrete footing is $\frac{100}{4} = 25$ sq. ft., say 5 ft. 3 in. square, allowing a margin for the weight of the footing itself. To disperse the load at 45 deg. as specified in By-law 33, the depth of the footing should not be less than $\frac{5 \text{ ft. } 3 \text{ in.} - 2 \text{ ft. } 6 \text{ in.}}{2} = 1$ ft. $4\frac{1}{2}$ in. The load producing punching shear on a perimeter of 4×2 ft. 6 in. $= 120$ in., is

$$\frac{5{\cdot}25^2 - 2{\cdot}5^2}{5{\cdot}25^2} \times 224,000 = 173,000 \text{ lb.}$$

Taking the safe punching shear stress on Mix IV (1 : 6) plain concrete at two-thirds of that on Mix III (1 : 2 : 4) ordinary concrete (the safe bearing pressure on the former being two-thirds of that on the latter), the working stress will be $\frac{2}{3} \times 2 \times 75 = 100$ lb. per square inch. The depth of the footing should not be less than $\frac{173,000}{100 \times 120} = 14{\cdot}5$ in. A reasonably proportioned base would be 5 ft. 3 in. square by 2 ft. 3 in. deep.

Sometimes large mass concrete footings of this type are strengthened by inserting a mesh of reinforcement in the form of bars, fabric, old rails, wire ropes, or other suitable material, this reinforcement being provided near the lower face of the mass concrete footing. A further precaution is sometimes taken to bond the column to the mass concrete footing by providing vertical splice bars, rails, or similar material extending upwards from the lower footing into the column shaft.

Reinforced Concrete Footing for Column.

As an example of the design of a column footing in the orthodox form of a truncated pyramid, the foundation of column A (*Fig.* 9 and Calculation Sheet No. 9) will be considered. This foundation is reproduced in *Fig.* 31, and in Chapter IV calculations are given for ground pressure, for the shear stress on planes $X-X$ and $Y-Y$ and for the depth required to provide sufficient bond length for the column bars.

It only remains to calculate the moment of resistance of the base and the amount of reinforcement required to resist the bending moments produced by the ground pressure. Considering the upward ground pressure on the shaded

area in *Fig.* 31, the total bending moment in inch-pounds about the line Z—Z for a square base is given by the expression

$$M = \frac{WA}{24}(2 + R)(1 - R)^2$$

where W = the column load in lb. (= 630,000 lb. in this example)

A = the length of the side of the footing in inches (= 8 ft. 6 in. = 102 in.)

$$R = \frac{\text{size of column}}{\text{size of base}}\left(= \frac{1 \text{ ft. } 9 \text{ in.}}{8 \text{ ft. } 6 \text{ in.}} = 0{\cdot}206\right).$$

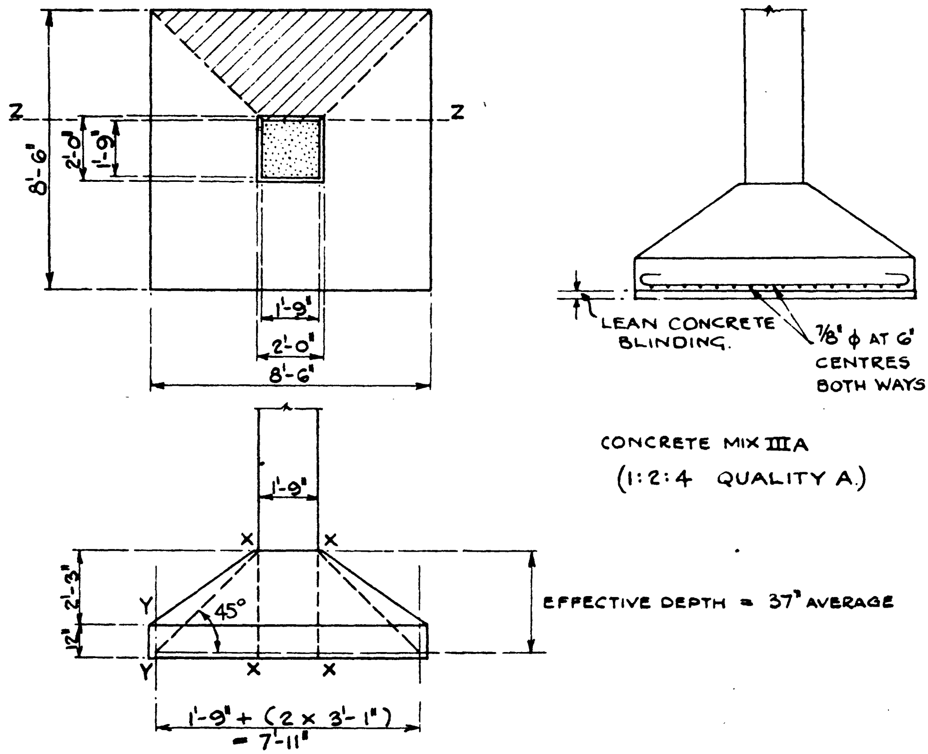


FIG. 31.—REINFORCED CONCRETE FOOTING.

$$\text{Hence } M = \frac{630{,}000 \times 102}{24} \times 2{\cdot}206 \times 0{\cdot}794^2.$$

$$= 3{,}720{,}000 \text{ in.-lb.}$$

The total area of reinforcement will be

$$A_T = \frac{3{,}720{,}000}{18{,}000 \times 0{\cdot}85 \times 37} = 7{\cdot}0 \text{ sq. in. approx.}$$

If this is provided within a width not exceeding the sum of the column width plus twice the effective depth, that is, 7 ft. 11 in., the area per foot width is 0·88 sq. in., which is amply met by $\frac{7}{8}$-in. bars at 6-in. centres. This arrangement will be

required in both directions. It is necessary to ensure that the bars have sufficient length of anchorage outwards from the section of maximum stress. In the present instance, the distance from the edge of the column to the beginning of the hook is 2 ft. 10½ in. According to *Table* 24, ⅞-in. bars with hooked ends, stressed at 18,000 lb. per square inch in 1 : 2 : 4 Quality A concrete, require a minimum length of adhesion of 2 ft. 9 in. The size of bar selected is therefore satisfactory. If, owing to the size of the base, insufficient length of bond were available it would be necessary to use smaller bars at closer pitch.

To check the compressive stress in the concrete, the moment of resistance of the central 2-ft. width of the base is

$$179 \times 24 \times 37^2 = 5{,}800{,}000 \text{ in.-lb.},$$

which is in excess of the calculated bending moment of 3,720,000 in.-lb. If this central section does not provide sufficient resistance, the resistance moments of the tapered sections on each side can be evaluated and added to that of the central section.

To provide a clean surface upon which to lay the reinforcement and to prevent the earth contaminating the structural concrete in the foundation, it is usual to spread a layer of lean concrete over the bottom of the excavation. This layer may be 2 in. or 3 in. thick and can be Mix V concrete (see *Table* 8).

Piled Foundation.

If the bearing resistance of the ground near the surface is so poor that it is necessary to resort to piling, a suitable base for the column in the previous example can be designed as follows. The total load to be carried is 630,000 lb. = 281 tons, and it would be advisable to carry this load on a group of six piles, each pile carrying 47 tons. This would require 14-in. square piles and, allowing a distance of 3 ft. 6 in. between adjacent piles and not less than 5 in. between any face of a pile and the edge of the pile cap, the overall size of the latter will be 9 ft. long by 5 ft. 6 in. wide as shown in *Fig.* 32. The thickness of the pile cap will, in the first instance, be controlled by the punching shear around the column and around the head of each pile; adopting a safe punching shear stress of 190 lb. per square inch for 1 : 2 : 4 Quality A concrete, (Mix IIIA, of the By-laws), the depth of the pile cap must not be less than

$$\text{At column base: } \frac{630{,}000}{4 \times 21 \times 190} = 39{\cdot}5 \text{ in.}$$

$$\text{At pile head: } \frac{47 \times 2{,}240}{4 \times 14 \times 190} = 10 \text{ in.}$$

If the cap is made 3 ft. 6 in. deep these conditions are complied with and sufficient embedded length is provided for the column and pile bars.

Transversely the bending effect can be neglected since, with the depth of cap selected, the load from the column when dispersed at 45 deg. will cover the centre pair of piles. Longitudinally provision must be made for the bending moment due to the cantilever effect of the load on two piles acting at a distance of 3 ft. 6 in. from the column centre. This moment is

$$2 \times 47 \times 2{,}240 \times 42 = 8{,}850{,}000 \text{ in.-lb.}$$

With an effective depth of 40 in. the amount of reinforcement required is 14·3

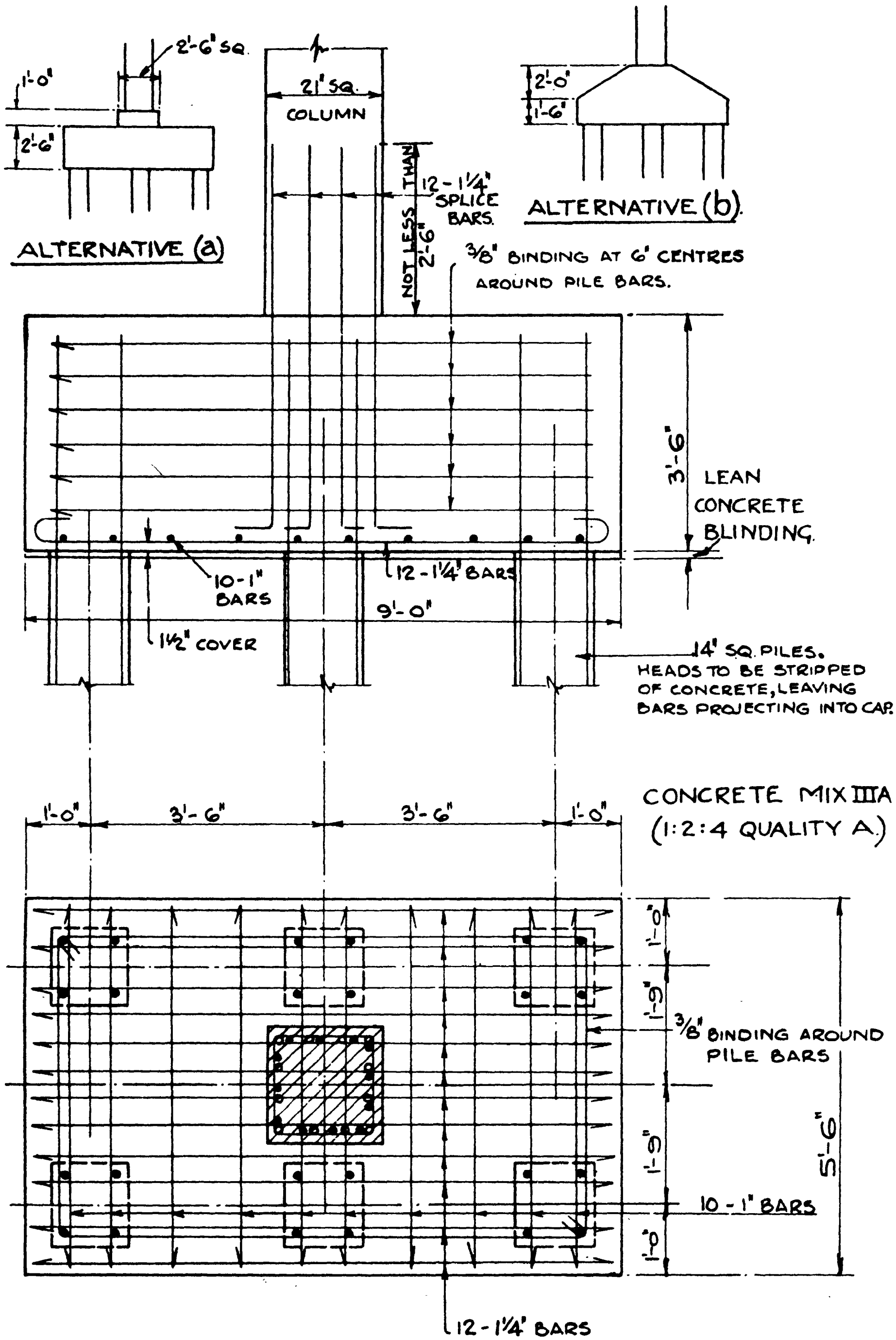


FIG. 32.—DETAILS OF PILE CAP.

sq. in. This is provided by twelve $1\frac{1}{4}$-in. bars arranged as in *Fig.* 32. The minimum length of bond required for a bar of this size with hooked ends is 3 ft. 11 in. (*Table* 24) for steel stressed at 18,000 lb. per square inch ; this length is slightly less than that provided.

The moment of resistance of the concrete should be checked ; it is $179 \times 40^2 \times 66 = 19{,}000{,}000$ in.-lb., which is ample compared with the applied bending moment. The allowable shear stress without shear reinforcement is 95 lb. per square inch and the maximum shear stress on the pile cap is

$$\frac{2 \times 47 \times 2{,}240}{66 \times 0{\cdot}85 \times 40} = 94 \text{ lb. per square inch.}$$

This stress is practically constant between the outer piles and the column. No shear reinforcement is required, and the remaining reinforcement shown on the detail in *Fig.* 32 is nominal.

Owing to the low compressive and shear stresses on the concrete section as detailed, alternative profiles for the pile cap, as shown in *Fig.* 32, might be considered. If the thinner cap (*a*) is adopted reinforcement to take the whole of the shear from two piles would be required, and it would be necessary to provide a block of reinforced concrete under the column to allow for punching shear around the edge of the column and for the embedment of the column bars. The main longitudinal reinforcement would have to be increased by 50 per cent. With the profile shown at (*b*) the main longitudinal reinforcement would be unaltered, but shear reinforcement would be necessary. In both alternatives the cost of the extra steel would, wholly or in part, be offset by the saving in concrete and shuttering.

The set to which the piles should be driven depends primarily on the working load, the weight, type, and fall of the hammer, the length of the pile, and the nature of the ground. The By-laws give no clauses concerning piling except to say in By-law 26 that piling shall be to the District Surveyor's approval. By the use of a reliable formula, such as Mr. Hiley's, which takes into account most of the variable factors, a fairly close assessment of the required set may be obtained. If it is assumed that the piles are 40 ft. long and are driven to a gravel sub-stratum with a $2\frac{1}{2}$-ton single-acting steam hammer dropping 3 ft. 6 in., the calculation would be as follows. The helmet, dolly, and packing are assumed to be in good condition at the end of the driving.

By adopting Mr. Hiley's formula and the relative coefficients in the modified form given in the author's "Reinforced Concrete Designers' Handbook," the settlement load is given by

$$\frac{wH_1en}{1 + cn} + w + P$$

The weight of the pile is $\dfrac{14^2 \times 40}{2{,}240} = 3\frac{1}{2}$ tons, to which should be added the weight of the helmet, dolly, packing, and stationary parts of the hammer, say an additional $\frac{1}{2}$ ton, making $P = 4$ tons. The weight w of the moving parts of the hammer is $2\frac{1}{2}$ tons. The effective drop H_1 for a single-acting steam hammer is taken as 90 per cent. of the actual drop, that is, $H_1 = 0{\cdot}90 \times 42 = 38$ in. The efficiency factor, e, depends on the relative value of $\dfrac{P}{w}$, and for $\dfrac{P}{w} = \dfrac{4}{2\frac{1}{2}} = 1{\cdot}6$, $e = 0{\cdot}42$. The

coefficient c takes account of the temporary elastic compression and is dependent upon the length of the pile, the severity of driving, the nature of the ground, and whether or not a dolly and packing are provided. The set (blows per final inch of penetration) is represented by n. To the load causing settlement a suitable factor of safety must be applied to give a margin between it and the working loads. A reasonable factor in the present case would be $2\frac{1}{4}$ as the driving will be "medium" to "hard." Thus with a working load of 47 tons, the calculated settlement load should not be less than $47 \times 2\frac{1}{4} = 105\frac{3}{4}$ tons. This gives a driving pressure of $\frac{105\frac{3}{4} \times 2{,}240}{14^2} = 1{,}210$ lb. per square inch. For this pressure, a 40-ft. pile, dolly provided, and gravel soil, the value of c can be taken as 0·36.

Substituting the known values in the formula, we get

$$105{\cdot}75 = \frac{2\frac{1}{2} \times 38 \times 0{\cdot}42 \times n}{1 + 0{\cdot}36n} + 2\frac{1}{2} + 4$$

from which $n = 24$ blows per inch. Thus the last ten blows should produce a penetration of not more than 0·42 in., say, $\frac{3}{8}$ in.

Strip Footing.

When the permissible ground pressures are low, or when columns are closely spaced, so that isolated footings would practically overlap, a line of columns can be conveniently carried on a strip footing. As an example, consider the base shown in *Fig.* 33, which is 50 ft. long by 5 ft. wide and carries five columns the loads on which are specified. The first step in the design is to find the centre of gravity of the loads. Taking moments about the right-hand end:

30 tons	×	5 ft.	=	150 ft.-tons.
35 ,,	×	18 ,,	=	630 ,, ,,
40 ,,	×	28 ,,	=	1,120 ,, ,,
45 ,,	×	37 ,,	=	1,665 ,, ,,
50 ,,	×	45 ,,	=	2,250 ,, ,,
200 tons				5,815 ft.-tons.

The distance of the centre of gravity from the right-hand end is $\frac{5{,}815}{200} = 29{\cdot}075$ ft.,

The centroid of the base will be 25 ft. from either end; hence the eccentricity of the loading is 4·075 ft. This being less than one-sixth of the length of the base, the maximum and minimum ground pressures are given by

$$\frac{200}{50 \times 5}\left(1 \pm \frac{6 \times 4{\cdot}075}{50}\right)$$

= 1·19 tons per square foot maximum (2,670 lb. per square foot)
and 0·41 ,, ,, ,, ,, minimum (920 ,, ,, ,, ,,)

These pressures will be distributed as indicated by the full line in *Fig.* 33. In a practical case the weight of the base would be added, the total maximum pressure then being compared with the safe pressure. If the maximum pressure was found to be excessive, the size of the base would have to be increased. Alternatively it might be possible to adjust the position of the base so that the centroid would coincide with the centre of gravity of the loading. This would ensure a uniform pressure throughout that would be less than the maximum pressure under the same

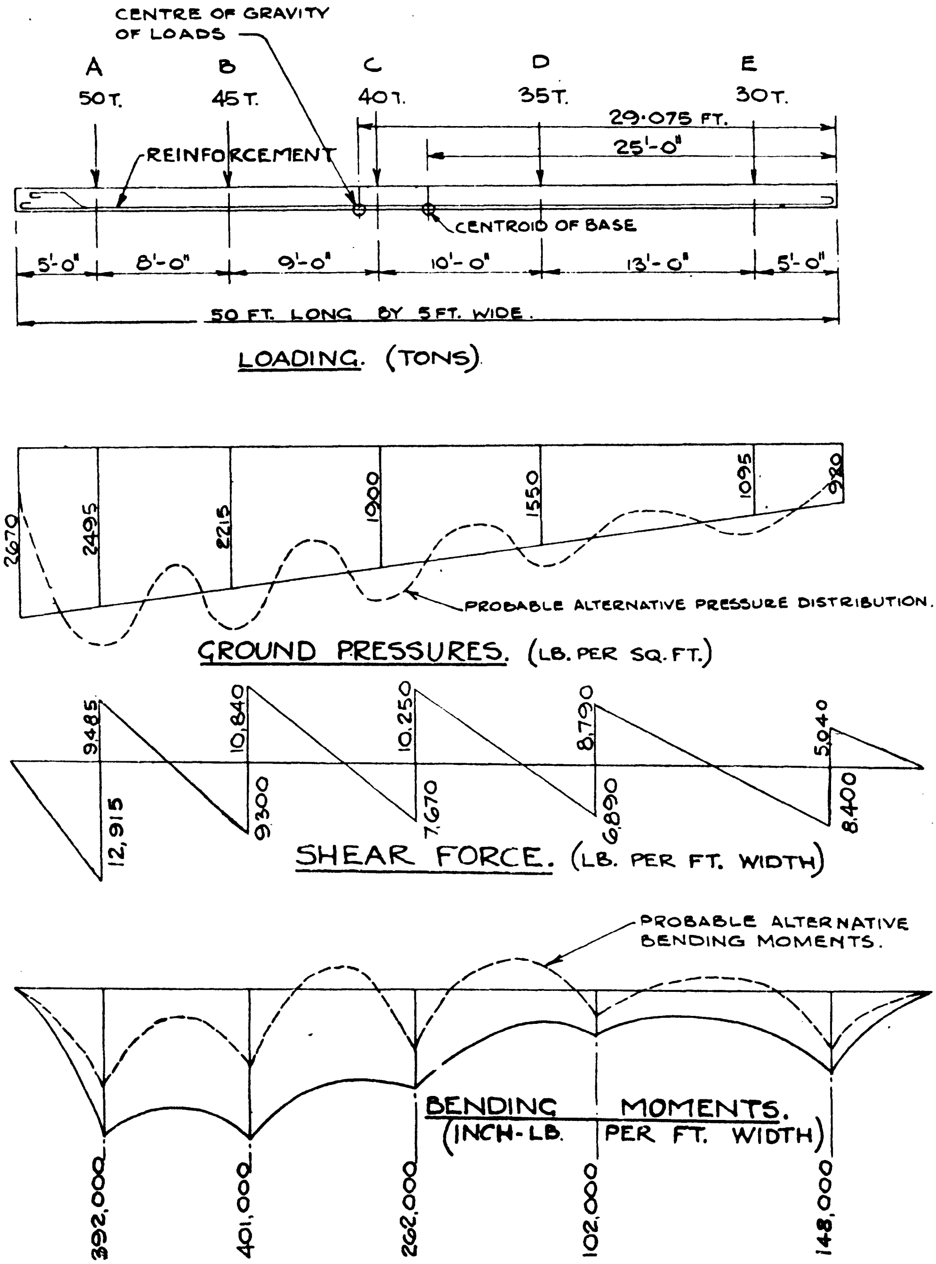


FIG. 33.—DESIGN OF STRIP FOOTING.

base eccentrically loaded with the same load. For the present example it will be assumed that the calculated pressures are not excessive and that the exigencies of the site do not permit the dimensions or position of the base to be altered.

The bending moment diagram and shear force diagram can be prepared by calculating the moments and shears at a number of sections in accordance with the following tabulated procedure; the pressures and loads are taken from *Fig.* 33 and the calculations apply to a 1-ft. width of base.

Section	Bending Moments (ft.-lb.)			Shears (lb.)		
		Positive	Negative		Positive	Negative
A (Loads on L.H.S.)	$-\ 2{,}495\times\frac{5^2}{2}$		$-\ 31{,}200$	$-2{,}495\times 5$		$-12{,}475$
	$-\ 175\times\frac{5^2}{3}$		$-\ 1{,}460$	$-175\times\frac{5}{2}$		$-\ 440$
			$-\ 32{,}660$			$-12{,}915$
			$=-392{,}000$ in.-lb.			
B (Loads on L.H.S.)	$+22{,}400\times 8$	$+179{,}200$				
	$-\ 2{,}215\times\frac{13^2}{2}$		$-187{,}000$		$+22{,}400$	
	$-\ 455\times\frac{13^2}{3}$		$-\ 25{,}600$	$-2{,}215\times 13$		$-28{,}800$
				$-\ 455\times\frac{13}{2}$		$-\ 2{,}960$
			$-212{,}600$			$-31{,}760$
			$+179{,}200$			$+22{,}400$
			$-\ 33{,}400$			$-\ 9{,}360$
			$=-401{,}000$ in.-lb.			
C (Loads on L.H.S.)	$+22{,}400\times 17$	$+381{,}000$			$+22{,}400$	
	$+20{,}200\times 9$	$+181{,}800$			$+20{,}200$	
	$-\ 1{,}900\times\frac{22^2}{2}$		$-460{,}000$	$-1{,}900\times 22$		$-41{,}800$
	$-\ 770\times\frac{22^2}{3}$		$-125{,}000$	$-\ 770\times\frac{22}{2}$		$-\ 8{,}470$
		$+562{,}800$	$-585{,}000$		$+42{,}600$	$-50{,}270$
			$+562{,}800$			$+42{,}600$
			$-\ 22{,}200$			$-\ 7{,}670$
			$=-264{,}000$ in.-lb.			
D (Loads on R.H.S.)	$+13{,}400\times 13$	$+174{,}500$			$+13{,}440$	
	$-\ 920\times\frac{18^2}{2}$		$-149{,}000$	$-\ 920\times 18$		$-16{,}560$
	$-\ 630\times\frac{18^2}{3}$		$-\ 34{,}000$	$-\ 630\times\frac{18}{2}$		$-\ 5{,}670$
			$-183{,}000$			$-22{,}230$
			$+174{,}500$			$+13{,}440$
			$-\ 8{,}500$			$-\ 8{,}790$
			$=\ 102{,}000$ in.-lb.			
E (Loads on R.H.S.)	$-\ 920\times\frac{5^2}{2}$		$-\ 11{,}500$	$-\ 920\times 5$		$-\ 4{,}600$
	$-\ 175\times\frac{5^2}{6}$		$-\ 730$	$-175\times\frac{5}{2}$		$-\ 440$
			$-\ 12{,}230$			$-\ 5{,}040$
			$=\ 148{,}000$ in.-lb.			

A check calculation of the bending moment at C is advisable at this stage to see whether any serious arithmetical error has occurred. Considering the loads on the right-hand side of C,

$$\begin{array}{lcrr}
-920 \times \dfrac{28^2}{2} & = & - & -361{,}000 \\[2ex]
-980 \times \dfrac{28^2}{6} & = & - & -128{,}000 \\[2ex]
+13{,}440 \times 23 & = & +309{,}000 & - \\
+15{,}680 \times 10 & = & +156{,}800 & - \\ \hline
 & & +465{,}800 & -489{,}000 \\
 & & & +465{,}800 \\ \hline
 & & & -23{,}200 = 279{,}000 \text{ in.-lb.}
\end{array}$$

The fact that this moment is not identical with that obtained by considering the loads on the left-hand side is due to the cumulative arithmetical or slide-rule approximations, but this small difference can be neglected in a problem of this character.

The thickness of the slab can be based on the maximum bending moment of 401,000 in.-lb., the reinforcement being varied at other sections. Thus, using 1 : 2 : 4 ordinary concrete (Mix III of the By-laws) the effective depth required is

$$\sqrt{\frac{401{,}000}{134 \times 12}} = 15{\cdot}6 \text{ in.}$$

Thus an 18-in. slab throughout will be satisfactory, the reinforcement being laid longitudinally in the bottom. At sections A and B the steel required is

$$\frac{401{,}000}{18{,}000 \times 0{\cdot}87 \times 16{\cdot}5} = 1{\cdot}55 \text{ sq. in., say, 1-in. bars at 6-in. centres.}$$

Throughout the remainder of the footing the reinforcement could be reduced to

$$\frac{264{,}000}{18{,}000 \times 0{\cdot}87 \times 16{\cdot}5} = 1{\cdot}02 \text{ sq. in., say, 1-in. bars at 9-in. centres.}$$

Owing to the length of the slab the bars would have to be provided in, say, two or three lengths, and lapped sufficiently to conform to the requirements for anchorage.

The maximum shear force as already determined is 12,915 lb., which results in a shear stress of

$$\frac{16{,}850}{12 \times 0{\cdot}87 \times 16{\cdot}5} = 98 \text{ lb. per square inch.}$$

Since the permissible shear stress without reinforcement is limited to 75 lb. per square inch for Mix III, it may be necessary to provide shear reinforcement to the left of A. The total shear to be resisted across the full width of the slab is $5 \times 12{,}915 = 64{,}575$ lb., which can be satisfactorily resisted by five 1-in. bars bent up at an angle of 30 deg. in double shear (see *Table* 23). The next highest shear occurs on the right-hand side of B (see *Fig.* 33) and results in a stress of

$$\frac{10{,}840}{12 \times 0{\cdot}87 \times 16{\cdot}5} = 64 \text{ lb. per square inch,}$$

which will not require reinforcement.

It will be noticed that a problem such as the foregoing involves, in the calculation of the moments, comparatively small differences between large quantities. It is therefore necessary, if the results are to be of any value, for all calculations to be made with great care, avoiding approximations (especially of the value of the distributed ground pressures) which, while reducing labour in computation, may give misleading results. Another factor to be borne in mind is that the conventional method of calculation as presented, assumes that the base is rigid enough to distribute the pressure with uniform variation throughout the length of the base. For the strains due to the calculated bending moments to be realised, there must be deformation of the base. This deflection will involve a redistribution of the ground pressures, the latter tending to increase under the load points and consequently to decrease between these points. This redistribution will in turn reduce the moments and in extreme cases may even reverse the sign of the moments between the load points. For the extreme case the moments under any column, say *C*, could be assessed as

$$\frac{40 \times 2{,}240}{5} \times 9{\cdot}5 \times \frac{12}{10} = 204{,}000 \text{ in.-lb. per foot width,}$$

where the load on column C is 40 tons and the mean of the spans C–B and D–C is 9·5 feet. If similar calculations are made for the other load points, a bending moment diagram as shown dotted in *Fig.* 33 may result, from which it will be seen that generally the peak moments (negative) are smaller, and positive moments, that did not occur in the " rigid base " analysis, appear. These smaller bending moments, however, are accompanied by higher local ground pressures, the variation of the latter being somewhat as indicated on the pressure diagram in *Fig.* 33. It is impossible to assess a close value for the maximum pressures, but if the ground can withstand these increased pressures without excessive settlement, the smaller moments will probably be realised. If these high local pressures were so much in excess of the safe pressure as to cause considerable settlement, a readjustment of the pressures would occur, so that in the case of poor soils with central loading, a uniformly distributed pressure would be approached everywhere. With eccentric loading the pressure distribution approached would be a uniformly varying pressure, similar to the original pressure indicated by the full line diagram in *Fig.* 33. This is the argument in favour of the method of analysis adopted in the detailed design in the foregoing example for cases where the bearing capacity of the ground is low.

Raft Foundation.

As an example of the design of a raft foundation for a building, consider the problem presented in *Fig.* 34, where the total load of 1,360 tons from the group of twelve columns and other loads on the raft is to be distributed so that the ground pressure does not exceed 12½ cwt. per square foot. If we assume that of the permissible ground pressure of 1,400 lb. per square foot, 400 lb. per square foot represents the weight of the raft and superimposed loading due to earth filling 3 ft. deep, the pressure available for supporting the column loads is 1,000 lb. per square foot. It is assumed that the distance from the centre line of each end row of columns to the edge of the raft is limited to 3 ft., and that

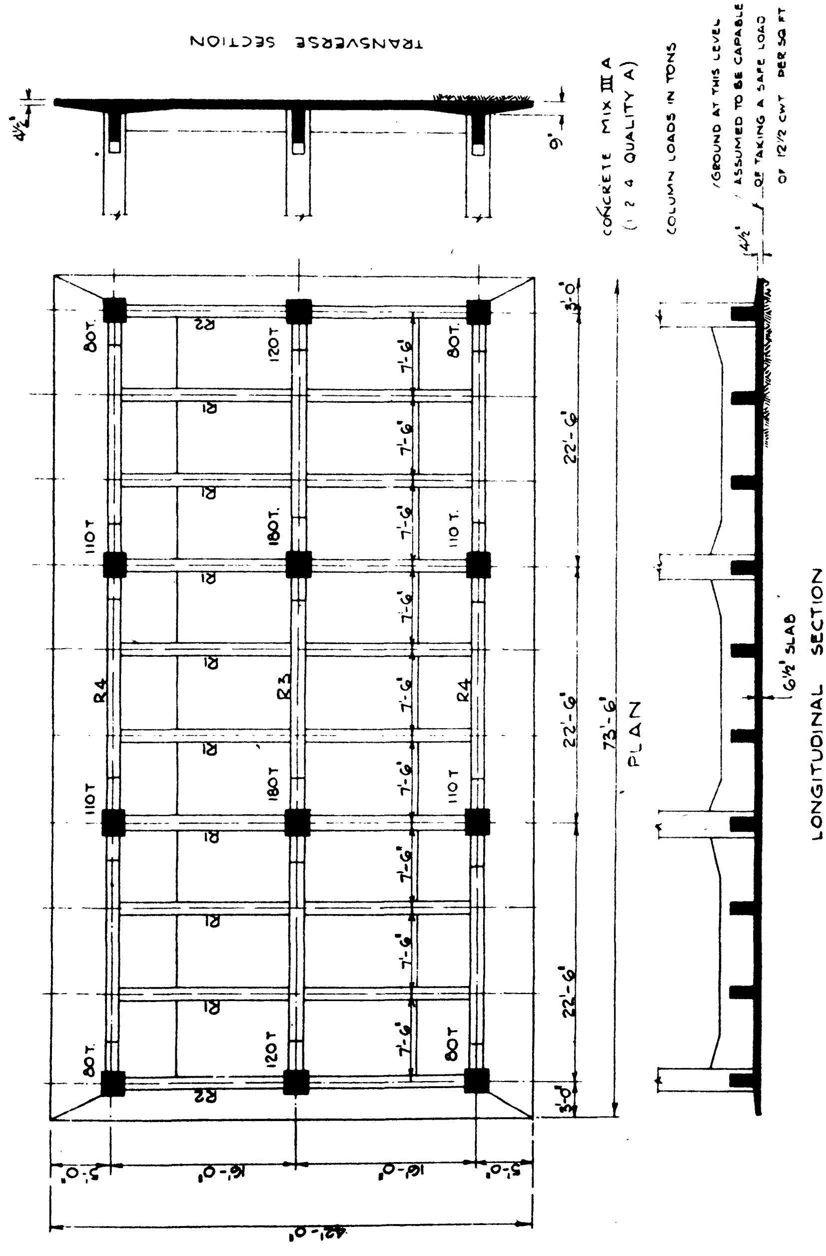


Fig. 34.—Raft Foundation.

there is no restriction on the width of the raft. Thus the maximum overall length of the raft is 73 ft. 6 in. and the minimum width is

$$\frac{1{,}360 \times 2{,}240}{1{,}000 \times 73 \cdot 5} = 41 \cdot 4 \text{ ft., say, } 42 \text{ ft.}$$

The raft will consist of a slab over the whole site, stiffened by transverse ribs which in conjunction with the longitudinal ribs, transmit the load from the columns to the ground. A suitable arrangement of ribs spaced at 7-ft. 6-in. centres is shown in *Fig.* 34. For the purpose of design, the raft is considered as an inverted floor carrying a uniformly distributed load ot 1,000 lb. per square foot, but with the difference that the column reactions are also known. This will affect the moments in the ribs as described later. Since the determination of the cross sections of the concrete and the areas of the reinforcement will follow the lines already discussed for slabs and tee-beams, the present consideration will be limited to the calculation of the bending moments and shearing forces to be resisted by the slabs and beams.

The bending moment in any interior panel of the slab is

$$1{,}000 \times 7 \cdot 5^2 \times \tfrac{12}{12} = 56{,}250 \text{ in.-lb.}$$

This requires a $6\tfrac{1}{2}$-in. slab if 1 : 2 : 4 Quality A concrete (By-laws' Mix IIIA) is used and $\tfrac{3}{4}$-in. cover is provided over the slab bars. The latter should be arranged to provide for tension in the top face of the slab between the transverse ribs, and in the bottom face under these ribs. The cantilever bending moment due to the 3-ft. projection beyond the end transverse ribs is

$$1{,}000 \times 3^2 \times \tfrac{12}{2} = 54{,}000 \text{ in.-lb.}$$

Hence the bending moment in the end 7-ft. 6-in. panels is practically the same as that in the interior panels, and the same thickness of slab and reinforcement can be used throughout.

Along the longitudinal edges of the raft where the effective projection is 4 ft. 6 in. the bending moment is

$$1{,}000 \times 4 \cdot 5^2 \times \tfrac{12}{2} = 121{,}500 \text{ in.-lb.,}$$

which requires a 9-in. slab at the face of the rib. The projecting slabs can be tapered to, say, $4\tfrac{1}{2}$ in. thick at the outer edges, as shown in *Fig.* 34.

The upward loading on the interior transverse ribs R1 is, as shown on *Fig.* 35, a uniform pressure of $7 \cdot 5 \times 1{,}000 = 7{,}500$ lb. per foot run, neglecting the relief due to the weight of the rib. The total upward load on this rib is $7{,}500 \times 32 = 240{,}000$ lb. The corresponding loads on the outer transverse ribs R2 are $\left(\frac{7 \cdot 5}{2} + 3\right)1{,}000 = 6{,}750$ lb. per foot run, and 216,000 lb. total. The sum of the reactions of the ribs R1 and R2 on the central longitudinal rib R3 must be equal to the sum of the column loads on this rib, that is, 600 tons = 1,344,000 lb. The load on the rib R2 being 90 per cent. of that on R1, the total number of ten ribs is equivalent to $8 + (2 \times 0 \cdot 9) = 9 \cdot 8$ ribs R1. Thus the reaction from each rib R1 is

$$\frac{1{,}344{,}000}{9 \cdot 8} = 137{,}150 \text{ lb.}$$

and from each rib R2

$$0 \cdot 9 \times 137{,}150 = 123{,}435 \text{ lb.}$$

The reaction from R1 on each of the outer longitudinal ribs R4 is

$$\frac{240{,}000 - 137{,}150}{2} = 51{,}425 \text{ lb.}$$

The corresponding figure for R2 is

$$\frac{216{,}000 - 123{,}435}{2} = 46{,}283 \text{ lb.}$$

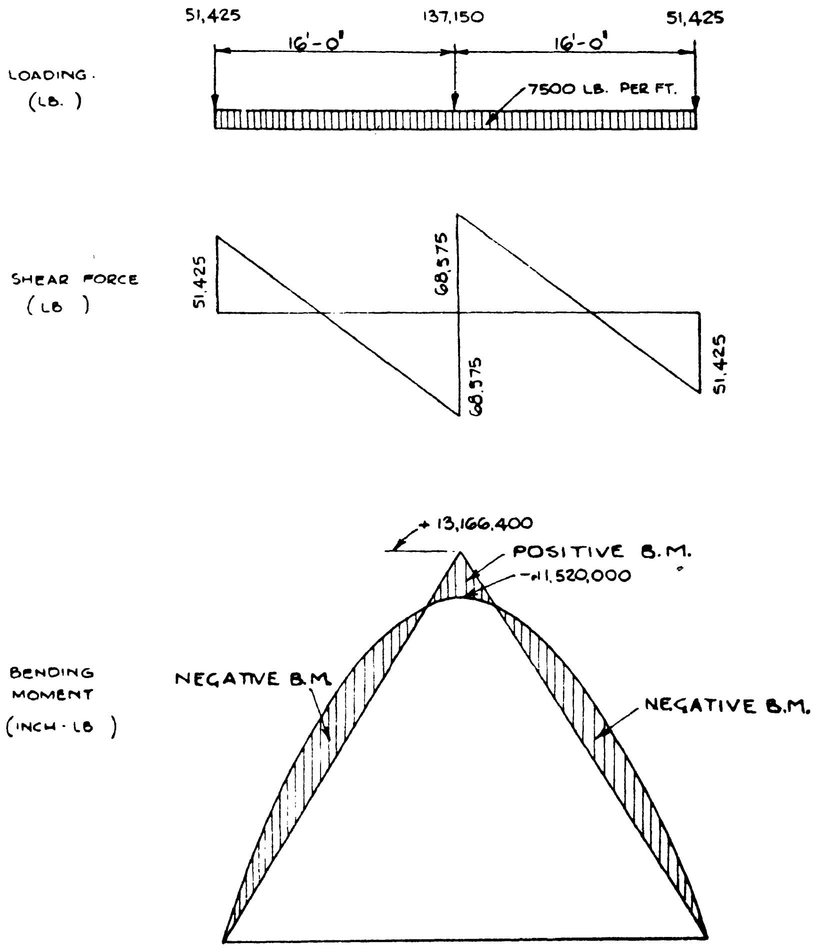


FIG. 35.—TRANSVERSE RIBS.

The loading and resulting bending moment and shear force diagrams for the rib R1 are shown in *Fig.* 35, the net bending moment diagram being the difference between a parabola of maximum height

$$-\frac{240{,}000 \times 32 \times 12}{8} = -11{,}520{,}000 \text{ in.-lb.,}$$

and a triangle of maximum height

$$+\frac{137{,}150 \times 32 \times 12}{4} = +13{,}166{,}400 \text{ in.-lb.}$$

Since the loading on the outer ribs R2 is only 10 per cent. less than that on R1, the detail adopted for the latter could be used for the former without much sacrifice of economy.

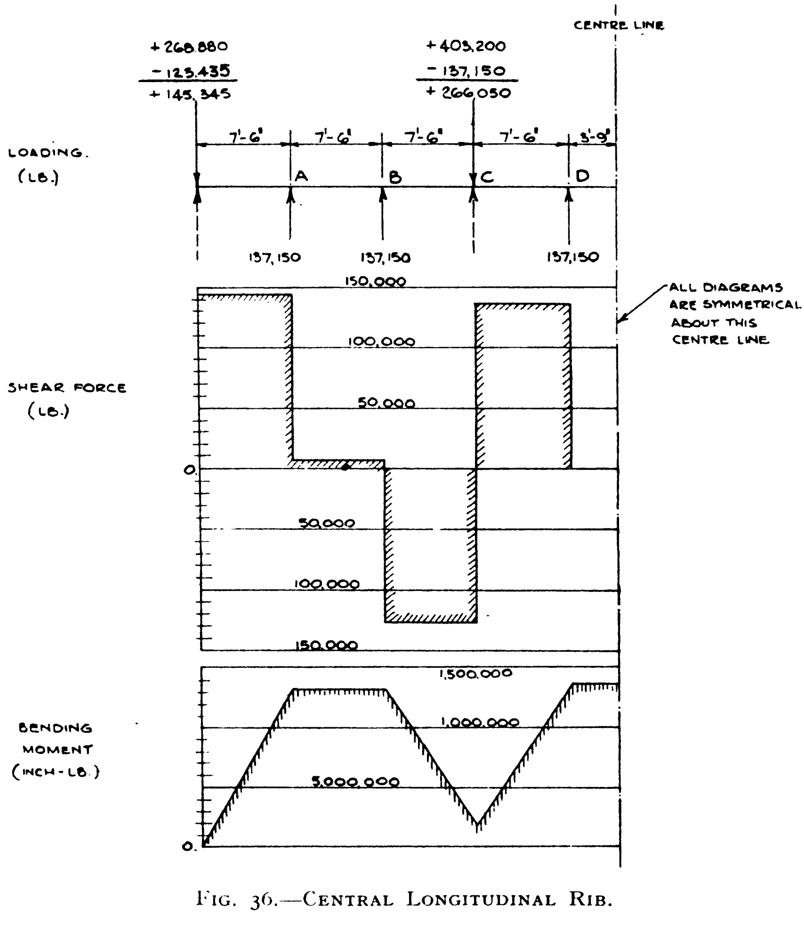


FIG. 36.—CENTRAL LONGITUDINAL RIB.

The loading on the longitudinal ribs R3 and R4 consists of the column loads acting downwards and the upward reactions from the transverse ribs, the weight of the ribs themselves being neglected. The diagrams of loading, shear force, and bending moment are given in *Fig.* 36 for the central longitudinal rib R3 and in *Fig.* 37 for the outer longitudinal rib R4. Since, in the example, the system is symmetrical, only one-half need be investigated. The calcula-

tion of the bending moments at a number of sections for the rib R3 is as follows :

				in.-lb.
Section A :	+ 145,345 × 7·5	=	+ 1,090,000	= 13,100,000
,, B :	+ 145,345 × 15·0	=	+ 2,180,000	
	− 137,150 × 7·5	=	− 1,030,000	
			+ 1,150,000	= 13,800,000
,, C :	+ 145,345 × 22·5	=	+ 3,270,000	
	− 137,150 × 22·5	=	− 3,090,000	
			+ 160,000	= 1,920,000
,, D :	+ 145,345 × 30·0	=	+ 4,360,000	
	+ 266,050 × 7·5	=	+ 1,995,000	
	− 137,150 × 37·5	=	− 5,150,000	
			+ 1,205,000	= 14,460,000

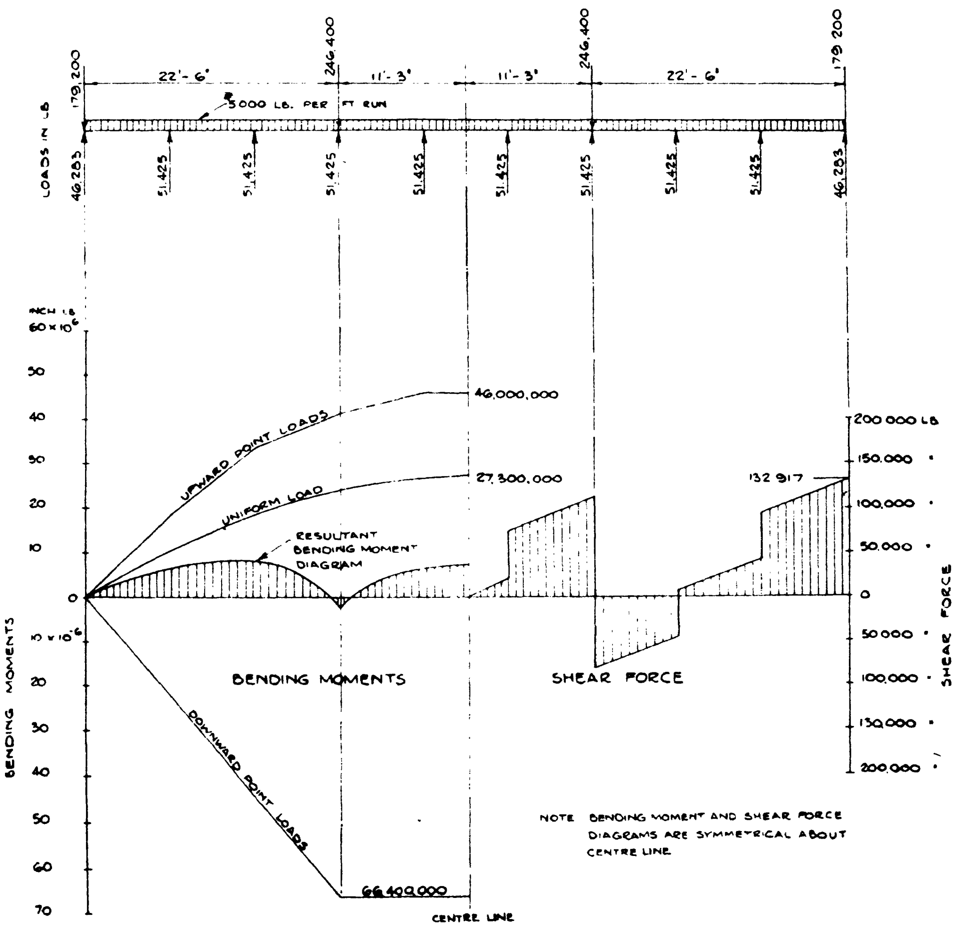


FIG. 37.—OUTER LONGITUDINAL RIBS.

In addition to the point loads, the outer longitudinal ribs R4 carry a uniformly distributed load of $5 \times 1{,}000 = 5{,}000$ lb. per foot run. The resultant bending moment diagram (*Fig.* 37) can be derived either by a similar analysis or by algebraical addition of moment diagrams due to the following system of loadings:

(1) Column reactions: a trapezoidal diagram having a maximum ordinate of

$$-\,246{,}400 \times 22\cdot5 \times 12 = -\,66{,}400{,}000 \text{ in.-lb.}$$

(2) Distributed load: a parabolic diagram having a maximum ordinate of

$$+\,\frac{5{,}000 \times 67\cdot5^2 \times 12}{8} = +\,34{,}150{,}000 \text{ in.-lb.}$$

(3) Reactions from transverse ribs: a diagram having the following ordinates

At A:	$+ 205{,}700 \times 7\cdot5$	$= + 1{,}543{,}000$	$= + 18{,}500{,}000$ in.-lb.
At B:	$+ 205{,}700 \times 15\cdot0$	$= + 3{,}085{,}000$	
	$- 51{,}425 \times 7\cdot5$	$= - 385{,}000$	
		$+ 2{,}700{,}000$	$= + 32{,}400{,}000$ in.-lb.
At C:	$+ 205{,}700 \times 22\cdot5$	$= + 4{,}628{,}000$	
	$- 51{,}425 \times 22\cdot5$	$= - 1{,}158{,}000$	
		$+ 3{,}470{,}000$	$= + 41{,}600{,}000$ in.-lb.
At D:	$+ 205{,}700 \times 30\cdot0$	$= + 6{,}170{,}000$	
	$- 51{,}425 \times 45\cdot0$	$= - 2{,}316{,}000$	
		$+ 3{,}854{,}000$	$= + 46{,}300{,}000$ in.-lb.
At E:	$+ 205{,}700 \times 33\cdot75$	$= + 6{,}941{,}500$	
	$- 51{,}425 \times 60\cdot0$	$= - 3{,}085{,}500$	
		$+ 3{,}866{,}000$	$= + 46{,}300{,}000$ in.-lb.

At this stage an arithmetical check on the whole of the preceding calculations can be made by equating the upward and downward loads on one of the outer ribs as given in *Fig.* 37.

Downward load :-	$2 \times 179{,}200$	$= 358{,}400$	lb.
	$2 \times 246{,}400$	$= 492{,}800$	,,
		$851{,}200$	lb.
Upward load :	$2 \times 46{,}283$	$= 92{,}566$	lb.
	$8 \times 51{,}425$	$= 411{,}400$	,,
	$73\cdot5 \times 5{,}000$	$= 367{,}500$	,,
		$871{,}466$	lb.

The difference can be accounted for by the difference between the ground pressure assumed at 1,000 lb. per square foot and the value obtained by dividing the total column load by the area of the raft.

The remarks made in the previous section regarding accuracy of computations, and the possible rearrangement of bending moments due to localised increases in ground pressures, apply equally to the present problem.

Raft Foundation for Cylindrical Structure.

A form of raft foundation met with in the construction of water towers, chimneys and similar structures, is circular or polygonal in plan and presents a

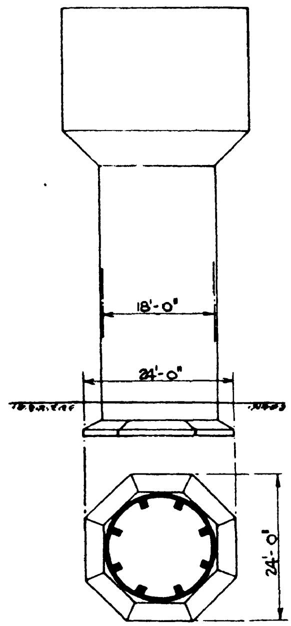


FIG. 38.—WATER TOWER ON RAFT FOUNDATION.

number of interesting features. As an example consider the foundation for the water tower outlined in *Fig.* 38, the principal data for which are:

	lb.
Weight of tower empty	570,000
Approximate weight of raft	80,000
Total dead weight	650,000
Weight of water and other superimposed loads . .	900,000
Total weight fully loaded	1,550,000

Wind moment at foundation level = 1,000,000 ft.-lb.

The conditions at the site are such that it is desirable to keep the ground pressure below 2 tons per square foot, for which reason a raft is required. For constructional convenience the raft is made octagonal in plan. If it is made 24 ft. across the flats the critical ground pressures are calculated as follows:

Area of 24-ft. octagon $= 0{\cdot}828 \times 24^2 = 477$ sq. ft.
Minimum Modulus of ditto $= 0{\cdot}1016 \times 24^3 = 1{,}400$ ft.^{3}
Maximum Modulus of ditto $= 0{\cdot}109 \times 24^3 = 1{,}510$ ft.^{3}

When the tank is empty the pressures on the ground are $\dfrac{650{,}000}{477} \pm \dfrac{1{,}000{,}000}{1{,}400}$,

giving a maximum pressure of 2,080 lb. per square foot and a minimum pressure of 650 lb. per square foot.

When the tank is full the pressures on the ground are $\frac{1,550,000}{477} \pm \frac{1,000,000}{1,400}$, giving a maximum value of 3,965 lb. per square foot and a minimum of 2,535 lb. per square foot.

The absolute maximum pressure of 3,965 lb. per square foot is less than the 2 tons allowable.

The possible pressure distribution across an axis through the corners of the raft is shown in *Fig.* 39, where the values in brackets represent the net upward pressures. The portion of the raft within the walls will be designed as a circular slab, first considered as freely supported along the line of the wall, an adjustment then being made for the reduction in the bending moment due to the overhanging portion.

The bending moment on a circular slab supported on its edges and carrying a uniformly distributed load can be variously assessed. If the slab is simply supported at the circumference, by subtracting the moment of the load on the semicircular area from the moment of the reaction around the circumference of the same semicircle, each moment being taken about the diameter of the semicircle, the total bending moment across the diameter is found to be $\frac{wD^3}{24}$ ft.-lb., where w is the intensity of loading in lb. per square foot and D is the diameter (ft.).

The average bending moment is therefore $\frac{wD^2}{24}$ ft.-lb. per foot width.

Since this moment varies from zero at the ends of the diameter to a maximum in the middle, a very conservative estimate of the maximum bending moment is to take double the average, that is $\frac{wD^2}{12}$ ft.-lb. per foot width. If the slab is fixed around its edges, the moment at the support can be assumed to be two-thirds of the free moment at the centre, that is $\frac{wD^2}{18}$, and the moment at midspan can be taken as $\frac{wD^2}{24}$. If partial fixity occurs, a maximum bending moment of $\frac{wD^2}{20}$ at the centre and at the support can be assumed. These coefficients are well on the safe side as they ignore the effect of spanning in two directions and the fact that the maximum bending moment is less than twice the average.

At the other extreme, some designers assume that the average free moment is $\frac{wD^2}{24}$ and multiply by a reduction factor of $\frac{1}{2}$ or $\frac{1}{3}$ to allow for the effect of spanning in two directions, thus obtaining moments such as $\frac{wD^2}{48}$ or $\frac{wD^2}{72}$ for freely supported slabs.

More logically the moments can be based on the analysis of the stresses in thin circular plates (see, for example, Morley's "Strength of Materials"). The

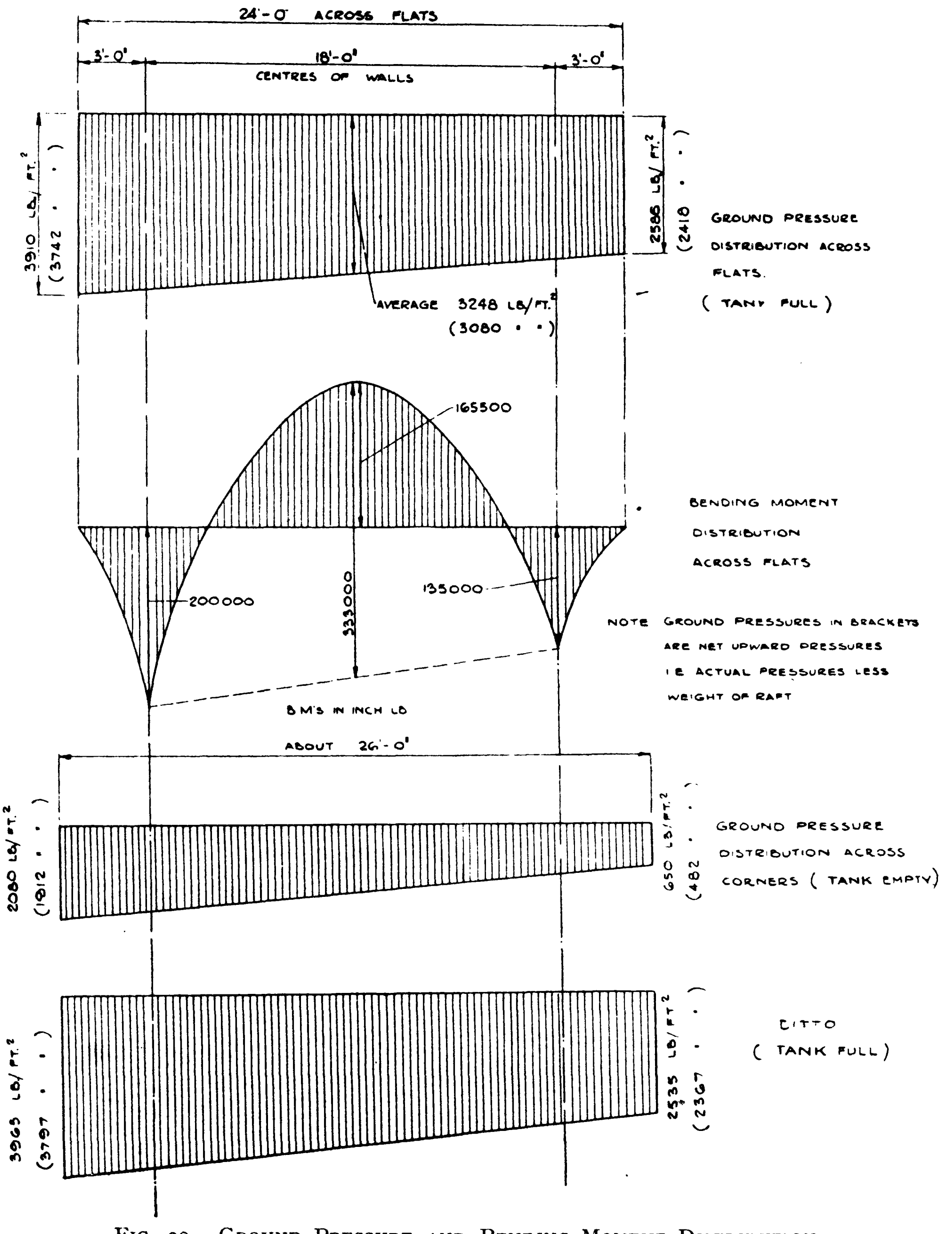


FIG. 39.—GROUND PRESSURE AND BENDING MOMENT DISTRIBUTION.

tensile and compressive stresses on a freely supported plate carrying a uniformly distributed load reach their maximum values at the centre, where the stress corresponding to the maximum strain is given by

$$t_{max.} = \frac{3(k-1)(3k+1)wD^2}{32k^2 \qquad d^2} \text{ lb. per square inch,}$$

in which d = thickness of plate (in.),
D = diameter of plate (ft.),

$k = \dfrac{1}{\text{Poisson's ratio}}$, and

w = load in lb. per square foot.

The bending moment corresponding to this stress is ($t_{max.} \times$ section modulus)

$$= t_{max.} \times \frac{d^2}{6} \text{ ft.-lb. per foot width.}$$

Thus the maximum bending moment can be considered as

$$\frac{(k-1)(3k+1)}{64k^2}wD^2 \text{ ft.-lb. per foot width.}$$

If we assume $k = \dfrac{1}{0{\cdot}15}$ for concrete, which is the value given by M. Pigeaud in his analyses of rectangular slabs, the expression for the maximum bending moment becomes $\dfrac{wD^2}{24}$ ft.-lb. per foot width. If we assume a parabolic distribution of the bending moment along a diameter (zero at the ends and a maximum in the middle) the total bending moment is

$$\frac{2}{3} \times \frac{wD^2}{24} \times D = \frac{wD^3}{36} \text{ ft.-lb.,}$$

and the average bending moment is $\dfrac{wD^2}{36}$ ft.-lb. per foot width.

Taking 80 per cent. of this moment to allow for slabs continuous or partly fixed around the circumference, the average bending moment is $\dfrac{wD^2}{45}$ per foot width, while for similar conditions the maximum bending moment would be $\dfrac{wD^2}{30}$ ft.-lb. per foot width.

On this basis there are two possible methods of design:

(1) Design for a maximum central bending moment of $\dfrac{wD^2}{24}$ (or $\dfrac{wD^2}{30}$ if continuous) and reduce the reinforcement or slab thickness towards the circumference.

(2) Design for an average bending moment of $\dfrac{wD^2}{36}$ (or $\dfrac{wD^2}{45}$ if continuous) and retain the same section throughout.

The latter method will be adopted in the example. The average net upward ground pressure (tank full) on the central circular portion of the raft is

$$\frac{1,470,000}{477} = 3,080 \text{ lb. per square foot.}$$

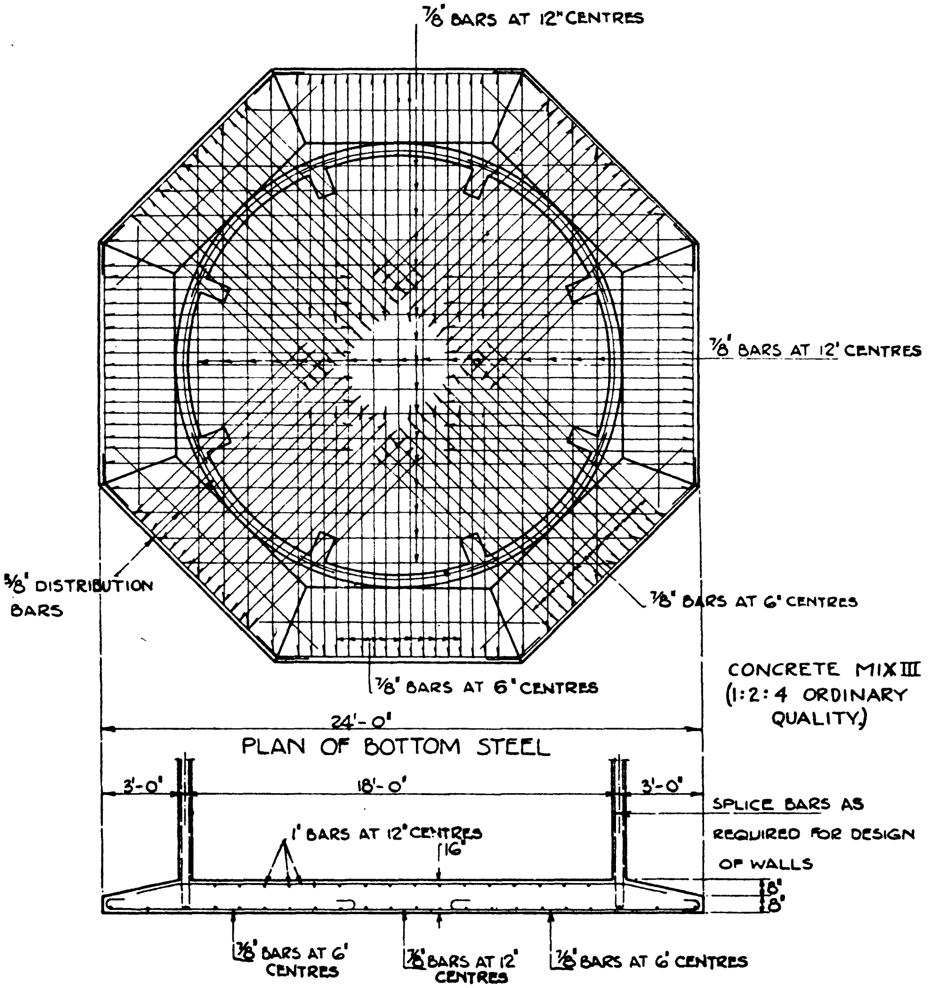


FIG. 40.—DETAIL OF FOUNDATION FOR CIRCULAR STRUCTURE.

Assuming free supports around the periphery whose diameter is 18 ft., the bending moment is $\frac{3,080 \times 18^2}{36} \times 12 = 333,000$ in.-lb. This moment is plotted on *Fig.* 39.

Along an axis across the flats of the octagon the variation in ground pressure due to wind moment is $\pm \frac{1,000,000}{1,510} = \pm$ 662 lb. per square foot, giving a

maximum net upward pressure of 3,742 lb. per square foot, and a minimum net upward pressure of 2,418 lb. per square foot. The approximate cantilever moments are therefore

$$\frac{3{,}660 \times 3^2 \times 12}{2} = 200{,}000 \text{ in.-lb. (maximum).}$$

and
$$\frac{2{,}500 \times 3^2 \times 12}{2} = 135{,}000 \text{ in.-lb. (minimum).}$$

From the approximate bending moment diagram (*Fig.* 39) the maximum bending moment at midspan is 165,500 in.-lb. Adopting a 1 : 2 : 4 ordinary quality concrete (Mix III in the By-laws) and a 16-in. slab, the reinforcement required is

$$\frac{165{,}500}{18{,}000 \times 0{\cdot}87 \times 14{\cdot}5} = 0{\cdot}73 \text{ sq. in.}$$

This is given by 1-in. bars at 12-in. centres, these bars being provided in the top of the slab in two directions at right-angles to each other as shown in *Fig.* 40.

The reinforcement in the bottom of the slab under the walls must be designed to resist the cantilever moment which attains a maximum value at a corner of the octagon on the leeward side when the wind blows normally to an axis across the corners. The total maximum pressure under the corner, as already calculated, is 3,965 lb. per square foot. Taking an average net upward pressure of 3,700 lb. per square foot and a cantilever span of 3 ft. 6 in., the bending moment is

$$\frac{3{,}700 \times 3{\cdot}5^2 \times 12}{2} = 272{,}000 \text{ in.-lb.}$$

The effective depth required is

$$\sqrt{\frac{272{,}000}{126 \times 12}} = 13{\cdot}4 \text{ in.}$$

If a 16-in. slab is provided with 1 in. cover of concrete on the earth face, the reinforcement required is 1·20 sq. in. With $\frac{7}{8}$-in. bars at 6-in. centres arranged as in *Fig.* 40 the cantilever bending moments are satisfactorily resisted.

In this and similar problems it is advisable to check the shear stress on the base slab. This will usually be a maximum at the outer periphery of the superstructure. In the present example, with a mean net projection of 3 ft. the approximate shear force per foot run is $3{,}700 \times 3{\cdot}0 = 11{,}100$ lb. Consequently the shear stress is $\frac{11{,}100}{12 \times 0{\cdot}87 \times 14{\cdot}6} = 72$ lb. per square inch, which will not require shear reinforcement.

CHAPTER XII

BASEMENT RETAINING WALLS

Basis of Design.

THE design of retaining walls for the basements of buildings depends upon the method of construction proposed. There are usually three conditions to investigate:

(1) When the wall only has been built; during this stage the wall may be subject to a minimum vertical load and a maximum overturning moment due to simple cantilever action, unless propping is retained against the wall.

(2) When the ground floor has been constructed but nothing above this level; in this case the floor acts as a prop at the top of the wall and the only vertical load, in addition to the weight of the wall itself, is the dead weight of the ground floor.

(3) When the building is completed and fully loaded.

Cases (1) or (3) may give the maximum ground pressures and either may give the maximum compressive stresses on the concrete, while Case (1) will give the maximum tensile stress in the vertical steel at the back of the wall. Case (2) will be investigated as producing tensile stresses on the front of the wall stem that may not be covered by either of the other cases. The requirements of the By-laws that can be related to retaining wall design are, apart from the general requirement of By-law 31, those concerning permissible working stresses and ground pressures, that is By-laws 30, 99, and 100.

As an illustration of the design of a basement retaining wall, the front wall of the building shown in *Fig.* 1, will be considered. The ground floor construction bearing on the wall is shown in *Fig.* 27. No special waterproof course will be provided, as it is assumed that the ground is not water-bearing. To guard against damp, greater impermeability will be secured by using a $1 : 1\frac{2}{3} : 3\frac{1}{3}$ concrete mix of Quality A for which the maximum compressive stress in bending (or in bending combined with direct compression) is 1,051 lb. per square inch as calculated in Chapter II. The allowable bond and shear stresses are 130 and 105 lb. per square inch respectively, and with a modular ratio of 15 the neutral axis factor is 0·47, the lever-arm factor is 0·84, and the resistance-moment factor Q is 208.

Although By-law 97 specifies a minimum cover of concrete of $\frac{1}{2}$ in. for slab work, it is preferable to allow, say, not less than 1 in. for bars near the face of the wall in contact with earth. The maximum allowable ground pressure in the present case is 4 tons per square foot.

Horizontal Pressures.

The horizontal pressures due to earth filling behind retaining walls are usually calculated by Rankine's formula, although a number of other expressions are in

satisfactory use for special conditions. If we assume an angle of repose of 35 deg. and a weight of 100 lb. per cubic foot for dry earth, the value of the Rankine factor $\frac{1 - \sin\theta}{1 + \sin\theta}$ is 0·271 and the pressure at any depth, h feet, is given by $p = 0{\cdot}271 \times 100h = 27{\cdot}1h$ lb. per square foot.

With a head of 11 ft., as in *Fig.* 41, the pressure at the bottom of the wall will be 298 lb. per square foot, decreasing uniformly to zero at the top of the wall. Some addition should be made to this pressure to allow for a surcharge of, say, 2 cwt. per square foot on the ground behind the wall. This will uniformly increase the pressures by $0{\cdot}271 \times 224 = 61$ lb. per square foot.

It has been assumed that the ground is dry, but to guard against unforeseen water-logging it is desirable to investigate the possible water pressures. If

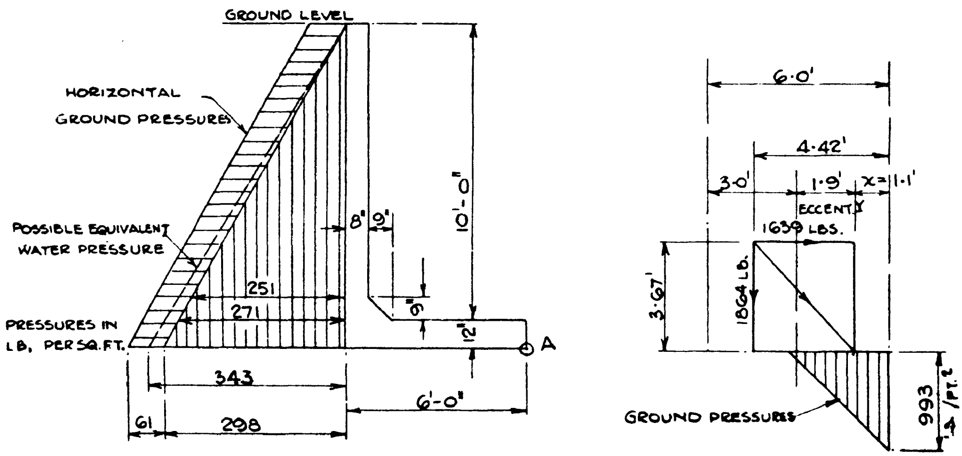


FIG. 41.—CONDITIONS DURING PRELIMINARY STAGE.

the whole height of the wall is subject to water pressure, the maximum at the bottom will be

$$11 \times 62{\cdot}4 = 686 \text{ lb. per square foot.}$$

As this case is unlikely to occur, it should be satisfactory to design for a factor of safety of not less than two, instead of the factor of four adopted for the normal condition of earth pressure. If the ground is continuously or intermittently water-bearing a factor of safety of four must be applied to water pressures. Thus, for comparative purposes in the present example, the minimum equivalent water pressure is $\frac{686}{2} = 343$ lb. per square foot compared with the maximum ground pressure of 359 lb. per square foot. Thus designing for normal ground pressures will cover the improbable case of water-logging. The latter effect can sometimes be avoided by the provision of land-drains or by providing weepholes through the wall to allow the water to escape. In buildings on city sites it is rarely possible to arrange an adequate system of land drains as can be done on an open site ;

also, weepholes are likely to be objectionable in the basements of buildings. For these reasons it is preferable to investigate the pressures due to water-logging.

Simple Cantilever Wall.

During the first stage already described the wall will temporarily act as a simple cantilever for which it is necessary to investigate the stability, the resistance to sliding, and the ground pressures, and to compute the bending moments. Assuming that a 6-ft. length of base, 12 in. thick, is constructed at the same time as the wall, there being a 9-in. splay at the foot of the wall stem, the latter being 8 in. thick, as shown in *Fig.* 41, the calculation of the stability of 1 ft. length of wall will proceed as follows:

		Load.		*Moment about A.*
Horizontal forces . . .	$\frac{298 \times 11}{2} =$	1,639 lb.	$\times \frac{11}{3}$	= 6,010 ft.-lb.
	$61 \times 11 =$	671 ,,	$\times \frac{11}{2}$	= 3,691 ,, ,,
		2,310 lb.		9,701 ft.-lb.
Vertical forces:				
Base	$6 \times 144 =$	864 lb.	$\times 3{\cdot}0$	= 2,592 ft.-lb.
Stem	$10 \times 96 =$	960 ,,	$\times 5{\cdot}67$	= 5,450 ,, ,,
Splay	$1 \times 40 =$	40 ,,	$\times 5{\cdot}08$	= 203 ,, ,,
		1,864 lb.		8,245 ft.-lb.

During the first stage it is safe to assume that the pressure due to the surcharge does not operate owing to the position of the hoarding around the site, and steps can be taken to prevent loads being deposited immediately behind the wall. In these circumstances the overturning moment is 6,010 ft.-lb. and is resisted by a counter-moment of 8,245 ft.-lb., giving a factor of safety of 1·37, which is satisfactory for a temporary condition. If precautions cannot be taken to prevent surcharge of the ground behind the wall during this preliminary stage, the overturning moment is 9,701 ft.-lb., which is in excess of the counter-moment. It would therefore be necessary to provide timber shores to the wall until the ground floor has been constructed.

Without external aids, the resistance to sliding is, say, $0{\cdot}4 \times 1{,}864 = 746$ lb. compared with the minimum horizontal force of 1,639 lb. It will therefore be necessary to provide struts to prevent possible forward movement of the wall. If shores are provided to resist overturning these can be arranged to give resistance against sliding. The earth in front of the base slab cannot be considered as providing any resistance as it may not be in contact with concrete and in any case will be largely removed when the excavation for the remainder of the basement slab is made. The coefficient of friction of 0·4 used in the calculations for resistance to sliding is a rational value for dry earths and gravels, but a lower coefficient should be taken for wet earths, approaching zero for wet clays.

The next step is to calculate the maximum ground pressure due to the horizontal force of 1,639 lb. acting at $\frac{11}{3} = 3{\cdot}67$ ft. above the base of the wall, and

the vertical load of 1,864 lb. acting at a distance of $\frac{8,245}{1,864} = 4{\cdot}42$ ft. from point A (*Fig.* 41). The point at which the resultant of these two forces strikes the underside of the base is at a distance x from point A, where

$$x = 4{\cdot}42 - \left(\frac{1,639}{1,864} \times 3{\cdot}67\right) = 1{\cdot}1 \text{ ft.}$$

Thus the eccentricity (e) is $\frac{6{\cdot}0}{2} - 1{\cdot}1 = 1{\cdot}9$ ft., and as this is greater than one-sixth of the width of the base the maximum ground pressure is

$$p_{\text{max.}} = \frac{4W}{3(L - 2e)} = \frac{4 \times 1,639}{3(6 - 3{\cdot}8)} = 993 \text{ lb. per square foot.}$$

This is well within the permissible value of 4 tons per square foot.

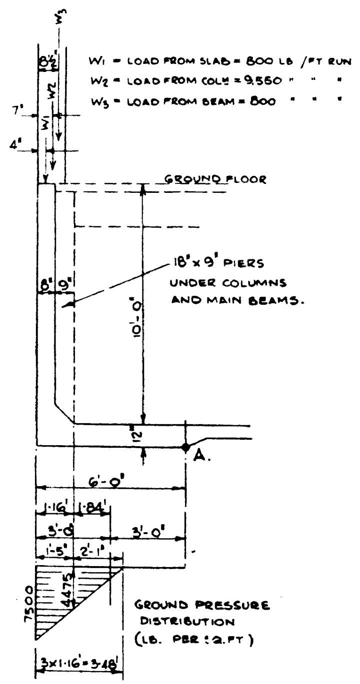


FIG. 42.—FINAL CONDITIONS.

It only remains to calculate the critical bending moments. Considering the base slab, the centre of pressure is 1·1 ft. from A, which is $4{\cdot}58 - 1{\cdot}1 = 3{\cdot}48$ ft. from the end of the splay. The bending moment at a section at the end of the splay is

$$\begin{aligned} -\ 1,864 \times 3{\cdot}48 \times 12 &= -\ 77,900 \text{ in.-lb.} \\ +\ 144 \times 4{\cdot}58 \times \frac{4{\cdot}58}{2} \times 12 &= +\ 18,100 \text{ ,, ,,} \\ \hline &\quad -\ 59,800 \text{ in.-lb.} \end{aligned}$$

The pressure at the level of the top of the splay is $0{\cdot}271 \times 9{\cdot}25 = 251$ lb. per

square foot, excluding the pressure of the surcharge. The bending moment at this section is

$$251 \times \frac{9{\cdot}25^2}{6} \times 12 = 42{,}800 \text{ in.-lb.}$$

Due to the surcharge the bending moment is

$$61 \times \frac{9{\cdot}25^2}{2} \times 12 = 31{,}400 \text{ ,, ,,}$$

$$\text{Total} = 74{,}200 \text{ in.-lb.}$$

These moments are transferred to *Fig.* 43.

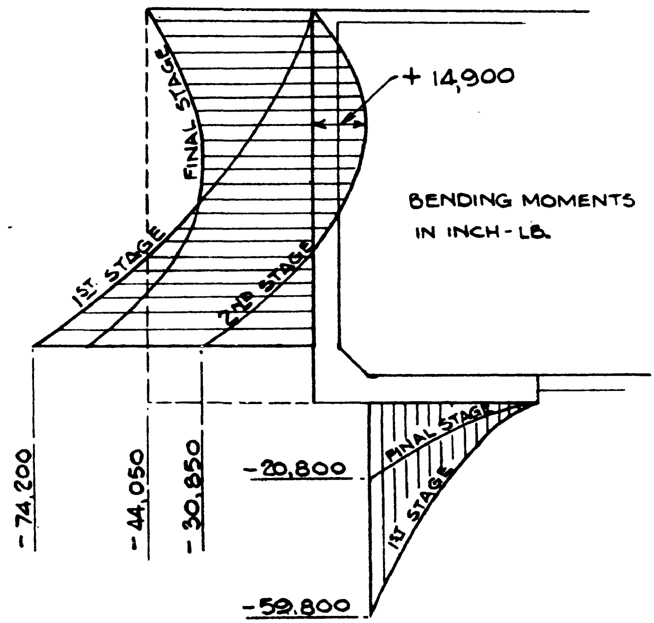


FIG. 43.—BENDING MOMENTS.

Wall as "Propped Cantilever."

At the intermediate stage of construction, when the ground floor only has been constructed, the floor acts as a prop at the head of the wall and the wall serves as a support for the floor. Considering the cross section of the basement shown in *Fig.* 27, the minimum vertical reaction on the front retaining wall is the dead weight only of the 4-in. slab spanning 8 ft., that is $\frac{8}{2} \times 48 = 192$ lb. per foot run. The effect of this reaction on the total vertical forces is

	Load.	*Moment about* A.
Ground floor	192 lb. × 5·67 ft. =	1,088 ft.-lb.
Wall (as before) . . .	1,864 ,,	8,245 ,, ,,
	2,056 ,,	9,333 ,, ,,

The resistance offered to sliding is

$$0{\cdot}4 \times 2{,}056 = 822 \text{ lb.}$$

For this stage we will assume that the full pressure of the surcharge acts but that the basement floor slab has not been constructed. Due to the horizontal propping effect of the ground floor the approximate horizontal force acting at the bottom of the wall is

$$\frac{4}{5} \times 1{,}639 = 1{,}311 \text{ lb.}$$

$$\frac{5}{8} \times 671 = 419 \text{ ,,}$$

$$1{,}730 \text{ lb.}$$

This is in excess of the resistance to sliding of 822 lb. Hence the props put in during the first stage must be retained until the basement floor has been constructed.

The ground pressures and bending moments in the base slab need not be considered in this case. The bending moments on the wall stem, assuming an effective span of 10 ft., are due to a total pressure, triangularly distributed, of $271 \times 10 \times \frac{1}{2} = 1{,}355$ lb. and a uniformly distributed pressure of $61 \times 10 = 610$ lb. The bending moment at the bottom of the wall is

$$-\frac{1{,}355 \times 10 \times 12}{7{\cdot}5} - \frac{610 \times 10 \times 12}{8}$$

$$= -21{,}700 - 9{,}150 = -30{,}850 \text{ in.-lb.}$$

At a point a little above the mid-height of the wall there will be a positive bending moment of

$$\frac{1{,}355 \times 10 \times 12}{16{\cdot}7} + \frac{610 \times 10 \times 12}{14{\cdot}2} = +14{,}900 \text{ in.-lb.}$$

These moments are transferred to the diagram in *Fig.* 43.

Permanent Conditions.

When the whole building is completed the wall is subjected to additional forces due to the live load on the ground floor and to the column loads, and there will be an additional moment from the restraint at the end due to the ground floor slab and beams. Basing our deductions on calculations given in previous chapters the following values will be given to the loads and bending moments. Load from external column = 229,590 lb.

$$= \frac{229{,}590}{24} = 9{,}550 \text{ lb. per foot run.}$$

Reaction from ground floor slab = 800 lb. per foot run.
Reaction from ground floor main beam

$$= 19{,}200 \text{ lb.} = \frac{19{,}200}{24} = 800 \text{ lb. per foot run.}$$

Bending moment from ground floor slab

$$= -12{,}800 \text{ in.-lb. per foot run.}$$

Bending moment from ground floor main beam

$$= -750{,}000 \text{ in.-lb.} = \frac{750{,}000}{24} = 31{,}250 \text{ in.-lb. per foot run.}$$

The total applied bending moment is therefore − 44,050 in.-lb. per foot run and will be constant throughout the height of the wall (see *Fig.* 43).

The additional vertical loads can be combined with the weight of the wall as follows (see *Fig.* 42).

Column	9,550 lb. × 5·42 ft. =	51,800 ft.-lb.
Slab	800 ,, × 5·67 ,, =	4,533 ,, ,,
Beam	800 ,, × 5·21 ,, =	4,170 ,, ,,
Wall (as before)	1,864 ,,	8,245 ,, ,,
	13,014 lb.	68,748 ft.-lb.

The centre of action of the vertical forces is $\frac{68{,}748}{13{,}014} = 5{\cdot}27$ ft. from A, and the eccentricity of the load about the centre of the 6-ft. base slab is therefore 2·27 ft. This causes a positive moment of $13{,}014 \times 2{\cdot}27 = 29{,}600$ ft.-lb. Combined with the negative bending moment of $\frac{44{,}050}{12} = -3{,}670$ ft.-lb., the resultant positive bending moment is 25,930 ft.-lb. At the bottom of the wall the bending moment due to the earth pressures as calculated in the previous stage is $\frac{30{,}850}{12} = -2{,}570$ ft.-lb., and to this extent the positive bending moment due to the eccentricity of the vertical loading will be relieved. As the pressure due to the surcharge, and perhaps part of the pressure due to the filling, may not be fully exerted all the time, an erroneous view of the conditions may be obtained if the full bending moment is deducted from the positive bending moment of 25,930 ft.-lb. Suppose, therefore, the latter moment is reduced only to 24,000 ft.-lb., which for finding the maximum ground pressure will be combined with a vertical load of 13,014 lb. This gives an eccentricity of $\frac{24{,}000}{13{,}041} = 1{\cdot}84$ ft. As this exceeds $\frac{6 \text{ ft.}}{6}$ the maximum ground pressure is given by

$$\frac{4 \times 13{,}014}{3[6 - (2 \times 1{\cdot}84)]} = 7{,}500 \text{ lb. per square foot}$$

$$= 3{\cdot}35 \text{ tons per square foot,}$$

which is within the allowable value of 4 tons per square foot.

From the distribution of ground pressure shown in *Fig.* 42 the bending moment at the edge of the splay on the base slab is

$$\frac{4{,}475 \times 2{\cdot}083^2 \times 12}{6} = 39{,}000 \text{ in.-lb. less}$$

18,200 in.-lb. due to the weight of the slab itself, which gives a net bending moment

of 20,800 in.-lb. The bending moments in the wall stem are the same as calculated for the intermediate stage.

Detail Design of Wall.

Having investigated the stability and ground pressure, it only remains to consider the internal resistance of the structure by checking the concrete stresses and calculating the amount of reinforcement required. The diagram (*Fig.* 43) gives the moments that may occur under the three conditions considered, and the envelope of the critical moments is indicated by a heavy line.

Treating the base first, the maximum bending moment, which occurs in the preliminary stage, is — 59,800 in.-lb. The minimum effective depth required is

$$\sqrt{\frac{59{,}800}{208 \times 12}} = 4 \cdot 9 \text{ in.}$$

The effective depth of the 12-in. slab provided is 10·69 in. (with 1-in. cover and ⅝-in. bars). The reinforcement required is

$$\frac{59{,}800}{18{,}000 \times 0 \cdot 84 \times 10 \cdot 69} = 0 \cdot 36 \text{ square inches.}$$

The reinforcement provided (*Fig.* 44) is ⅝-in. bars at 9-in. centres (bars " b ") plus part of the value of bars " a ".

The critical bending moment for determining the reinforcement on the back face at the bottom of the wall stem is — 74,200 in.-lb., requiring an effective depth of 5·4 in. For an 8-in. slab with an effective depth of 6·94 in. the area of steel required is 0·69 sq. in. The arrangement of bars shown in *Fig.* 44, which provides ⅝-in. bars at 4½-in. centres, will therefore be suitable.

During the second stage a positive bending moment of 14,900 in.-lb. is developed in the wall stem. At an approximate effective depth of 7·25 in. the vertical reinforcement required on the inside face is 0·132 sq. in., which is provided by ⅝-in. bars at 9-in. centres.

The compressive stress in the concrete of the wall stem, found by combining the bending moment and vertical loading occurring in the final stage, should be investigated. At the top of the splay the bending moment is approximately — 64,000 in.-lb. and the central direct load is

Wall stem . . .	9·25 × 96 lb.	=	880 lb. per foot run.
Ground floor slab . .		=	800 ,, ,, ,, ,,
	Total	=	1,680 ,, ,, ,, ,,

The vertical loads from the columns and ground floor beams will be considered separately. The magnitude of the resultant eccentricity of $\frac{64{,}000}{1{,}680} = 38$ in. suggests that the following approximate method can be used for calculating the compressive stress in the concrete. With an effective depth of 6·94 in. and an area of tensile reinforcement of 0·818 sq. in. per foot width, the percentage of steel is

$$\frac{0 \cdot 818}{12 \times 6 \cdot 94} \times 100 = 0 \cdot 98.$$

From the curves on *Table* 10 the corresponding neutral-axis factor (with $m = 15$)

is 0·415 and the lever-arm factor is therefore 0·86. Due to the bending moment acting alone, the compressive stress on the concrete (neglecting the small amount of compression steel) is

$$c_1 = \frac{2M}{n_1 a_1 b d^2}$$

$$= \frac{2 \times 64{,}000}{0{\cdot}415 \times 0{\cdot}86 \times 12 \times 6{\cdot}94^2} = 620 \text{ lb. per square inch.}$$

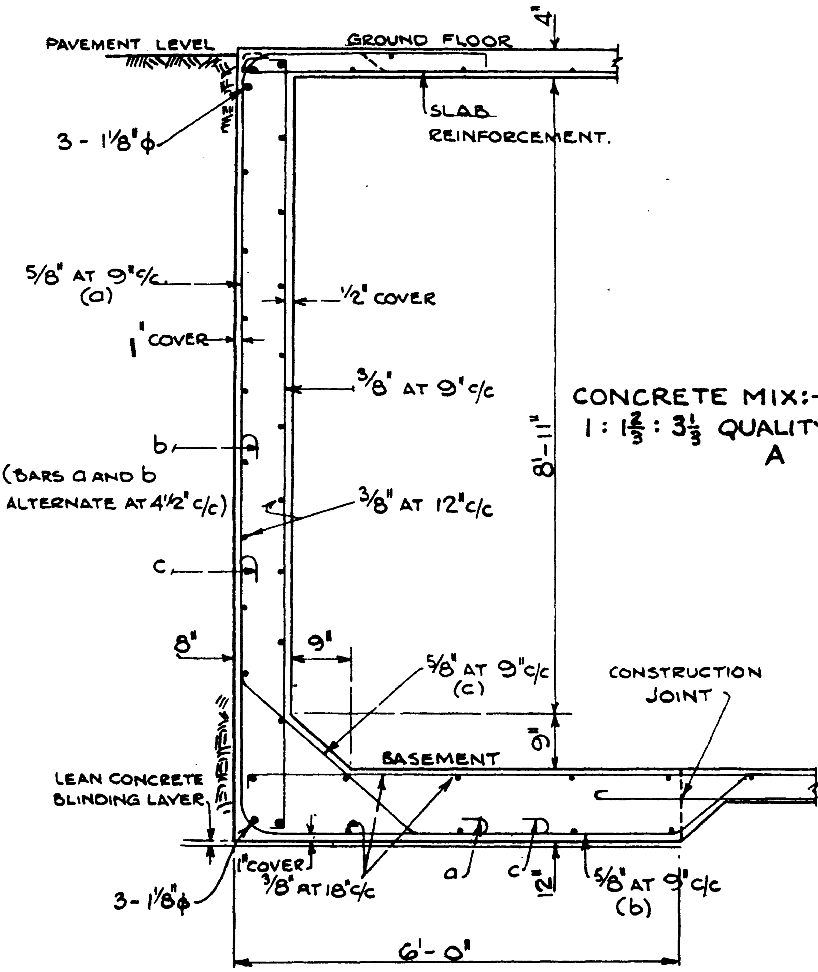


FIG. 44.—REINFORCEMENT.

To this should be added a stress

$$c_2 = \frac{N}{n_1 b d + m A_T}$$

$$= \frac{1{,}680}{(0{\cdot}415 \times 12 \times 6{\cdot}94) + (15 \times 0{\cdot}818)} = 47 \text{ lb. per square inch.}$$

The maximum concrete stress under these conditions is $c_1 + c_2 = 667$ lb. per square inch, which is well within the maximum allowable stress of 1,051 lb. per square inch for bending.

The piers provided under each of the columns will transmit the load from the columns and main beams to the base slab which, with the wall stem, will act as a distributing beam to transfer the load to the ground. The total load on each pier is

Column load	=	229,590 lb.
Main beam	=	19,200 ,,
		248,790 lb.

If the piers are 18-in. wide and project 9 in. from the wall face and are reinforced with six $1\frac{1}{8}$-in. bars and two $\frac{3}{4}$-in. bars ($A_c = 6{\cdot}84$ sq. in.) which will project above the ground floor level to act as splice bars for column D (see *Fig.* 14), the load-carrying capacity at a working direct stress of $1{,}051 \times 0{\cdot}80 = 841$ lb. per square inch will be

Concrete	$841 \times 18 \times 17$	= 257,000 lb.
Steel	$6{\cdot}84 \times 841 \times 14$	= 87,000 ,,
		344,000 lb.

Thus the pier is able to carry the vertical loading without assistance from the wall stem, but actually the load will be distributed between the pier and the adjacent parts of the 8-in. wall stem.

The bending moment due to spreading the column and beam load over 24 ft. can be taken as $248{,}790 \times 24 \times \frac{12}{12} = 5{,}970{,}000$ in.-lb. The effective depth of the wall acting as a beam is, say, 10 ft. 6 in., and the compressive resistance moment due to the 8-in. stem alone is $208 \times 8 \times 126^2 = 26{,}500{,}000$ in.-lb., which is ample without considering the effect of the ground-floor slab at sections near the piers or the base slab at sections midway between the piers. The longitudinal reinforcement required is

$$\frac{5{,}970{,}000}{18{,}000 \times 0{\cdot}84 \times 126} = 3{\cdot}10 \text{ sq. in.}$$

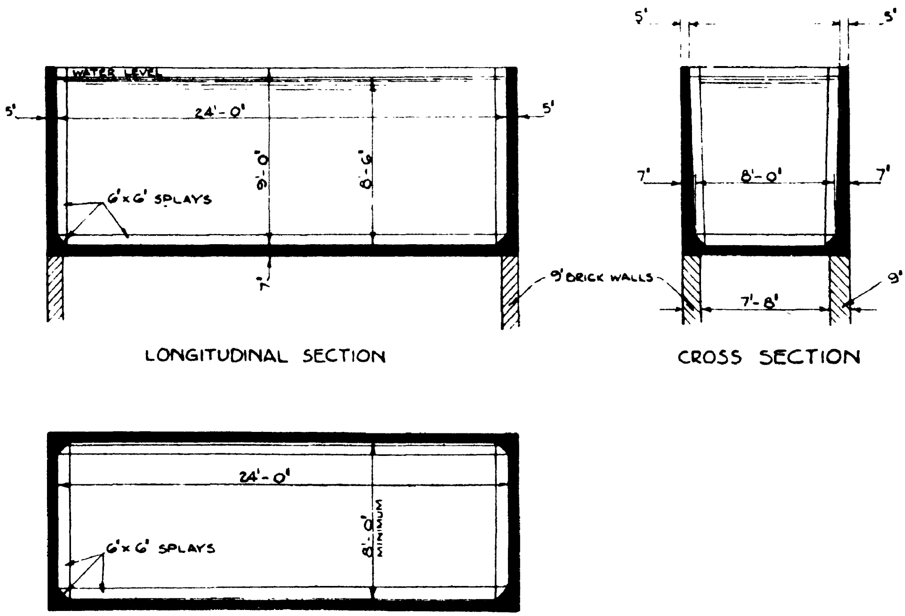
which can be provided by three $1\frac{1}{8}$-in. bars at the bottom and top of the wall stem. The horizontal steel in the wall stem can be provided in accordance with the requirements of the By-laws for distribution steel (*Table* 17) or in conformity with conventional retaining wall design ; however, the longitudinal bars will be not less than $\frac{3}{8}$-in. in diameter spaced at not more than 12-in. centres on both faces of the wall.

CHAPTER XIII

RECTANGULAR TANK

Design Data and Stresses.

THE example in this chapter is an open 10,000-gallon rectangular tank to be constructed on the roof of a building. It is assumed that the tank will be supported on brick walls as shown in *Fig.* 45 and that a convenient internal size is 24 ft. long by 8 ft. wide. To give the required capacity the minimum depth of water is $\frac{10,000}{6{\cdot}24 \times 24 \times 8} = 8{\cdot}35$ ft.



PLAN

FIG. 45.—RECTANGULAR TANK: 10,000 GALLONS CAPACITY.

By making the tank 9 ft. deep and fixing the overflow to give an 8-ft. 6-in. depth of water the required capacity is obtained with sufficient clearance between the top of the walls and the water level. In all horizontal and vertical corners 6-in. by 6-in. splays are provided. The longitudinal walls will be designed as cantilevers and the end walls as slabs carrying the greater part of their loads in a horizontal direction.

For structures to contain water it is usual to provide a richer mix of concrete than in normal building work, and a mix of nominally 1 : 1⅔ : 3⅓ parts by volume is commonly used. To ensure an impermeable concrete it is better to increase the sand content of the mixture relative to the coarse aggregate content, and in the present case a mix of 2½ cu. ft. of fine aggregate and 3¾ cu. ft. of coarse aggregate to each 112-lb. bag of cement is used. In any given case, the ratio of fine to coarse aggregate that gives the maximum density (that is, the least combined volume for given volumes of separate materials), should be determined and adopted with a small increase to the quantity of fine aggregate.

With ordinary quality concrete of the mix specified By-law 99 permits a working stress in compression on the concrete in bending of 817 lb. per square inch, but for structures to contain water, it is advisable to use lower stresses than in building work. In this case the maximum stresses will be taken as:

	lb. per square inch.
Concrete in compression (c)	750
Reinforcement in tension:	
On water face of slabs	12,000
On faces not in contact with water	16,000

With a modular ratio of 15 the design factors for bending will be:

Steel stress (lb. per square inch) (t) . . .	12,000	16,000
Neutral-axis factor $\left(n_1 = \dfrac{1}{1 + \dfrac{t}{mc}}\right)$	0·49	0·42
Lever-arm factor $\left(a_1 = 1 - \dfrac{n_1}{3}\right)$	0·84	0·86
Resistance-moment factor $\left(Q = \dfrac{a_1 n_1 c}{2}\right)$	152	134

For bars on the water-face of the walls and bottom slab a minimum cover of concrete of ¾ in. will be provided; on faces not in contact with water a minimum cover of ½ in. or the diameter of the bar (whichever is greater) will be provided.

Longitudinal Walls.

The pressure on the walls varies as shown in *Fig.* 46 from zero at water level to a maximum of $8·5 \times 62·4 = 530$ lb. per square foot at the bottom. Designing the longitudinal walls to act as vertical cantilevers, the critical section is at the top of the splay where the pressure is $8·0 \times 62·4 = 499$ lb. per square foot. The bending moment at this section is

$$\frac{499 \times 8^2 \times 12}{6} = 64{,}000 \text{ in.-lb.}$$

As this moment produces tension on the face next the water the effective depth required is

$$\sqrt{\frac{64{,}000}{12 \times 152}} = 5·9 \text{ in.}$$

Thus a 7-in. slab will be required immediately above the splay and the thickness of the wall can be tapered to 5 in. at the top. The reinforcement required at the bottom is

$$\frac{64{,}000}{12{,}000 \times 0{\cdot}84 \times 5{\cdot}875} = 1{\cdot}08 \text{ sq. in. per foot run of wall.}$$

Use $\frac{3}{4}$-in. bars at $4\frac{1}{2}$-in. centres. This amount of steel need not be carried to the full height of the wall, and it will be satisfactory if the $\frac{3}{4}$-in. bars are stopped at the mid-height of the wall and if $\frac{1}{2}$-in. diameter bars lapped with each alternate

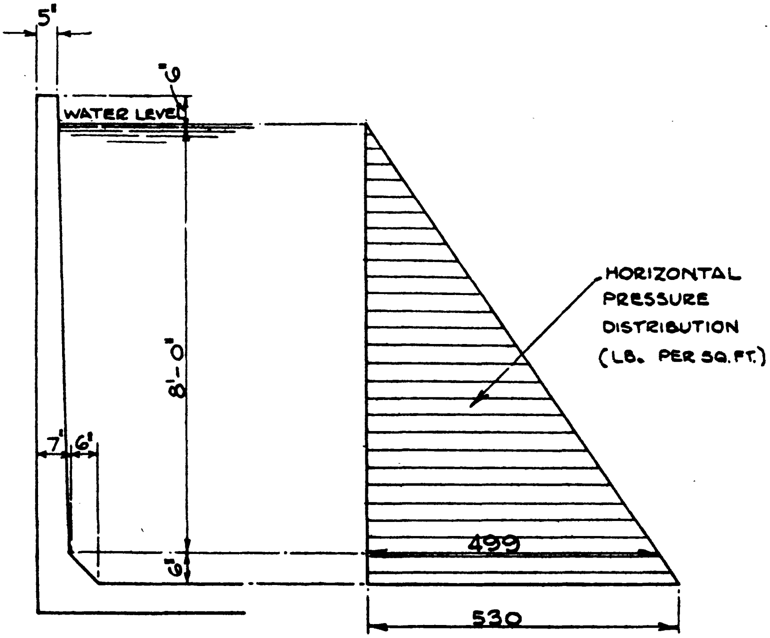


FIG. 46.—PRESSURES ON LONGITUDINAL WALLS.

bar are carried to the top as shown in *Fig.* 47. The bending moment at mid-height of the wall is

$$\frac{62{\cdot}4 \times 4^3 \times 12}{6} = 8{,}000 \text{ in.-lb.}$$

The effective depth being a little over 5 in., the area of steel required per foot run is a little less than

$$\frac{8{,}000}{12{,}000 \times 0{\cdot}84 \times 5{\cdot}00} = 0{\cdot}16 \text{ sq. in.}$$

Half-inch bars at 9-in. centres are sufficient.

The $4\frac{1}{2}$-in. spacing of the vertical bars need not be continued into the corners where, to a considerable degree, the longitudinal walls tend to span horizontally. Except in the corners, which will be given further consideration later, the horizontal steel in the longitudinal walls must be sufficient to take the reaction

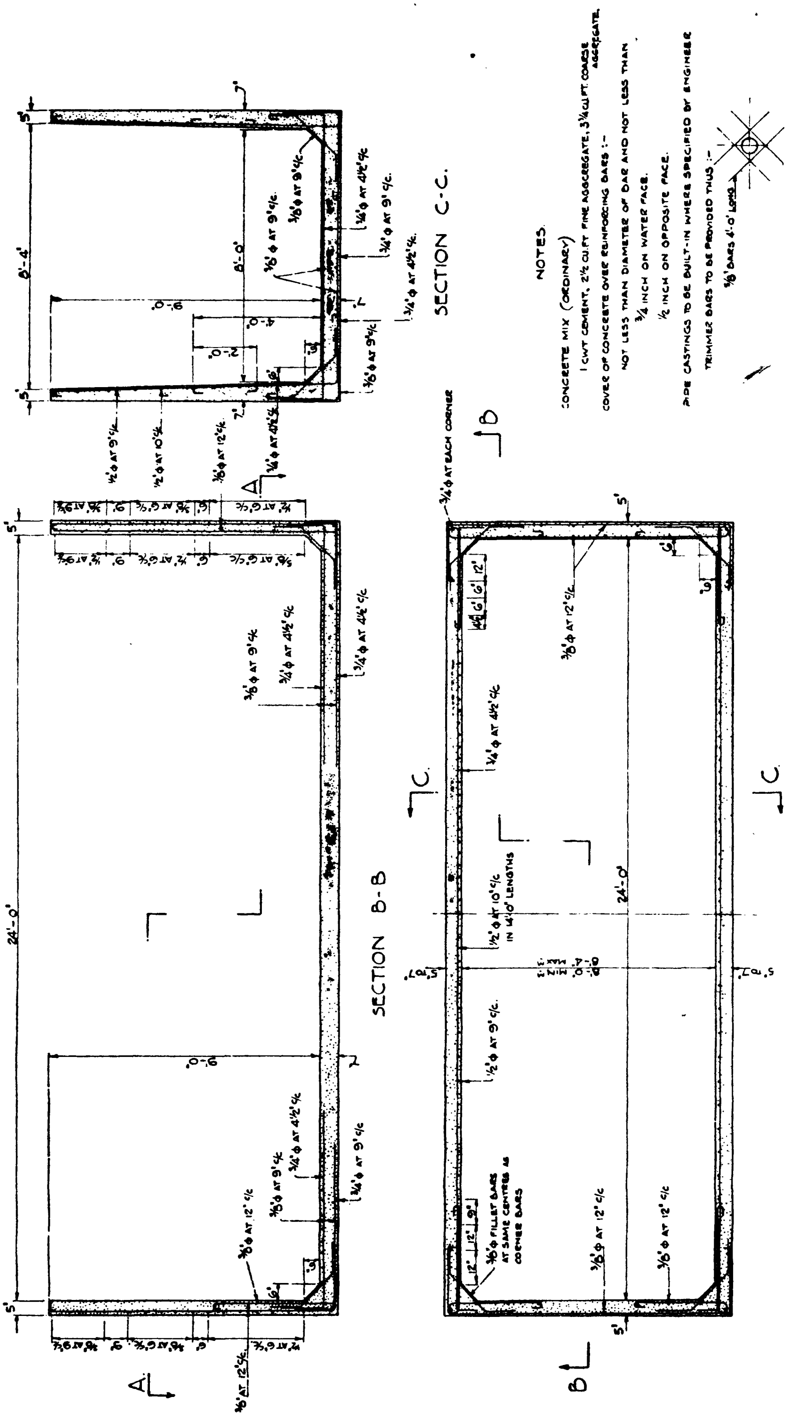


Fig. 47.—Arrangement of Reinforcement.

from the end walls. If it is assumed that this reaction reaches a maximum at 6 ft. below the water level, the tensile force to be resisted in 1-ft. height of longitudinal wall is

$$6 \times 62{\cdot}4 \times \frac{8}{2} = 1{,}498 \text{ lb. per foot.}$$

Since the thickness of the wall is about $6\frac{1}{2}$ in. at a section 6 ft. from the top, the tensile stress in the concrete, neglecting any horizontal reinforcement, is $\frac{1{,}498}{12 \times 6{\cdot}5} = 19{\cdot}2$ lb. per square inch, which is well within the value of 200 lb. per square inch that is commonly allowed for direct tension in impermeable construction. The area of steel required to resist this horizontal tension, assuming that the concrete is neglected, is $\frac{1{,}498}{12{,}000} = 0{\cdot}125$ sq. in. The minimum amount of horizontal steel often recommended for water-containing structures is 0·3 per cent. of the concrete area, which in this case at mid-height is

$$\frac{0{\cdot}3}{100} \times 12 \times 6{\cdot}0 = 0{\cdot}216 \text{ sq. in.}$$

If $\frac{1}{2}$-in. bars at 10-in. centres are provided as shown in *Fig.* 47 this requirement is fulfilled and the direct tension requirements are also amply covered.

Transverse Walls.

In designing the end walls to span principally in a horizontal direction, allowance will be made for continuity around the corners. At any depth h ft. below the water level the maximum bending moment, positive or negative, is:

At midspan: $$M = \frac{62{\cdot}4h \times 8{\cdot}5^2 \times 12}{16} = 3{,}370h \text{ in.-lb. per foot of height.}$$

At the support: $$M = \frac{62{\cdot}4h \times 8{\cdot}5^2 \times 12}{12} = 4{,}500h \quad ,, \quad ,, \quad ,,$$

The bending moment coefficients of $\frac{1}{16}$ and $\frac{1}{12}$ are obtained by considering the bending moment diagram in *Fig.* 48. The free-moment diagram is the parabola whose base $L = 8$ ft. 6 in. and of which the maximum ordinate is $0{\cdot}125pL^2$. If the ends of the span were completely fixed the maximum negative bending moment would be $0{\cdot}0833pL^2$ and the maximum positive bending moment would be $0{\cdot}0417pL^2$. The critical section for the negative bending moment is at the beginning of the splays; in the present case the bending moment there is $0{\cdot}0781pL^2$. Various factors operate to amend these theoretical bending moments. To allow for the possibility of not obtaining complete fixity at the ends, the maximum positive bending moment is increased by 50 per cent., that is, to $0{\cdot}0625pL^2 = +\frac{pL^2}{16}$. On the other hand, given complete fixity, the increased moment of inertia at the corners due to the splays tends to increase the negative bending moment. If an increase of about 6 per cent. is made in the negative moment coefficient, the resulting bending moment is $-\frac{pL^2}{12}$. The

foregoing method of adjustment is only justified if L is the full span between the centres of the supports, that is 8 ft. 6 in. in the example.

To secure the continuity at the corners it is necessary to provide reinforcement horizontally along the inside face of the longitudinal walls for a distance not less than x (*Fig.* 48). Although the reaction from the pressure on the longitudinal walls is provided by the bottom of the tank throughout the greater part of the length of these walls, the reaction from the end portion x is mainly taken on the transverse walls. It is therefore reasonable to assume that the direct tension on each transverse wall is due to the water pressure on, say, 3 ft. of the longitudinal wall. Thus at any depth h ft. below the free surface of the water the direct tension is

$$T = 62{\cdot}4h \times 3 = 187{\cdot}2h \text{ lb. per foot of height.}$$

The thickness of the end walls can be determined by considering the maximum bending moment. At the bottom the wall spans vertically as a canti-

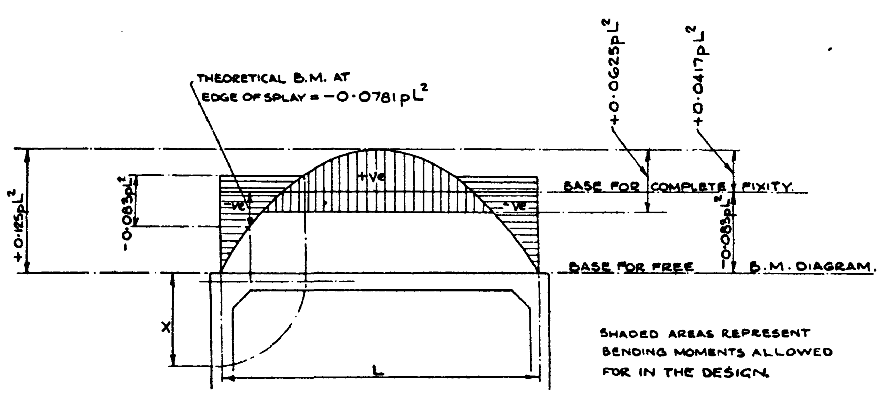


FIG. 48.—BENDING MOMENTS ON END WALLS.

lever, fixity with the bottom slab prohibiting any horizontal deformation. At some point up the wall, say point Y_1 (*Fig.* 49), this restriction due to the bottom slab is negligible, and from this point upwards the wall spans horizontally. The horizontal and vertical distribution of pressure will be as shown on *Fig.* 49. If point Y is 6 ft. below the water level, the maximum horizontal bending moment is $4{,}500 \times 6 = -27{,}000$ in.-lb. With $Q = 152$, the effective depth required is 3·8 in., for which a 5-in. slab is satisfactory. This thickness will be retained throughout the end walls.

The critical bending moments and direct tensions can be combined as follows :
At the corners :

$$\text{Eccentricity} = \frac{M}{T} = \frac{4{,}500h}{187{\cdot}2h} = 24 \text{ in.}$$

$$\text{Eccentricity about the centre of the tension steel} = 24 - \frac{5}{2} + 1 = 22{\cdot}5 \text{ in.}$$

$$\text{Lever arm} = 0{\cdot}84 \times 4 = 3{\cdot}36 \text{ in.}$$

Area of reinforcement required at 12,000 lb. per square inch

$$= \frac{187 \cdot 2h}{12{,}000}\left(1 + \frac{22 \cdot 5}{3 \cdot 36}\right)$$

$$= 0 \cdot 121h \text{ sq. in. per foot.}$$

At midspan: Eccentricity $= \frac{3{,}370h}{187 \cdot 2h} = 18$ in.

Eccentricity about steel $= 18 - \frac{5}{2} + 0 \cdot 75 = 16 \cdot 25$ in.

Lever arm $= 0 \cdot 86 \times 4 \cdot 25 = 3 \cdot 65$ in.

Area of steel at 16,000 lb. per square inch

$$= \frac{187 \cdot 2h}{16{,}000}\left(1 + \frac{16 \cdot 25}{3 \cdot 65}\right)$$

$$= 0 \cdot 0639h \text{ sq. in. per foot.}$$

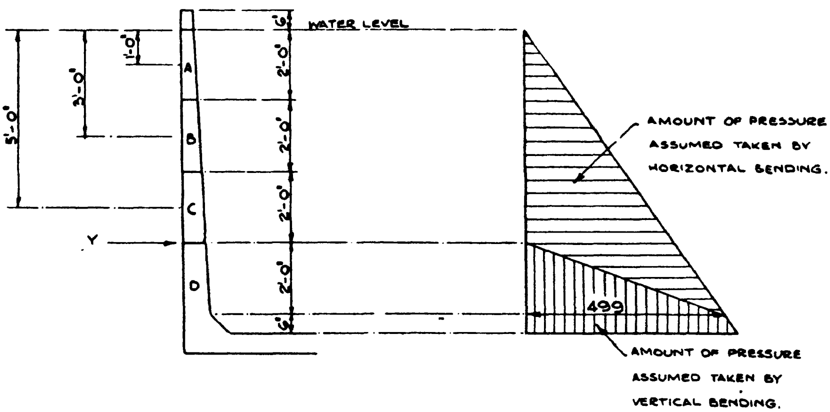


FIG. 49.—PRESSURE ON END WALLS.

If the wall is now divided into a convenient series of horizontal strips, say 2 ft. each in height, as in *Fig.* 49, the area of reinforcement can be determined by substituting for h the mean depth of each strip below the water level. Thus

Strip	Value of h (ft.)	At Corners		At Midspan	
		Area (sq. in.)	Bars	Area (sq. in.)	Bars
A	1·0	0·121	½-in. 9-in. crs.	0·064	⅜-in. 9-in. crs.
B	3·0	0·363	½-in. 6-in. crs.	0·192	⅜-in. 6-in. crs.
C	5·0	0·605	⅝-in. 6-in. crs.	0·320	½-in. 6-in. crs.

This arrangement of reinforcement provides the areas required in a practicable manner. The spacing in strip A is maintained at a maximum of 9 in.

although less reinforcement than this is actually required. The spacing in strip C is continued throughout strip D.

The area of the vertical steel at the bottom of the end walls is calculated from the cantilever moment. Due to the amount of pressure shown as spanning vertically (*Fig.* 49) the approximate bending moment is

$$\frac{499 \times 2^2}{6} \times 12 = 4{,}000 \text{ in.-lb.}$$

With an effective depth of $3\frac{1}{2}$ in. and a stress of 12,000 lb. per square inch, the area of reinforcement required is 0·114 sq. in. The vertical bars provided, $\frac{3}{8}$-in. at 12-in. centres, are therefore sufficient.

Bottom Slab.

As shown in *Fig.* 45 the walls of the tank are supported directly on brick walls between which the slab forming the bottom of the tank spans transversely. Since the longitudinal walls have been designed as cantilevers it is necessary to provide a resistance in the bottom slab at the foot of the walls equal to that at the base of the wall stem. Thus a 7-in. bottom slab is required, and when the tank is full the loading on this slab is

	lb. per square foot
Water:	$8{\cdot}5 \times 62{\cdot}4 = 530$
Slab:	$7 \times 12 = 84$
	Total $= 614$

The free bending moment is

$$\frac{614 \times 8{\cdot}583^2 \times 12}{8} = 68{,}000 \text{ in.-lb.}$$

The negative moment at each end of the slab due to the cantilever action of the walls is

$$\frac{530 \times 8{\cdot}5^2 \times 12}{6} = 76{,}600 \text{ in.-lb.}$$

As will be seen from the bending moment diagram (*Fig.* 50), when the tank is full a negative moment exists throughout the base slab, this moment having a minimum value at midspan, where the bending moment is 76,600 − 68,000 = 8,600 in.-lb. This condition does not always occur, for if the tank is only partly full it can be shown that positive bending moments are produced in the bottom slab. Thus with any depth h ft. of filling, the free positive bending moment due to the water alone is

$$\frac{62{\cdot}4h \times 8{\cdot}583^2 \times 12}{8} = 6{,}900h \text{ in.-lb.}$$

The negative bending moment is $\dfrac{62{\cdot}4h^3 \times 12}{6} = 125h^3$ in.-lb. Hence the net bending moment at midspan is $6{,}900h - 125h^3$. By differentiating this expression and equating the result to zero, we can determine the value of h that gives the maximum positive bending moment in the slab. Thus

$$6{,}900 - 375h^2 = 0$$

from which $$h = 4{\cdot}3 \text{ ft.}$$

Substituting this value in the expression for the bending moment at midspan, we have

$$(6{,}900 \times 4{\cdot}3) - (125 \times 4{\cdot}3^3) = 19{,}950 \text{ in.-lb.}$$

Adding to this bending moment the positive moment due to the dead weight of the slab, that is $\dfrac{84 \times 8{\cdot}583^2 \times 12}{8} = 9{,}300$ in.-lb., the total positive bending moment is $19{,}950 + 9{,}300 = 29{,}250$ in.-lb.

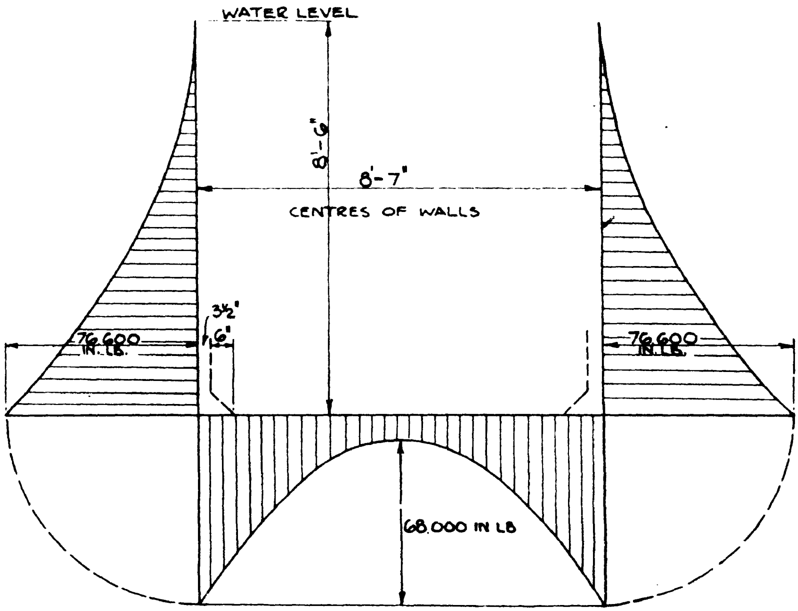


FIG. 50.—BENDING MOMENTS (TANK FULL).

The direct tension acting along with this bending moment is that due to the reaction from a pressure of a 4-ft. 3-in. head of water on the longitudinal walls.

Thus $$T = \frac{4{\cdot}25^2}{2} \times 62{\cdot}4 = 563 \text{ lb. per foot.}$$

$$e = \frac{29{,}250}{563} = 51{\cdot}8 \text{ in.}$$

With a 7-in. slab having an effective depth of 6·06 in.

$$e_s = 51{\cdot}8 - 3{\cdot}5 + 0{\cdot}94 = 49{\cdot}24 \text{ in.}$$

The area of steel required at a stress of 16,000 lb. per square inch is

$$\frac{563}{16{,}000}\left(1 + \frac{49{\cdot}24}{0{\cdot}86 \times 6{\cdot}06}\right) = 0{\cdot}368 \text{ sq. in.}$$

This is amply provided by the ¾-in. bars at 9-in. centres extending from the longitudinal walls.

The maximum amount of steel required in the top face of the bottom slab at the edge of the splays on the bottom of the longitudinal walls is calculated by combining a negative bending moment of about 64,000 in.-lb. with a direct tension of $\frac{530 \times 8\cdot5}{2} = 2{,}250$ lb. (see *Figs*. 45 and 50).

$$e = \frac{64{,}000}{2{,}250} = 28\cdot4 \text{ in.}$$

$$e_s = 28\cdot4 - 3\tfrac{1}{2} + 1\cdot125 = 26\cdot025 \text{ in.}$$

$$A_T = \frac{2{,}250}{12{,}000}\left(1 + \frac{26\cdot025}{0\cdot84 \times 5\cdot875}\right) = 1\cdot185 \text{ sq. in.}$$

This area is provided by $\frac{3}{4}$-in. bars at $4\frac{1}{2}$-in. centres.

The area of longitudinal steel in this slab is

$$\frac{0\cdot3}{100} \times 7 \times 12 = 0\cdot252 \text{ sq. in. per foot,}$$

which is provided by two layers of $\frac{3}{8}$-in. bars at 9-in. centres.

The bearing pressure on the brickwork should be checked. The load per foot run is approximately:

Walls:	$9\cdot0 \times 72$ lb. average =	648 lb.
Water:	$8\cdot5 \times 4\cdot0 \times 62\cdot4$ lb. =	2,130 ,,
Bottom slab:	$4\cdot58 \times 84$ lb. =	386 ,,
		3,164 ,,

On 9-in. walls the bearing pressure is $\frac{3{,}164}{2{,}240 \times 0\cdot75} = 1\cdot88$ tons per square foot, which is well within the permissible bearing stress allowed on good brickwork laid in Portland cement, according to By-law 60.

APPENDIX I *

[BY-LAWS FOR THE CONSTRUCTION AND CONVERSION OF BUILDINGS IN REINFORCED CONCRETE MADE BY THE LONDON COUNTY COUNCIL IN PURSUANCE OF THE LONDON BUILDING ACT (AMENDMENT) ACT, 1935.]

Attention is drawn to the Council's Power of Modification or Waiver under Section 9 of the Act.

Interpretation.

1. In these by-laws except where expressly stated to the contrary or the context otherwise requires, the following expressions have the meanings hereby respectively assigned to them, that is to say:

" Aggregate " means all material, other than cement and water, used in the making of concrete.

" Base " in relation to a wall or pier means (1) the underside of the course immediately above the footings, if any, or in the case of a wall carried by a beam above the beam ; (2) in any other case, the bottom of such wall or pier.

" Beam " means any part of construction which will, by its resistance to bending, support or transmit loading.

" Column " in relation to reinforced concrete means any part of construction which will by its resistance to compression in the direction of its length and to bending actions induced by such compression, support or transmit loading.

" Dead loading " means the weight of all walls, floors, roofs, partitions and other like permanent construction.

" Height " in relation to a wall or pier means the vertical dimension measured from the base of such wall or pier to the top thereof, or if the top be shaped as a gable, to midway between the base of the gable and the top thereof.

" Lateral support " in relation to a wall or pier means such support in the direction of the thickness, length or breadth of such wall or pier as prevents movement thereof at the level and in the line of the direction of such support.

" Length " in relation to any part of a wall means the greater horizontal dimension. The expression " wall " implies a length exceeding six times the thickness.

If the horizontal section of a wall at any level is not a rectangle then the length means the average length at that level.

" Lintel " means a beam not exceeding eight feet in length measured from end to end supporting walling over an opening or recess.

" Load-bearing " in relation to any part of a building (including the foundation) means any such part bearing a load other than that due to its own weight and to wind pressure on its own surface.

" Mesh " in relation to the measurement of materials means the mesh of a sieve complying with the British Standard Specification for Test Sieves No. 410—1931.

" Partition wall " means any internal wall not being a division wall nor a party structure.

" Plain concrete " means concrete which is not reinforced for the purpose of compliance with these by-laws.

* The abstract of the By-laws in this Appendix gives only the clauses relating to design in reinforced concrete.

"The principal Act" means the London Building Act, 1930.

"Reinforced concrete" means concrete reinforced for the purpose of compliance with these by-laws.

"Required thickness" in relation to a wall means the thickness required in order to comply with these by-laws.

"Storey-height" in relation to a wall or pier means that part of a wall or pier which is between the level of one lateral support and the level of the lateral support parallel thereto next above, or (if there be no such lateral support above) the top of such wall or pier.

"Superimposed loading" means all loading other than dead loading.

"Thickness" in relation to any part of a wall or pier means the lesser horizontal dimension. If the horizontal section of a wall or pier at any level is not a rectangle then the thickness at that level shall be the average thickness at that level.

"Width" in relation to any part of a pier, means the greater horizontal dimension. If the horizontal section of a pier at any level is not a rectangle then the width means the average width at that level. The expression "pier" implies a width not exceeding six times the thickness.

PART I.—LOADING.

2. Every part of a building shall be so constructed as to be capable of safely sustaining and transmitting all the dead and superimposed loading thereon without exceeding the appropriate limitations of permissible stresses provided in these by-laws.

3. For the purpose of calculating dead loading the weights of materials shall be taken to be as set forth in B.S.S. (Schedule of Unit Weights of Building Materials) No. 648—1935 unless otherwise agreed with the district surveyor.

4. (*a*)—*Schedule of loading*—The minimum superimposed load on each floor and on the roof shall be estimated as equivalent to the following dead loads :

Class No.	Type of building or floor	Slabs. Lb. per sq. ft. of floor area	Beams. Lb. per sq. ft. of floor area
1	Rooms used for residential purposes ; and corridors, stairs and landings within the curtilage of a flat or residence.	50	40
2	Offices, floors above entrance floor	80	50
3	Offices, entrance floor and floors below entrance floor ; retail shops ; and garages for private cars of not more than two and one quarter tons net weight.	80	80
4	Corridors, stairs and landings not provided for in class 1.	Loading to be provided for to be ascertained to the satisfaction of the district surveyor, but not less than :— 100	100
5	Workshops and factories ; and garages for motor vehicles other than private cars of not more than two and one quarter tons net weight.	Loading to be provided for to be ascertained to the satisfaction of the district surveyor, but not less than :— 150	120
6	Warehouses, book stores, stationery stores and the like.	Loading to be provided for to be ascertained to the satisfaction of the district surveyor, but not less than :— 200	200
7	Any purpose not herein specified	Loading to be provided for to be ascertained to the satisfaction of the district surveyor.	

Beams and ribs not spaced further apart than 2 ft. 6 in. between centres shall be designed for slab loads.

Class No.	Roofs	Slabs. Lb. per sq. ft. of covered area	Beams. Lb. per sq. ft. of covered area
8	Flat-roofs and roofs inclined at an angle with the horizontal of not more than twenty degrees.	50	30

Subject to the provision of paragraph (*d*) of this by-law, all columns, piers, walls, foundations, and other supports to beams shall be calculated for the superimposed loads tabulated above in this paragraph for beams.

On roofs inclined at an angle with the horizontal of more than 20 deg. a minimum superimposed load (deemed to include the wind load) of 15 lb. per square foot of surface shall be assumed acting normal to the surface inwards on the windward side, and 10 lb. per square foot of surface acting separately and not simultaneously outwards on the leeward side. This requirement shall apply only in the design of the roof construction, and a vertical superimposed load of 10 lb. per square foot of covered area shall be substituted for it in estimating the vertical superimposed roof load upon all other parts of the construction.

(*b*) In all cases of floors where the positions of partitions are not definitely located in the design, a uniformly distributed load sufficient to allow for them shall be added to the dead floor load. For all floors of rooms used for offices the minimum total allowance for internal partitions shall be at the rate of 20 lb. per square foot of floor area.

(*c*) Slabs and beams shall be capable of carrying in accordance with these by-laws the following superimposed loads in any position on an otherwise unloaded floor:

Class of floor	Minimum superimposed load	
	Slabs	Beams
Floors tabulated under class 1.	$\frac{1}{4}$ ton uniformly distributed per foot width.	1 ton uniformly distributed.
All floors tabulated under classes 2 to 6 except garage floors tabulated under class 5.	$\frac{3}{8}$ ton uniformly distributed per foot width.	2 tons uniformly distributed.
Garage floors tabulated under class 5.	$1 \cdot 5 \times$ maximum possible combination of wheel loads, but each wheel load not less than 1 ton.	

Provided that beams and ribs spaced not further apart than 2 ft. 6 in. between centres shall be calculated for the slab loads tabulated in this paragraph; provided also that non-load-bearing beams such as beams solely employed as ties to columns shall be exempt from any load calculation under this paragraph. The reactions due to the superimposed loads tabulated in this paragraph need not be allowed for in calculating the loads on columns, piers, walls or foundations.

(*d*) For the purpose of calculating the total load to be carried on columns, piers, walls and foundations in buildings of more than two storeys in height, and in which the loads and stresses are transmitted through each storey to the foundations, (i) wholly by a skeleton framework of structural steel, *or* (ii) partly by a skeleton framework of structural steel and partly by a party wall or party walls, *or* (iii) wholly by a skeleton framework of reinforced concrete, *or* (iv) partly by a skeleton framework of reinforced concrete and partly by a party wall or party walls, the superimposed loads for the roof and topmost storey shall be calculated in full in accordance with the schedule of loading in paragraph (*a*) of this by-law, but for the lower storeys a

reduction of the superimposed loads may be allowed in accordance with the following table :

Next storey below topmost storey .	10 per cent. reduction of its superimposed load.
Next storey below . . .	20 ,, ,, ,,
Next storey below . . .	30 ,, ,, ,,
Next storey below . . .	40 ,, ,, ,,
Each succeeding storey below .	50 ,, ,, ,,

The above reduction may be made by estimating the proportion of floor area carried by each foundation, column, pier and wall. No such reductions shall be allowed on any floor tabulated in this by-law for a superimposed beam loading exceeding 100 lb. per square foot.

(*e*) In any case where the superimposed load on any floor or roof is to exceed that hereinbefore specified for the floor or roof, such greater load shall be provided for in accordance with By-law 2. In the case of any floor intended to be used for a purpose for which a superimposed load is not specified herein, the superimposed load to be carried on that floor shall be provided for in accordance with By-law 2.

(*f*) In cases where a superimposed load may move, proper provision in accordance with these by-laws to the satisfaction of the district surveyor shall be made for all effects of such movement, including vibration, impact, acceleration and deceleration.

5. In every storey the floor of which is constructed for superimposed loading exceeding 100 lb. per square foot there shall be exhibited by the owner permanently in a conspicuous position in every room a notice in the following form stating the superimposed loading for which the floor has been constructed.

"LONDON BUILDING ACT (AMENDMENT) ACT, 1935.
"NOTICE.

"The floor of this room is constructed for superimposed loading to
"an intensity not exceeding pounds on any square foot
"of its surface."

6. A building shall be so constructed as to resist a wind pressure in each horizontal direction of not less than 15 lb. per square foot on the upper two-thirds of its surface up to the general roof level or ridge which is or which may be exposed to wind pressure and an additional pressure in each horizontal direction of not less than 10 lb. per square foot upon all projections above the general roof level or ridge. If the height of a building be less than twice the width (measured in a direction parallel with that of the wind pressure) of the base upon which the building depends for its resistance to the overturning action of the wind pressure in that direction and provided the district surveyor be satisfied that all loading due to wind pressure will be transmitted safely to the earth, the above wind pressure need not be calculated for the building as a whole ; but provision shall be made in accordance with By-law 2 for all the local loading due to wind pressure.

7. Where loading is transmitted through plain concrete, brickwork or other material of similar consistency, the angle of dispersion of such loading through such material shall be taken as not more than 45 deg. with the direction of such loading.

8. A building or any part thereof shall not be subjected to loading beyond its proper load-bearing capacity at the time when such loading is applied. This provision shall not apply to any loading which may be required by the district surveyor for the purpose of testing.

PART II.—MATERIALS OF CONSTRUCTION.

9. The following provisions shall apply to the aggregates for reinforced concrete :

Aggregate shall be sand and gravel, or crushed natural stone. It shall be hard, strong and durable and shall be reasonably clean and free from clay, organic matter, coal and coal residues (including clinker, ashes, coke-breeze, pan-breeze, slag and other similar material), copper slag, forge breeze, dross (and other similar material), soluble sulphates (including gypsum and other similar material), porous material and other

materials liable to reduce the strength or durability of the concrete, or to attack the steel reinforcement.

Fine aggregate shall be of such a size that it will pass through a $\frac{3}{16}$-in. mesh. Not more than 5 per cent. by weight shall pass a No. 100 mesh.

Coarse aggregate shall be of such a size that it will be retained on a $\frac{3}{16}$-in. mesh and will pass a mesh of a size $\frac{1}{4}$ in. less than the minimum lateral distance between the reinforcing bars.

Aggregate shall be so graded between the limits as to make a dense concrete of the specified proportions and consistency that will work readily into position without segregation and without the use of an excessive water content.

10. Aggregate for plain concrete shall consist of such materials as are specified in By-law 9 or of hard well-burned brick, hard well-burned tile, pumice or other material which the district surveyor may approve as of like suitability and shall be so graded and contain sand in such proportion as to produce a dense concrete.

11. Sand shall be clean and shall be composed of hard siliceous grains reasonably free from clay or any animal, vegetable or bituminous matter. The grains shall be of such a size as to pass through a $\frac{3}{16}$-in. mesh.

12. Cement shall be Portland cement, Portland blastfurnace cement, or high-alumina cement, but no two of such cements shall be used in combination.

Portland cement shall comply with the B.S.S. for Portland Cement, No. 12—1931.

Portland blastfurnace cement shall comply with the B.S.S. for Portland Blastfurnace Cement, No. 146—1932.

High alumina cement shall consist of aluminous and calcareous materials which have been fused to a molten state and ground to such a degree of fineness that the cement will not leave a residue of more than 12 per cent. by weight on a No. 170 mesh and not more than 1 per cent. on a No. 72 mesh. The cement shall contain not less than 35 per cent. by weight of alumina and the ratio of percentage by weight of alumina to that of lime shall be not less than 0·9. When gauged with 22 per cent. by weight of water it shall not begin to set before the expiration of two hours but shall begin to set within 6 hours of gauging and the final setting shall take place within 2 hours of the initial setting. The strength of high alumina cement shall be such that when a mortar is composed of one part by weight of high alumina cement to three parts by weight of white Leighton Buzzard sand graded to pass a No. 18 mesh and stop on a No. 25 mesh, and the whole is gauged with a weight of water equal to 8 per cent. of the dry materials, such mortar shall have a tensile strength of not less than 475 lb. per square inch within 24 hours after gauging and within seven days its tensile strength shall have increased and shall be not less than 550 lb. per square inch.

Wherever cement is used it shall not be moved or disturbed after one hour from the time it has come into contact with water until it has set hard.

13. Water used shall be clean and free from deleterious matter.

14. Concrete shall consist of aggregate mixed with cement. The proportions of fine aggregate to coarse aggregate to cement shall be as set out in Tables I and II or in any intermediate proportions in which the volume of coarse aggregate is twice that of the fine aggregate. Provided that in any particular case where specially authorised by the district surveyor the proportion of coarse aggregate may be varied within the limits of one-and-a-half and two-and-a-half times the fine aggregate. If in any particular case the district surveyor so requires, the proportion of coarse aggregate shall be varied within the aforesaid limits. Where the proportion of coarse aggregate to fine aggregate is so varied the requirements of this by-law shall be based on the ratio of the sum of the volumes of fine and coarse aggregates, each measured separately, to the quantity of cement and shall be obtained by proportion to the two nearest specified mixes. If the district surveyor so requires, the builder shall make such tests in accordance with Schedules I and III of this Part of these by-laws as may be necessary to prove the quality of the concrete. The grades of concrete designated I, II and III in column 1 of Table I and mixed in the proportions set out in column 2 of such Table against each such designation shall be deemed to be "ordinary concrete" and shall, within the period of twenty-eight days, possess the respective

minimum resistance to crushing set out in column 3 of such Table against each such designation. Where intermediate proportions of cement to aggregate are used, as heretofore provided, the minimum crushing strength shall be in proportion to the prescribed minimum crushing strengths of the two nearest specified mixes.

TABLE I.

(1)	(2)		(3)
Designation of concrete	Cubic feet aggregate per 112 lb. of cement		Minimum resistance to crushing in lb. per square inch within 28 days after mixing
	Fine aggregate	Coarse aggregate	
I	$1\frac{1}{4}$	$2\frac{1}{2}$	2,925
II	$1\frac{7}{8}$	$3\frac{3}{4}$	2,550
III	$2\frac{1}{2}$	5	2,250
IV	$7\frac{1}{2}$		1,480
V	10		1,110
VI	$12\frac{1}{2}$		740
VII	15		370

The grades of concrete designated IA, IIA and IIIA in Table II shall be deemed to be "Quality A concrete" and when such concrete is to be used, if the district surveyor so requires, preliminary tests shall be made in accordance with Schedule II of this Part of these by-laws before the commencement of the work and subsequently whenever any change is to be made in the materials or in the proportions of the materials to be used. Works tests shall be made in accordance with Schedules I and III of this Part of these by-laws as and when the district surveyor shall require to prove the quality of the concrete. A record of such tests identifying them with the part of the work executed shall be kept by the builder on the works.

The grades of concrete designated IA, IIA and IIIA in column 1 of Table II and mixed in the proportions set out in column 2 of such Table against each such designation shall, within the period of twenty-eight days, possess the respective minimum resistance to crushing set out in column 3 of such Table against each such designation. Provided that such concrete may possess a lesser strength (but not less than that required for ordinary concrete in Table I) subject to the appropriate maximum permissible stresses in such concrete being proportionately less than the maximum stresses specified in By-laws 99 and 101 of these by-laws.

TABLE II.

1	2		3	
Designation of concrete	Cubic feet of aggregate per 112 lb. of cement		Minimum resistance to crushing in lb. per square inch within 28 days after mixing	
	Fine aggregate	Coarse aggregate	Preliminary test	Works test
IA	$1\frac{1}{4}$	$2\frac{1}{2}$	5,625	3,750
IIA	$1\frac{7}{8}$	$3\frac{3}{4}$	4,950	3,300
IIIA	$2\frac{1}{2}$	5	4,275	2,850

Concrete containing more than 10 cub. ft. of aggregate per 112 lb. of cement, or having resistance to crushing less than 1,110 lb. per square inch, shall not be used in the construction of a building or any part thereof.

Concrete containing more than 15 cub. ft. of aggregate per 112 lb. of cement, or having resistance to crushing less than 370 lb. per square inch, shall not be used for any purpose in connection with the construction of a building or chimney shaft.

The quantity of water for making reinforced concrete shall be sufficient but not more than sufficient to ensure that the concrete shall surround, cover, embed and grip adequately all the reinforcement.

The quantity of water for making Portland cement concrete and Portland blast-furnace cement concrete shall be sufficient but not more than sufficient to bring the entire mixture to a uniform colour and to ensure that the concrete shall be suitable for its intended purpose.

The quantity of water for making high alumina cement concrete shall be sufficient but not more than sufficient to produce a sound concrete and the concrete shall be kept wet for 24 hours after gauging.

Material shall be so mixed as to secure uniformity throughout the mixture.

The concrete shall be deposited without segregation of the materials and shall be properly consolidated by punning, rodding, vibrating, or other means after depositing and before the cement begins to set. After such consolidation, the concrete shall remain undisturbed and shall be protected from frost, heat, running water, evaporation, vibration or any other cause which may reduce its strength or tend to form voids in it. During mixing, depositing and setting, the temperature of concrete shall not fall below 40 deg. Fah.

Where formwork is employed it shall be sufficiently rigid to retain the concrete in position and shape during depositing, punning and consolidating.

15. Structural steel shall comply with the B.S.S. for Structural Steel for Bridges, etc., and General Building Construction, No. 15—1936.

Provided that, in any particular case, reinforcement (for slabs only) complying with B.S.S. No. 165—1929, structural steel complying with B.S.S. No. 548—1934, or structural steel of other quality or other structural metal may be used in accordance with conditions prescribed by the Council in that case.

25. The district surveyor may for the purpose of due supervision by notice require the builder or other person causing or directing the work, to furnish him with proof by means of adequate tests or otherwise that the materials used or to be used conform to the requirements of these by-laws, and no material shall be so used unless such material conforms to such requirements.

SCHEDULE I.

Standard method of test for consistency of concrete.

The test is to be used both in the laboratory and during the progress of the work for determining the consistency of concrete.

The test specimen shall be formed in a mould in the form of the frustrum of a cone with internal dimensions as follows: Bottom diameter, 8 in., top diameter, 4 in., and height, 12 in. The bottom and the top shall be open, parallel to each other, and at right angles to the axis of the cone. The mould shall be provided with suitable foot pieces and handles. The internal surface shall be smooth. Care shall be taken to ensure that a representative sample is taken. The internal surface of the mould shall be thoroughly clean, dry and free from set cement before commencing the test. The mould shall be placed on a smooth, flat, non-absorbent surface, and the operator shall hold the mould firmly in place, while it is being filled, by standing on the foot pieces. The mould shall be filled to about one-fourth of its height with the concrete which shall then be puddled, using 25 strokes of a $\frac{5}{8}$-in. rod, 2 ft. long, bullet pointed at the lower end. The filling shall be completed in successive layers similar to the first and the top struck off so that the mould is exactly filled. The mould shall then be removed by raising vertically, immediately after filling. The moulded concrete shall then be allowed to subside and the height of the specimen measured after coming to rest. The consistency shall be recorded in terms of inches of subsidence of the specimen during the test, which shall be known as the slump.

SCHEDULE II.

Standard method of making preliminary cube tests of concrete.

The method described applies to compression tests of concrete made in a laboratory where accurate control of materials and test conditions is possible.

Materials and proportioning—The materials and the proportions used in making the preliminary tests shall be similar in all respects to those to be employed in the work. The water content shall be as nearly as practicable equal to that to be used in the work, but shall be not less than the sum of 30 per cent. by weight of the cement and 5 per cent. by weight of the aggregate unless specially authorised by the district surveyor. For porous aggregates additional water shall be used to allow for the amount absorbed by the aggregates.

Materials shall be brought to room temperature (58 deg. to 64 deg. F.) before beginning the tests. The cement on arrival at the laboratory shall be mixed dry either by hand or in a suitable mixer in such a manner as to ensure as uniform a material as possible, care being taken to avoid intrusion of foreign matter. The cement shall then be stored in air-tight containers until required. Aggregates shall be in a dry condition when used in concrete tests.

The quantities of cement, aggregate and water for each batch shall be determined by weight to an accuracy of 1 part in 1,000.

Mixing concrete—The concrete shall be mixed by hand or in a small batch mixer in such a manner as to avoid loss of water. The cement and fine aggregate shall first be mixed dry until the mixture is uniform in colour. The coarse aggregate shall then be added and mixed with the cement and sand. The water shall then be added and the whole mixed thoroughly for a period of not less than two minutes and until the resulting concrete is uniform in appearance.

Consistency—The consistency of each batch of concrete shall be measured, immediately after mixing, by the slump test made in accordance with the method of test for consistency of concrete given in Schedule I. Provided that care is taken to ensure that no water is lost the material used for the slump tests may be re-mixed with the remainder of the mix before making the test specimen.

Size of test cubes—Compression tests of concrete shall be made on 6-in. cubes. The mould shall be of metal with inner faces accurately machined in order that the opposite sides of the specimen shall be plane and parallel. Each mould shall be provided with a metal base, having a smooth machined surface. The interior surfaces of the mould and base shall be slightly oiled before concrete is placed in the mould.

Compacting—Concrete test cubes shall be moulded by placing the fresh concrete in the mould in three layers, each layer being rammed with a steel bar 15 in. long and having a ramming face of 1 in. square and a weight of 4 lb. For mixes of $1\frac{1}{2}$ in. slump or less, 35 strokes of the bar shall be given for each layer; for mixes of wetter consistency the number may be reduced to 25 strokes per layer.

Curing—All test cubes shall be placed in moist air of at least 90 per cent. relative humidity and at a temperature of 58 deg. F. to 64 deg. F. for 24 hours ($\pm \frac{1}{2}$ hour) commencing immediately after moulding is completed. After 24 hours the test cubes shall be marked, removed from the moulds, and placed in water at a temperature of 58 deg. F. to 64 deg. F. until required for test.

Method of testing—All compression tests on concrete cubes shall be made between smooth plane steel plates, without end packing, the rate of loading being kept approximately at 2,000 lb. per square inch per minute. One compression plate of the machine shall be provided with a ball seating in the form of a portion of a sphere, the centre of which falls at the central point of the face of the plate. All test cubes shall be placed in the machine in such a manner that the load shall be applied to the sides of the cubes as cast.

Distribution of specimens and standards of acceptance—For each age at which tests are required, three cubes shall be made and each of these shall be taken from a different batch of concrete. The acceptance limits are a difference of 15 per cent. of the average strength between the maximum and minimum recorded strengths of the three cubes. In cases where this is exceeded repeat tests shall be made, excepting where the minimum strength test result does not fall below the strength specified.

SCHEDULE III.

Standard method of making works cube tests of concrete.

The method described applies to compression tests of concrete sampled during the progress of the work.

Size of test cubes and moulds—The test specimens shall be 6-in. cubes. The moulds shall be of metal, with inner faces accurately machined in order that opposite sides of the specimen shall be plane and parallel. Each mould shall be provided with a metal base plate, having a smooth machined surface. The interior surfaces of the mould and base shall be slightly oiled before concrete is placed in the mould.

Sampling—Wherever practicable concrete for the test cubes shall be taken immediately after it has been deposited in the work. Where this is impracticable samples shall be taken as the concrete is being delivered at the point of deposit, care being taken to obtain a representative sample. All the concrete for each sample shall be taken from one place. A sufficient number of samples, each large enough to make one test cube, shall be taken at different points so that the test cubes made from them will be representative of the concrete placed in that portion of the structure selected for tests. The location from which each sample is taken shall be noted clearly for future reference.

In securing samples the concrete shall be taken from the mass by a shovel or similar implement and placed in a large clean pail or other receptacle, for transporting to the place of moulding. Care shall be taken to see that each test cube represents the total mixture of concrete from a given place. Different samples shall not be mixed together, but each sample shall make one cube. The receptacle containing the concrete shall be taken to the place where the cube is to be moulded as quickly as possible and the concrete shall be slightly re-mixed before placing in the mould.

Consistency—The consistency of each sample of concrete shall be measured, immediately after re-mixing, by the slump test made in accordance with the method of test for consistency of concrete given in Schedule I. Providing that care is taken to ensure that no water is lost the material used for the slump tests may be re-mixed with the remainder of the mix before making the test cube.

Compacting—Concrete test cubes shall be moulded by placing the fresh concrete in the mould in three layers, each layer being rammed with a steel bar 15 in. long and having a ramming face of 1 in. square and a weight of 4 lb. For mixes of $1\frac{1}{2}$ in. slump or less, 35 strokes of the bar shall be given for each layer ; for mixes of wetter consistency the number may be reduced to 25 strokes per layer.

Curing—The test cubes shall be stored at the site of construction at a place free from vibration, under damp sacks for 24 hours ($\pm\ \frac{1}{2}$ hour), after which time they shall be removed from their moulds, marked, and buried in damp sand until the time for sending to the testing laboratory. They shall then be well packed in damp sand or other suitable damp material and sent to the testing laboratory, where they shall be similarly stored until the date of test. Test cubes shall be kept on the site for as long as practicable and for at least three-fourths of the period before test, except for tests at ages less than seven days. The temperature of the place of storage on the site shall not be allowed to fall below 40° F., nor shall the cubes themselves be artificially heated.

Record of temperatures—A record of the maximum and minimum day and night temperatures at the place of storage of the cubes shall be kept during the period the cubes remain on the site.

Method of testing—All compression tests on concrete cubes shall be made between smooth plane steel plates without end packing, the rate of loading being kept approximately at 2,000 lb. per square inch per minute. One compression plate of the machine shall be provided with a ball seating in the form of a portion of a sphere, the centre of which falls at the central point of the face of the plate. All test cubes shall be placed in the machine in such a manner that the load shall be applied to the sides of the cubes as cast.

PART III.—FOUNDATIONS AND SITES OF BUILDINGS AND EXCAVATIONS ADJACENT THERETO.

26. Every foundation shall be constructed to sustain and transmit safely all the loading imposed thereon, and without exceeding the limitations of permissible stresses provided in these by-laws. Piling shall be to the satisfaction of the district surveyor.

27. Before the erection of any part of a building is commenced : (i) The site of such part shall be cleared of all material impregnated with faecal or offensive animal or offensive vegetable matter (unless it has become or been rendered innocuous) and such other material as in the opinion of the district surveyor would, if not removed, affect adversely the building or some part thereof ; and (ii) All excavations, voids or cavities in the said site shall be filled or otherwise treated as the district surveyor may require for the purpose of ensuring the stability of the said building and of every part thereof. Filling for the purpose of compliance with this by-law shall be of such material as the district surveyor may approve as ensuring the stability of the building and of every part thereof ; and if of plain concrete shall be as regards composition and quality not inferior to that designated VII in By-law 14.

28. The site of every part of a building shall be covered with concrete, as regards composition and quality not inferior to that designated V in By-law 14 at least 6 in. in thickness and smoothed on the upper surface. Provided that where such site is covered with reinforced concrete which complies with the requirements of these by-laws relating thereto the thickness of such reinforced concrete shall be at least 4 in.

29. All excavations, voids or cavities in the earth outside a building and less than 3 ft. measured horizontally in any direction from the external face of any wall or pier of such building shall be filled or otherwise treated to the satisfaction of the district surveyor. Filling for the purpose of compliance with this by-law shall be of such material as the district surveyor may approve ; and if of plain concrete shall be as regards composition and quality not inferior to that designated VII in By-law 14.

30. The pressure upon earth to support any part of a building shall be calculated if so required by the district surveyor ; and the intensity of such pressure shall not exceed that allowed by the district surveyor.*

31. Where the earth adjacent to any building will or may exert pressure upon, or otherwise cause the application of loading to, any part of such building, adequate provision shall be made in the design and construction of every part of the building to ensure that such pressure or loading will be supported and transmitted properly and safely and without exceeding the appropriate limitations of permissible stresses specified in these by-laws.

32. Where metal is used in combination with the concrete required for compliance with By-law 33, or in combination with the corresponding lowermost portion of any other part of a building, proper protection shall be provided to prevent damage to such metal.

33. Unless supported on a beam every wall or pier or the footings thereof (if any) shall rest on concrete, and such concrete shall extend horizontally beyond each of the side and end faces of the wall or pier to a distance not less than the thickness of the wall or pier at the underside of the course immediately above the footings, or, if there be no footings, the thickness of such wall or pier where it rests upon the concrete. Provided that the distance to which such concrete shall extend horizontally beyond each of the side and end faces of the wall or pier may be less than that hereinbefore in this by-law specified if the district surveyor is satisfied that the

* The following loads are given only as a general guide to the safe bearing capacity of various subsoils :—

	Load on ground. Tons per square foot
Alluvial soil, made ground, very wet sand	½
Soft clay, wet or loose sand	1
Ordinary fairly dry clay, fairly dry fine sand, sandy clay	2
Firm dry clay	3
Compact sand or gravel, London blue or similar hard compact clay	4

standard of stability in each part of the building will not thereby be rendered inferior to that required otherwise by these by-laws. Provided also that where an external wall is built against another external wall or against a party wall, the district surveyor may allow such concrete on the side of the first mentioned external wall next to the other external wall or next to the party wall to be omitted.

Concrete for compliance with this by-law shall: (i) If plain, be, as regards composition and quality, not inferior to that designated V in By-law 14.

The angle of dispersion through such plain concrete shall be taken as not less than 45 deg. with the horizontal.

(ii) If reinforced, comply with the requirements of these by-laws relating to the use of reinforced concrete.

34. The intensity of pressure upon plain concrete filling shall not exceed that shown as appropriate for each designation of concrete in Table IV.

TABLE IV.

Designation of concrete	Maximum permissible intensity of pressure in tons per square foot
IV	20
V	15
VI	10
VII	5

35. The pressure upon concrete which is to support walls or piers shall be calculated. Such concrete shall, if plain, be restrained to the satisfaction of the district surveyor, against horizontal movement at its upper and lower extremities; and if it be further restrained to his satisfaction against horizontal movement between those extremities, the intensity of such pressure shall not exceed the maximum set against each designation of concrete in Table V.

TABLE V.

Designation of concrete	Maximum permissible intensity of pressure in tons per square foot
I	40
II	35
III	30
IV	20
V	15

If such plain concrete, although restrained to the satisfaction of the district surveyor against horizontal movement at its upper and lower extremities, be not

TABLE VI.

Ratio of height to least horizontal dimension	Maximum permissible intensity of pressure, in tons per square foot, appropriate for the respective designations of concrete				
	I	II	III	IV	V
Up to 2	40	35	30	20	15
3	36	31·5	27	18	13·5
4	32	28	24	16	12
5	28	24·5	21	14	10·5
6	24	21	18	12	9
7	20	17·5	15	10	7·5
8	16	14	12	8	6
9	12	10·5	9	6	4·5
10	8	7	6	4	3
11	4	3·5	3	2	1·5
12 and more . . .	0	0	0	0	0

further restrained to his satisfaction against horizontal movement between those extremities, the intensity of such pressure shall not exceed the maximum set against each ratio of its height to its least horizontal dimension in Table VI appropriate for the respective designations of concrete.

For any ratio of height to least horizontal dimension between 2 and 12 not shown in Table VI, the maximum permissible intensity of pressure shall be determined on the basis that the maximum permissible intensity of pressure varies in proportion with the ratio of height to least horizontal dimension between those shown as consecutive in Table VI.

36. Plain concrete shall not be relied upon to resist tensile stresses otherwise than in accordance with the requirements of By-law 33.

37. Where the concrete laid for the purpose of compliance with By-law 28 forms or may form a floor of a building or any part thereof, such floor shall be so constructed as to ensure that the building will not be affected adversely by moisture from adjoining earth. The materials employed, and the extent and manner of their employment, shall be to the satisfaction of the district surveyor.

38. Throughout the period of preparation for the construction or conversion of a building or part of a building the builder shall on the site take all precautions necessary for the purpose of securing the stability of the building and of every part thereof.

PART IV.—WALLS AND PIERS.

Section 1—*General Requirements.*

39. Every building shall be enclosed with walls. Provided that openings may be made in such walls subject to the following conditions—(1) That the total elevational area of openings in any such wall above the soffit of the first floor do not exceed one-half the elevational area of such wall measured from the soffit of the first floor of the building to the roof; (2) that the total elevational area of openings in any storey-height of such wall above the soffit of the first floor of the building do not exceed two-thirds of the total area of such wall within such storey-height; (3) that the total width of openings at any level above the soffit of the first floor do not exceed three-quarters of the total length of the wall at that level. For the purposes of this by-law, the expression " walls " shall be deemed to include piers and for the purpose of this by-law and of By-laws 43 and 51 (*g*) any glazing or glass in the thickness of such walls shall be deemed to be an opening.

40. Every wall or pier of a building shall be constructed of bricks or blocks laid in horizontal courses properly bonded, bedded and jointed with mortar or of plain concrete or of reinforced concrete or (except in the case of party walls) of such materials in combination with metal framework. Where any walls of a building meet, or where such walls meet piers, they shall be properly bonded or otherwise securely and permanently bound together.

41. No hollow bricks or hollow blocks shall be used in the construction of a wall or pier of a building (other than a non-load bearing partition wall) unless evidence has been produced to the satisfaction of the Council showing that such wall or pier will be equal as regards fire-resistance to that of a wall or pier constructed of solid bricks or solid blocks or of plain or reinforced concrete in accordance with the requirements of these by-laws.

42. No timber or other combustible material (other than the ends of beams, joists, purlins and rafters, the horns of door frames and of window frames, fixing blocks and plugs and pole plates bearing rafters and supporting no walling other than windpinning) shall be built into the required thickness of a wall or pier and when the end of a beam, joist, purlin, rafter or other timber is built into the required thickness of a party wall, it shall not extend beyond the middle of the wall and shall be encased in brickwork or other solid incombustible material not less than 4 in. in thickness.

43. No external wall, party wall or buttressing wall constructed of bricks or blocks or plain concrete shall be of less thickness in any part than 8½ in., exclusive of plastering, rendering, rough cast or other applied covering. No reinforced concrete external wall nor reinforced concrete part or panel of an external wall shall be of less thickness

in any part than 4 in. exclusive of plastering, rendering, rough cast or other applied covering. No reinforced concrete party wall shall be of less thickness in any part than 8 in., and no such party wall forming part of a building of the warehouse class, such building being constructed otherwise than as a reinforced concrete building, shall be of less thickness in any part than 13 in. exclusive of plastering, rendering, roughcast or other applied covering in each case.

Provided that: (i) a building of not more than one storey in height, not being a dwelling house, and the width of which (measured in the direction of the span of the roof) does not exceed 30 ft. and the height of the walls of which does not exceed 10 ft.; or (ii) an erection situated above the level of the roof of a building and intended for the protection of a tank or motor or for a like purpose, and not intended for or adapted to use for habitable purposes or as a work room, such erection being adequately supported to the satisfaction of the district surveyor, and not exceeding 10 ft. in either length or width and not exceeding 8 ft. in height measured from the level of the roof of the building to the top of the walls of such erection; may be enclosed with external walls constructed of bricks or blocks and not less than 4 in. thick subject to the following conditions: (*a*) That any such wall be bonded into piers of the size required by calculations based on the loads and stresses specified in these by-laws, but not less than 8½ in. square in horizontal section. (*b*) That such pier be provided at each end of such external wall. (*c*) That in the case of (i) further similar piers be provided if any such wall exceeds 10 ft. in length, as may be necessary so to divide the wall that the length of each portion of such wall shall not exceed 10 ft. measured in the clear between such piers. (*d*) That all bedding and jointing be in cement mortar. (*e*) That the roof be so constructed that the walls are not subject to any thrust therefrom. (*f*) That no load other than a distributed load of the roof be borne by the walls.

Every party wall which exceeds 30 ft. in height shall have a thickness of solid material in every part thereof of not less than 13 in. Provided that this requirement shall not apply to a wall which does not serve to enclose a building of the warehouse class, and (*a*) which does not exceed 40 ft. in height; or (*b*) which does not exceed 35 ft. in length and 50 ft. in height.

Where any part of a party wall serves to divide basements, such part shall not have a less thickness of solid material than 13 in. Provided that this requirement shall not apply to a party wall not forming part of a building of the warehouse class and not exceeding 25 ft. in height and not exceeding 30 ft. in length.

46. Every building shall be so constructed as to ensure that it will not be affected adversely by moisture from adjoining earth.

48. No earth, concrete, brickwork, stonework or any other material supporting or aiding in the support of any superstructure shall be disturbed within two clear days of the district surveyor having received notice in writing giving particulars of the nature of the work and the date of its commencement.

49. A wall shall not be thickened except after two clear days' notice being served on the district surveyor of the intention to thicken, and the thickening shall be executed to the satisfaction of the district surveyor and such wall so thickened shall be of the required thickness.

50. A wall or pier shall not be deemed to sustain and transmit all the dead and superimposed loading as required by By-law 2 unless such wall or pier is in conformity with either (*a*) the prescribed conditions; dimensions and other requirements set out in Section 2 of this part of these by-laws (applicable to brick walls only); or (*b*) the limits of stress and other requirements set out in Section 3 of this part of these by-laws, and in Parts V and VI of these by-laws.

Section 3—Rules as to the Permissible Stresses in Walls and Piers for the purpose of Calculation where the Thicknesses are not Determined under Section 2.

58. If in a storey-height, part of a wall is borne by a pier or a pier is borne by part of a wall, such pier together with the part of the wall of the storey-height directly above or below shall be deemed to be a pier extending throughout such storey-height.

Where a wall and a pier are horizontally in structural combination and the pier projects from one or both faces of the wall, if such projection from one face of the wall does not exceed one-quarter the thickness of such wall, or if the sum of two projections from the two faces does not exceed one-third the thickness of the wall, such combination shall, for the purposes of this Section of these by-laws be deemed to be a wall. If such projection or the sum of such projections exceed one-quarter or one-third the thickness of such wall respectively, and if such additional thickness or width constitutes part of the required thickness, such combination shall be deemed to be a pier the thickness or width of which shall be measured from the face of the projection on the one side of the wall to the face of the other side of the wall, or if such pier project also from the other side of the wall, to the face of the projection on the other side of the wall.

59. For the purpose of By-law 60 the slenderness ratio of any storey-height of a wall or pier constructed of bricks or blocks or of plain concrete shall be the ratio of the effective height to the horizontal dimension lying in the direction of the lateral support determining such storey-height. For the purpose of this by-law, the effective height shall be: In the case of a storey-height of a wall without lateral support at the top thereof, one and a half times such storey-height. In the case of a storey-height of a wall with lateral support at the top thereof, three-quarters of such storey-height. In the case of a storey-height of a pier without lateral support at the top thereof, twice such storey-height. In the case of a storey-height of a pier with lateral support at the top, the actual storey-height.

60. If any storey-height of a wall or pier not having a slenderness ratio exceeding 6 be constructed of bricks or blocks, the total compressive stress due to the vertical load, horizontal pressure and to any other forces shall not exceed the maximum pressure indicated in Table VII in respect of the designation of the bricks or blocks employed as regards strength or of the proportions of the mortar employed, whichever is the weaker. If such storey-height be constructed of plain concrete, such compressive stress shall not exceed the maximum pressure indicated in Table VIII.

TABLE VII—MAXIMUM PERMISSIBLE PRESSURES ON WALLS AND PIERS OF BRICKS AND BLOCKS.

Designation of bricks or blocks as regards strength (as specified in By-law 19)	Proportion of Mixture of Mortar (in Volumes)			Maximum pressure in tons per square foot
	Cement	Lime	Sand	
Special	1	—	2	$\frac{1}{500}$ of the number of lb./sq. in. of the resistance of the bricks or blocks to crushing + 10, but not exceeding 40.
1st	1	—	$2\frac{1}{2}$	30
2nd	1	—	$2\frac{1}{2}$	23
3rd	1	—	3	16
4th	1	—	3	$13\frac{1}{2}$
5th	1	—	4	11
5th	1	1	6	10
6th	1	—	4	8
6th	1	1	6	7
6th	1	2	9	6
6th	1	3	12	$5\frac{1}{2}$
6th	1	4	15	5
6th	1	5	18	$4\frac{1}{2}$
6th	—	1	3	4

If, in any case, the designation of the materials is not determined for the purposes of Tables VII or VIII, the maximum permissible pressure shall be as approved by the district surveyor. In all cases of thickening of walls, or the combination of new work with old in any other manner, the permissible stresses shall

TABLE VIII—MAXIMUM PERMISSIBLE PRESSURES ON WALLS AND PIERS OF PLAIN CONCRETE.

Designation of concrete as regards strength (as specified in By-law 14)	Cubic feet of aggregate per 112 lb. of cement		Maximum pressure in tons per square foot
	Fine aggregate	Coarse aggregate	
I	1¼	2½	40
II	1⅞	3¾	35
III	2½	5	30
IV	7½		20
V	10		15
VI	12½		10
VII	15		5

be as approved by the district surveyor. If in any wall or pier and in the same storey-height materials differing in designation be employed, the weakest shall be deemed to be employed.

If such wall or pier or any storey-height thereof have a slenderness ratio of 12 such total compressive stress shall not exceed 40 per cent. of the corresponding maximum pressure indicated in Table VII or Table VIII as the case may be. If in such wall or pier or in any storey-height thereof, the greater slenderness ratio be between 6 and 12, the total compressive stress in such wall or pier or such storey-height thereof shall not exceed a pressure correspondingly proportionate to the pressure appropriate to the slenderness ratios of 6 and 12.

No load-bearing wall or pier constructed of bricks or blocks or of plain concrete nor any storey-height thereof shall have a slenderness ratio exceeding 12 except a cavity wall or a partition wall constructed under the prescribed conditions and in accordance with By-law 45 or 53 respectively.

Provided that where a wall or pier constructed of bricks or blocks or of plain concrete supports a beam or column or is otherwise subjected to local loading of a like nature, and the stresses resulting from such loading are immediately distributed through adjacent material not so stressed, the compressive stress in the material so subjected to local loading may exceed the appropriate maximum pressure indicated in Table VII or Table VIII as the case may be, by not more than 20 per cent.

61. No account shall be taken of resistance to shearing or tensile stresses in any wall or pier of bricks or blocks or of plain concrete, and such materials shall not be relied upon to resist such stresses except in the case of arches, lintels, corbelling, footings and the like constructions wherein the resistance of the bricks, blocks or plain concrete to shearing and tensile stresses may be deemed to be one-tenth of that to compression. Provided that if a wall constructed of bricks or blocks or of plain concrete be laterally supported by buttressing walls, piers or other constructions to the satisfaction of the district surveyor and the length of wall between such supports does not exceed 45 ft. and does not exceed forty-five times the thickness of such wall, then such wall may be deemed to transmit to such supports a horizontal load equal to 25 per cent. of the wind-pressure on such wall, and for the purpose of calculating the overturning moment on such wall, the horizontal force due to wind-pressure shall be deemed to be reduced thereby to 75 per cent. of that specified in By-law 6.

62. Underpinning shall be carried out to the satisfaction of the district surveyor and in such manner that the permissible stresses prescribed by these by-laws will not be exceeded.

PART VI—THE USE OF REINFORCED CONCRETE.

92. Concrete shall comply with these by-laws and shall not be relied upon to support, collect, or transmit loading otherwise than as provided in these by-laws. Reinforced

concrete shall be, as regards composition and quality, not inferior to that designated III in By-law 14.

93. Construction which will support or transmit loading supported, collected or transmitted by reinforced concrete shall comply with the requirements of these by-laws.

94. Loading supported, collected, or transmitted by reinforced concrete shall be distributed upon the earth by concrete which shall (*a*) comply with the requirements of By-law 32 in the same manner as is required for concrete which is to support walls or piers; (*b*) if plain, be of composition and quality not inferior to that designated V in By-law 14. The angle of dispersion through such plain concrete shall be taken as not less than 45 deg. with the horizontal; and such plain concrete shall not be relied upon to resist shearing or tensile stresses otherwise than in accordance with this by-law; and (*c*) if reinforced, comply with the requirements of these by-laws. The pressure upon such distributing concrete shall be calculated, and such concrete, if plain, shall comply in all respects with the requirements of By-law 35 in the same manner as is required for plain concrete which is to support walls or piers.

95. Where metal is used in combination with concrete which supports, collects or transmits loading in a building or part thereof, or which distributes such loading upon the earth, proper protection shall be provided to prevent such damage to the metal as would, in the opinion of the district surveyor, affect adversely the stability of such building or of any part thereof.

96. Reinforcement shall be of structural steel complying with these by-laws so combined with the concrete that the reinforcement will be sufficient to provide, in accordance with these by-laws, all necessary (*a*) resistance to tension; (*b*) assistance for the concrete to resist shearing actions; and (*c*) assistance for the concrete to resist compression. Reinforcement shall, immediately before being placed in the concrete, be free from loose mill scale, loose rust, oil or other matter which might affect adversely the proper combination of such reinforcement with such concrete.

97. Reinforcement shall have concrete cover, and the thickness of such cover (exclusive of plaster or other decorative finish) shall be (*a*) for each end of a reinforcement rod or bar which is anchored otherwise than by means of a hook, not less than 2 in., nor less than twice the diameter of such rod or bar beyond such anchorage; (*b*) for a longitudinal reinforcement rod or bar in a column, not less than 1½ in., nor less than the diameter of such rod or bar; (*c*) for a longitudinal reinforcement rod or bar in a beam, not less than 1 in. nor less than the diameter of such rod or bar; (*d*) for tensile, compressive, shear or other reinforcement in a slab, not less than ½ in., nor less than the diameter of such reinforcement; (*e*) for any other reinforcement (not being a binding), not less than ½ in., nor less than the diameter of such reinforcement.

98. The following By-laws 99 to 112 inclusive shall relate only to the use of reinforced concrete in a building wherein the loads and stresses are transmitted through each storey to the foundations wholly by a skeleton framework of reinforced concrete or partly by a skeleton framework of reinforced concrete and partly by a party wall or party walls.

99. The compressive, shearing and bond stresses in reinforced concrete shall be calculated, and, subject to the requirements of By-law 101 such stresses shall not exceed those shown as appropriate for each designation of concrete in Tables IX and X.

Where other proportions of fine to coarse aggregate are used the permissible concrete stresses shall be based on the ratio of the sum of the volumes of the fine and coarse aggregates, each measured separately, to the quantity of cement, and shall be obtained by proportion from the two nearest designations. Provided that where reinforcement in the form of plain bars is used to resist tensile stresses induced by bending action the calculated bond stress due to a variation in tensile stress shall

not exceed twice that shown as appropriate for each designation of concrete in Tables IX and X in this by-law. Provided also that the "punching shear" stress in a footing (or similar construction) of reinforced concrete which complies with the

TABLE IX—*Ordinary concrete.*

Designation of concrete	Modular ratio	Permissible concrete stresses. lb. per square inch			
		Compression		Shear	Bond
		Due to bending	Direct		
I	15	975	780	98	123
II	15	850	680	85	110
III	15	750	600	75	100

TABLE X—*Quality A concrete*

Designation of concrete	Modular ratio	Permissible concrete stresses. lb. per square inch			
		Compression		Shear	Bond
		Due to bending	Direct		
IA	15	1,250	1,000	125	150
IIA	15	1,100	880	110	135
IIIA	15	950	760	95	120

requirements of these by-laws shall be not more than twice the permissible shearing stress shown as appropriate for each designation of concrete in Tables IX and X in this by-law.

100. The tensile and compressive stresses in steel reinforcement of reinforced concrete shall be calculated, and, subject to the requirements of By-law 101, such stresses shall not exceed those shown as appropriate for each designation of stress in Table XI.

TABLE XI.

Designation of stress in steel reinforcement	Maximum permissible stress, in pounds per square inch
Tension in helical reinforcement in a column	13,500
Tension other than in helical reinforcement in a column .	18,000
Longitudinal compression in a beam where the compressive resistance of the concrete is not taken into account.	18,000
Longitudinal compression, direct or due to bending where the compressive resistance of the concrete is taken into account.	The calculated compressive stress in the surrounding concrete multiplied by the modular ratio.

101. The maximum permissible stresses in a reinforced concrete column or part thereof having a ratio of effective column length to least radius of gyration not exceeding 50 shall be as specified in By-laws 99 and 100. The maximum permissible stresses in a reinforced concrete column or part thereof having a ratio of effective column length to least radius of gyration between 50 and 120 as shown in Table XII

TABLE XII.

Ratio of effective column length to least radius of gyration	Coefficient
50	1·0
60	0·9
70	0·8
80	0·7
90	0·6
100	0·5
110	0·4
120	0·3

shall not exceed those which result from the multiplication of the appropriate maximum permissible stresses specified in By-laws 99 and 100 by the coefficient shown as appropriate for each ratio of effective column length to least radius of gyration in Table XII. For any ratio of effective column length to least radius of gyration between 50 and 120 not shown in Table XII, the appropriate coefficient shall be determined on the basis that the coefficient varies in proportion with the ratio of effective column length to least radius of gyration between those shown as consecutive in Table XII above. A reinforced concrete column or part thereof shall not have a ratio of effective column length to least radius of gyration more than 120.

102. The effective column length to be assumed in determining the working load per square inch in accordance with By-laws 99, 100 and 101 shall be as follows :

	Type of column	Effective column length
Columns of one storey	Properly restrained at both ends in position and direction.	0·75 of the actual column length.
	Properly restrained at both ends in position but not in direction.	Actual column length.
	Properly restrained at one end in position and direction and imperfectly restrained in both position and direction at the other end.	A value intermediate between the actual column length and twice that length depending upon the efficiency of the imperfect restraint.
Columns continuing through two or more storeys.	Properly restrained at both ends in position and direction.	0·75 of the distance from floor level to floor level.
	Properly restrained at both ends in position and imperfectly restrained in direction at one or both ends.	A value intermediate between 0·75 and 1·00 of the distance from floor level to floor (or roof) level, depending upon the efficiency of the directional restraint.
	Properly restrained at one end in position and direction and imperfectly restrained in both position and direction at the other end.	A value intermediate between the distance from floor level to floor (or roof) level and twice that distance, depending upon the efficiency of the imperfect restraint.

The effective column length values given above are in respect of typical cases only and embody the general principles which shall be employed in assessing, to the satisfaction of the district surveyor, the appropriate value for any particular column.

103. The maximum permissible stresses in reinforced concrete and in its reinforcement may exceed those specified in By-laws 99 and 100 respectively by not more than $33\frac{1}{3}$ per cent., provided such excess is solely due to stresses induced by wind loading, and provided that such excess shall not apply to secondary floor beams, nor to the stresses in roof construction above the topmost floor level in a building.

104. A reinforced concrete column shall have longitudinal steel reinforcement, and the cross-sectional area of such reinforcement shall not be less than 0·8 per cent., nor more than 8 per cent., of the gross cross-sectional area of the column required to transmit all the loading in accordance with these by-laws. A reinforced concrete column having helical reinforcement shall have also at least six bars of longitudinal reinforcement within such helical reinforcement. Such longitudinal bars shall be in contact with such helical reinforcement and equidistant around its inner circumference. At a splice in a longitudinal reinforcement, the spliced bars shall overlap longitudinally through a distance not less than 24 times the diameter of the upper bar, or a sufficient distance to develop the force in the bar by bond, whichever is the lesser.

105. A reinforced concrete column shall have transverse or helical reinforcement so disposed as to provide all necessary restraint against buckling of the longitudinal reinforcement; and the ends of such transverse reinforcement shall be anchored properly. The diameter of such transverse reinforcement shall be not less than $\frac{1}{4}$ in. The pitch of such transverse reinforcement shall be not more than the least of the three following distances: (1) the least lateral dimension of such column; (2) twelve times the diameter of the smallest longitudinal reinforcement in such column; (3) 12 in.

Helical reinforcement shall be of regular formation, with the turns of the helix spaced evenly; and its ends shall be anchored properly. The pitch of the helical turns shall be not more than 3 in. nor more than one-sixth of the core-diameter of such column; and such pitch shall be not less than 1 in. nor less than three times the diameter of the steel forming such helix.

106. The diameter of a steel reinforcement in reinforced concrete shall be not more than 2 in. The diameter of a longitudinal steel reinforcement in a reinforced concrete column shall be not less than $\frac{1}{2}$ in. The diameter of a main steel reinforcement in a reinforced concrete beam or slab shall be not less than $\frac{1}{4}$ in. The diameter of a steel reinforcement in reinforced concrete other than a longitudinal reinforcement in a column or a main reinforcement in a beam or slab, and the diameter of steel forming a tie, helix, stirrup or the like, shall be not less than $\frac{3}{16}$ in. The diameter of steel forming a mesh-reinforcement for the purpose of resisting tension in reinforced concrete shall be not less than $\frac{1}{10}$ in.

107. The distance between two steel reinforcements in reinforced concrete shall be not less than the greatest of the three following distances: (*a*) the diameter of either bar if their diameters are equal; (*b*) the diameter of the larger bar if their diameters be unequal; (*c*) $\frac{1}{4}$ in. more than the greatest size of the coarse aggregate comprised in such concrete. Provided that the vertical distance between two horizontal main steel reinforcements, or the corresponding distance at right angles to two inclined main steel reinforcements, shall be not less than $\frac{1}{2}$ in. except at a splice and except where one of such reinforcements is transverse to the other. The pitch of distributing bars in a reinforced concrete solid slab shall be not more than four times the effective depth of such slab. A mesh-reinforcement shall be of such dimensions, shape, proportions and arrangement as will afford proper combination of such reinforcement with such concrete.

108. Where the concrete alone is not sufficient to resist, in accordance with By-law 99, the shearing action in reinforced concrete, the whole of such shearing action shall be provided for by the tensile resistance of shear reinforcement acting in proper conjunction with the compressive resistance of the concrete: but the magnitude

of such shearing action to be so provided for shall not exceed four times that which the concrete alone could resist in accordance with By-law 99.

109. A stirrup in reinforced concrete shall pass round, or be secured adequately otherwise to, the appropriate tensile reinforcement; and such stirrup shall have both its ends anchored properly.

110. A reinforced concrete solid slab spanning in one direction shall have distributing bars at right angles to the main tensile reinforcement of such slab; and the aggregate cross-sectional area of such distributing bars shall be not less than one-tenth of the aggregate cross-sectional area of such main tensile reinforcement associated therewith.

111. Where the compressive resistance of concrete in beams is taken into account, the compression reinforcement where it is required shall be effectively anchored over the distance where it is required at points not further apart centre to centre than twelve times the diameter of the anchored bar. Where the compressive resistance in concrete is not taken into account the compressive reinforcement shall be effectively anchored laterally and vertically over the distance where it is required at points not further apart centre to centre than eight times the diameter of the anchored bar. The subsidiary reinforcement used for this purpose shall pass round or be hooked over both the compression and tensile reinforcement.

112. Hooks and other anchorages of reinforcement in reinforced concrete shall be of such form, dimensions and arrangement as will ensure their adequacy without overstressing the concrete, or any other material.

113. Where reinforced concrete is used in the construction of a building wherein the loads and stresses are not transmitted through each storey to the foundations wholly by a skeleton framework of reinforced concrete nor partly by a skeleton framework of reinforced concrete and partly by a party wall or party walls, the standard of stability shall to the satisfaction of the district surveyor, be not inferior to that required for compliance with By-laws 99 to 112 inclusive.

114. Reinforcement in reinforced concrete shall not be connected by welding, except in accordance with conditions prescribed by the Council in each particular case. Provided that round or square bars not more than 0·4 in. diameter and transverse to each other forming a mesh reinforcement in a solid slab may be connected by electrically fusing the metal of such rods at their points of contact if such fusion be executed at the works where such mesh is fabricated and if the district surveyor be satisfied as to its suitability.

115. Reinforced concrete subjected to bending actions in a building shall possess adequate stiffness to prevent such deflection or deformation as might, in the opinion of the district surveyor, affect adversely the stability of such building or of any part thereof.

116. Reinforced concrete subjected to compression in a building shall possess adequate stiffness, or be provided with adequate restraint, to prevent such lateral flexure as might, in the opinion of the district surveyor, affect adversely the stability of such building or of any part thereof.

117. The fabrication and erection of reinforced concrete shall be such as will ensure that the assumptions upon which the stresses in such concrete and in its reinforcement have been calculated shall be fulfilled adequately at all times in the building of which such reinforced concrete forms part.

118. Where the district surveyor finds substantial reason for doubt as to the sufficiency or suitability of the reinforced concrete for its purposes under these by-laws, the builder shall make such test or tests on such concrete as the district surveyor may require; and if such testing proves, in the opinion of the district surveyor, that such concrete is insufficient or unsuitable for its purposes under these by-laws, such concrete shall be removed and replaced with reinforced concrete which complies with these by-laws.

150. All floors and staircases (together with their enclosing walls) in buildings wherein the loads and stresses are transmitted through each storey to the foundations (i) wholly by a skeleton framework of structural steel, or (ii) partly by a skeleton

framework of structural steel and partly by a party wall or party walls, or (iii) wholly by a skeleton framework of reinforced concrete, or (iv) partly by a skeleton framework of reinforced concrete and partly by a party wall or party walls, shall be constructed throughout of fire-resisting materials carried upon supports of fire-resisting materials. Provided that in the case of self-supporting flights of stairs constructed of reinforced concrete or steel no additional support shall be required if such support is not necessary for the purpose of stability.

151. Structural metal shall not be used for conducting electrical currents. Provided that such metal may be used as part of a sufficient and properly earthed apparatus for protection of the building against damage by lightning.

159. Insofar as these by-laws control the use of metal skeleton or reinforced concrete construction, they relate to the stability of a building or part of a building. In pursuance of the provisions of sub-section (5) of section 4 of the London Building Act (Amendment) Act, 1935, any person dissatisfied with the refusal of the Council to modify or waive the requirements hereinafter specified (in so far as they relate to the use of reinforced concrete construction) of these by-laws, or dissatisfied with any term or condition attached by the Council to any such modification or waiver, may appeal to the Tribunal of Appeal:

By-law 3 so far as regards the average weight of reinforced concrete together with plastering, tiles, mosaic, granite or other similar finishing material for such concrete.

By-laws 4 and 6, so far as regards the pressure of wind, and the extent of surface subjected thereto.

By-law 14 so far as regards the relation between the weight and volume of Portland cement complying with the B.S.S. No 12—1931.

By-law 30 so far as regards the permissible pressure of foundations on the natural ground.

By-law 39—Conditions (2) and (3) to the proviso.

By-law 43 so far as regards (*a*) the thickness of portions of external walls between columns and beams, and (*b*) the thickness of party walls.

By-laws 59 and 60, so far as regards brickwork which supports steel or reinforced concrete construction.

By-law 97.

By-law 104, so far as regards the relation between the overlap and the diameter of reinforcement.

By-law 151 so far as regards reinforcement.

APPENDIX II

WELDING—APPLICATIONS FOR MODIFICATIONS OR WAIVERS.*

REGULATIONS MADE BY THE COUNCIL ON 7TH DECEMBER, 1937, UNDER SECTION 9 (2) OF THE LONDON BUILDING ACT (AMENDMENT) ACT, 1935, RELATING TO APPLICATIONS FOR MODIFICATIONS OR WAIVERS OF BUILDING BY-LAWS NOS. 104 AND 114, SO AS TO PERMIT THE USE OF ELECTRIC (METAL) ARC WELDING INSTEAD OF LAPPING REINFORCED BARS.

1. Each application for permission to use electric (metal) arc welding (instead of lapping as required and provided for in the building by-laws) should be accompanied by adequate particulars, calculations and plans relating to the character and quality of the welding proposed and to the manner in which it is proposed to be used.

If such application be granted, the Council will prescribe such conditions as it may deem proper to the use of the welding in the manner proposed for that case; and such conditions will apply only in respect of the building to which such permission relates.

The Council will, however, be prepared to consider preliminary applications for approval in principle of the adoption of welding in relation to the construction of a building. If such application is granted it will be necessary for the detailed consent of the Council to be obtained in due course to the methods to be adopted.

2. Structural steel parts for connection by welding should comply with the requirements of the British Standard Specification No. 15—1936.

3. Provision in accordance with the building by-laws should be made for all the effects (e.g., of continuity or rigidity) consequent upon the use of welding instead of lapping.

4. Terms relating to electric (metal) arc welding used in applications to the Council for the use of such welding should bear the meanings assigned to them in the B.S.S. No. 499—1933. The terms used herein bear the same meanings as in that Specification.

5. The applicant should furnish in each case evidence to the satisfaction of the Council as to the strength, ductility and other essential properties of electrodes and of weld metal. Such evidence should be suitable and sufficient to enable the Council to decide whether—and, if so, the conditions under which—the welding proposed may be used in that case.

The applicant shall furnish also in each case evidence to the satisfaction of the Council as to the means proposed for (*a*) ensuring that the welding will be executed by competent and reliable operatives; (*b*) supervising the work of each operative welder during progress; and (*c*) ensuring that defective work will not be incorporated in the building to which the consent of the Council relates.

6. The Council will determine in each case the maximum permissible stresses, the detail arrangement of connections and such other restrictions as the Council may deem proper for the use of such welding in the manner proposed. The following table may, however, be taken as a general indication of the probable maximum stresses which will be permitted by the Council:

* In this abstract the clauses which relate only to the welding of structural steel are omitted.

Classification of stress in welded connections.	*Maximum permissible stress, in tons per square inch.*
Tension and compression in butt welds . .	8
Shearing in butt welds other than webs of plate girders and joists	5
Stress in end fillet welds	6
Stress in side fillet welds, diagonal fillet welds and tee fillet welds.	5

7. A square butt weld should not be used when the thickness of the parts to be jointed exceeds $\frac{3}{16}$ in.

8. When a J or bevel butt weld must be used, the maximum permissible stresses should be reduced to three-fourths of those specified in clause 6.

9. Subject to the provisions of clause 7, any of the forms of butt weld specified in clause (ii), except a square butt weld, may be used provided the parts to be joined be not less than $\frac{3}{16}$ in. in thickness ; but the form and dimensions of the weld surfaces should be such as will provide access for the electrode to the surfaces to be welded, and enable the welder to see clearly the work in progress.

10. Steel parts less than $\frac{3}{8}$ in. in thickness should, before butt welding, be separated by a gap not less than $\frac{1}{16}$ in. Steel parts not less than $\frac{3}{8}$ in. in thickness should, before butt welding, be separated by a gap not less than $\frac{1}{8}$ in. Provided that in bevel welds the gaps above specified should be not less than $\frac{1}{8}$ in. and $\frac{3}{16}$ in. respectively.

11. A root face (if any) in a butt weld should be not more than $\frac{1}{16}$ in. in width, for steel parts not more than $\frac{1}{2}$ in. in thickness, nor more than $\frac{1}{8}$ in. in width for steel parts more than $\frac{1}{2}$ in. in thickness. In the case of a double V or a double bevel butt weld there should be no root face.

12. The included angle of a V butt weld should be not less than 70 deg. nor more than 100 deg.

13. In a bevel butt weld, the angle of bevel should be not less than 45 deg. nor more than 50 deg. ; and the edges of the steel parts should, before welding, be separated by a gap not less than $\frac{1}{8}$ in. if the parts are less than $\frac{3}{8}$ in. in thickness, and by a gap not less than $\frac{3}{16}$ in. if the parts are not less than $\frac{3}{8}$ in. in thickness.

14. In a U butt weld, the radius at the bottom of the U should be not less than $\frac{1}{8}$ in., and the angle of bevel on each face shall be at least 10 deg.

15. In a J butt weld, the radius at the bottom of the J should be not less than $\frac{3}{16}$ in. ; and the angle of bevel should be not less than 20 deg. nor more than 30 deg.

17. (*a*) Single V, U, J or bevel butt welds should be reinforced wherever practicable by depositing a run of weld metal on the back of the joint. Where this is not done, the maximum stress in the weld should be (except as provided in paragraph (*b*) of this clause) not more than one-half of the corresponding stress indicated in clause 6. (*b*) Where it is not practicable to deposit a run of weld metal on the back of the joint, then, provided another steel part is in contact with the back of the joint, and provided also the steel parts are bevelled to an edge with a gap of at least $\frac{1}{8}$ in. to ensure fusion into the bottom of the V and the steel part at the back of the joint, and provided further that the first run is made with an electrode not larger than No. 8 (S.W.G.), the working stress should not exceed that indicated in clause 6.

18. (*a*) A butt weld should be reinforced so that the thickness at the centre of the weld is at least 10 per cent. more than the thickness of the steel parts joined. (*b*) Where a flush surface is required, the butt weld should be first reinforced as in paragraph (*a*) of this clause, and then dressed flush. (*c*) Where a butt weld is dressed flush in accordance with paragraph (*b*) of this clause, the working stress in the weld metal should not exceed that specified in clause 6.

19. The throat thickness of a butt weld should be taken as the thickness of the thinner of the steel parts joined.

21. The strength of a fillet weld should be calculated on a dimension of 0·7 of the size. The effective length of a fillet weld (for the purpose of stress calculation) should be deemed to be the overall length of the weld minus twice the weld size.

22. The minimum effective length of a fillet weld required to transmit loading should be not less than 2 in. nor less than six times the size of the weld. Fillet welds connecting steel parts the surfaces of which form an angle less than 60 deg. or more than 110 deg. should not be relied upon to transmit loading.

23. A side fillet weld is a fillet weld stressed in longitudinal shear, i.e. a fillet weld the axis of which is parallel with the direction of the applied load.

24. An end fillet weld is a fillet weld stressed in transverse shear, i.e. a fillet weld the axis of which is at right angles to the direction of the applied load.

25. A diagonal fillet weld is a fillet weld inclined to the direction of the applied load.

27. The actual lengths, sizes, and types of welds should be clearly specified on the particulars, calculations and plans to be submitted to the Council. Symbols used should be as specified in B.S.S. No. 499—1933.

28. The effective cross section of a weld should be taken as the effective length of the weld multiplied by the throat thickness as specified in clause 19 for butt welds.

29. The effective section modulus (Z) of a weld or of a group of welds in a plane of a connection should be taken as the moment of inertia of the effective cross section of the weld or group about the neutral axis of the weld or group divided by the distance between the neutral axis and the edge of the effective cross-section farthest from it.

30. The direct stress (f) in fillet or in butt welds of connections stressed in tension, compression or shear should be computed by the following formula

$$f = \frac{P}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where P is the load to be transmitted by the connection, and A is the effective sectional area of the weld or welds transmitting such load.

31. The stress in the weld or welds of a connection due to bending should be computed by the following formula

$$f_b = \frac{M}{Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

where f_b is the stress due to bending, M is the bending moment transmitted by the connection and Z is the effective section modulus of the weld or welds.

32. When the weld or welds in a connection are subjected to the action of bending combined with direct stress due to shear, tension or compression, the direct stress should be computed by formula (i), the stress due to bending should be computed by formula (ii), and the resultant direct stress f_r should be determined therefrom. The resultant tensile or compressive stress f_r should not exceed the maximum permissible stress specified for tension or compression in clause 6.

33. The arrangement of welds at a joint should be such that uncertainty as to the distribution of stress is avoided as far as practicable. Where an eccentric connection cannot be avoided the bending effect should be computed and proper allowance made.

34. Members and connections should be so designed that component parts may be readily assembled and securely held in place by means of clamps or other devices. The welds should be so located as to be readily accessible for welding, inspection, painting and maintenance.

35. Connections for bracing members, of which the sections are not determined by calculated stresses, should be designed to develop the effective strength of the member.

36. In all cases where welded joints may be exposed to weather, the joining edges of the contact surfaces should be sealed by welding, or the parts should be effectively connected by welding so that the contact surfaces are securely held in contact to prevent the entry of moisture.

37. (*a*) Intermittent fillet welds may be used when continuous welds are not required for strength ; intermittent butt welds should not be used. (*b*) The longitudinal space between intermittent fillet welds should not exceed 16 times the thickness of the thinner plate in tension members or 12 times the thickness of the thinner plate in compression members.

38. For the purpose of extending the length of fillet welds within the space occupied by a joint, slots or holes may be made through one or more of the plates forming the joint ; the slot or hole should not be filled with weld metal nor partially filled in such a manner as to form a direct weld metal connection between opposite sides of the slot. The dimensions of the slot or hole should comply with the following limits in terms of the thickness of the steel part in which the slot or hole is formed : (*a*) Width to be not less than twice the thickness, with a minimum of 1 in. (*b*) Corners to be rounded with a radius not less than the thickness, with a minimum of ½ in. (*c*) Distance from the edge of the member of slot or hole to be not less than twice the thickness.

39. Combination of side and end fillet welds should be used in preference to side or end fillet welds alone.

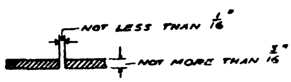


FIG. 1. SQUARE BUTT WELD

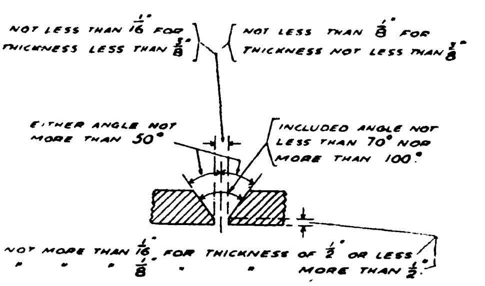


FIG. 2. SINGLE V. BUTT WELD.

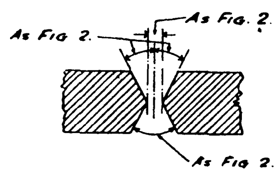


FIG. 3. DOUBLE V. BUTT WELD

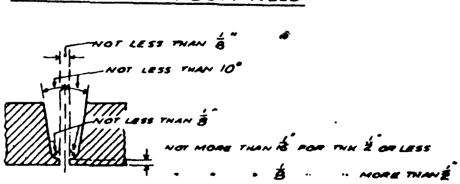


FIG 4. SINGLE U. BUTT WELD

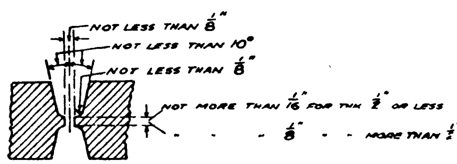


FIG. 5. DOUBLE U. BUTT WELD.

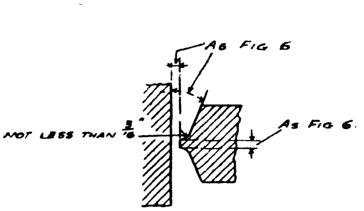


FIG. 7.—DOUBLE J. BUTT WELD

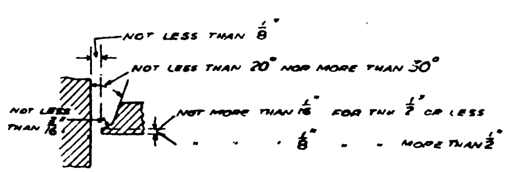


FIG. 6.—SINGLE J. BUTT WELD

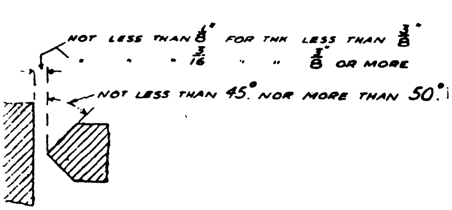


FIG. 9.—DOUBLE BEVEL BUTT WELD

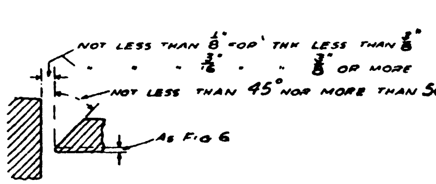


FIG. 8.—SINGLE BEVEL BUTT WELD

40. If side fillet welds are used in end connections, the length of each side fillet weld should be not less than the distance between them. The side fillet welds may be either at the edges of the members or in slots or holes.

41. In end connections a single end fillet weld should not be used without side fillet welds. Where two or more end fillet welds are used without side fillet welds, the end of each fillet weld should, wherever possible, be returned as a side fillet weld for a length of at least one inch, and in this case the full length of the end weld may be used for the purpose of calculating its strength, the return welds being disregarded.

Information for the guidance of applicants.

The Council may include in the conditions upon which a waiver or modification is granted the following requirements:

(i) Electrodes for welding should comply with the requirements for Class A electrodes in the B.S.S. No. 639—1935.

(ii) Butt welds should be made in one of the following forms: (*a*) Square butt joint (as shown in Fig. 1); (*b*) Single V butt joint (Fig. 2); (*c*) Double V butt joint (Fig. 3); (*d*) Single U butt joint (Fig. 4); (*e*) Double U butt joint (Fig. 5); (*f*) Single J butt joint (Fig. 6); (*g*) Double J butt joint (Fig. 7); (*h*) Single bevel butt joint (Fig. 8); (*i*) Double bevel butt joint (Fig. 9).

(iii) The applicant should furnish in each case, to the satisfaction of the district surveyor, as and when he may require, evidence that welding used or to be used is in accordance with the conditions prescribed by the Council in that case.

(iv) The surfaces to be welded and the surrounding material for a distance of at least ½ in. should be freed from scale and cleaned so as to remove dirt, grease, paint, heavy rust or other surface deposit, wire brushing being used if necessary. A coating of linseed oil applied for the purpose of preventing corrosion may be disregarded.

(v) Fusion faces which require to be cut to a special form or shape may be cut by shearing, clipping, machining or by a gas cutting machine. Hand cutting by gas may be substituted for machine cutting only when cutting by machine is, in the opinion of the district surveyor, impracticable, and should be so carried out that the effect of cutting is uniform. If the prepared fusion face is irregular, it should be dressed, to the satisfaction of the district surveyor, by chipping, filing or grinding.

(vi) The pieces to be welded should be securely held in their correct relative positions during welding. The welding sequence adopted should be such that distortion is reduced to a minimum.

(vii) The deposition of the weld metal should be carried out so as to ensure that: (*a*) Welds will be of good clean metal, deposited by a process which will ensure uniformity and continuity of the weld; and (*b*) the surfaces of the weld will have an even contour and regular finish indicating proper fusion with the parent metal. Welds showing cavities, or in which the weld metal tends to fall over on the parent metal without proper fusion, should be cut out and rewelded. Care should be taken to avoid undercutting; and where serious undercutting occurs, the reduction at that point should be made good by an additional run of weld metal if the district surveyor so requires. All slag should be removed after making each run, and for this purpose light hammering followed by wire brushing (or other methods which will not disturb the weld) may be used. The electric current used in making welds should be within the range defined by the manufacturer of the electrodes used.

(viii) Finished welds and adjacent parts should be coated with clean linseed oil immediately all slag has been removed. The welds and adjacent parts should not be painted until the district surveyor has had two working days in which to approve them.

(ix) Welders should be provided with such staging as will enable them properly to perform the welding operations. For site welding, shelter should be provided to protect welders and the parts to be welded from the weather.

(x) Adequate steps shall be taken to ensure that the work is of the highest quality and thoroughly reliable and that all work is done under competent and skilled supervision.

NOTE.—The above information is intended only as a general guide for applicants as the Council will deal with each application on its merits.

APPENDIX III

SPECIAL REINFORCEMENT.

REGULATIONS UNDER SECTION 9 (2) OF THE LONDON BUILDING ACT (AMENDMENT) ACT, 1935, RELATING TO APPLICATIONS FOR MODIFICATIONS OR WAIVERS OF BUILDING BY-LAWS NOS. 15, 92, 94, 96, 97, 99, 100 AND 103 TO 114 (INCLUSIVE), SO AS TO PERMIT THE USE OF STEEL REINFORCEMENT (FOR REINFORCED CONCRETE) OTHER THAN THE REINFORCEMENT REQUIRED AND PROVIDED FOR IN THE BUILDING BY-LAWS.

1. Each application for permission to use steel reinforcement (for reinforced concrete) other than the reinforcement required and provided for in the building by-laws should be accompanied by adequate particulars, calculations and plans relating to the character and quality of the reinforcement proposed and to the manner in which it is proposed to be used. If such application is granted, the Council will prescribe such conditions as it may deem proper to the use of the reinforcement in the manner proposed for that case; and such conditions will apply only in respect of the building to which such permission relates. The Council will, however, be prepared to consider preliminary applications for approval in principle of the use of steel reinforcement other than that required and provided for in the building by-laws in relation to the construction of a building. If such application is granted it will be necessary for the detailed consent of the Council to be obtained in due course to the methods to be adopted.

2. Steel reinforcement should comply with the appropriate British Standard Specification. If application is made for permission to use steel reinforcement for which there is no appropriate B.S.S., the Council may prescribe special conditions specifying the circumstances in which the reinforcement proposed may be used in that case.

3. The applicant should furnish in each case evidence to the satisfaction of the Council as to the yield-point stress, ductility, ultimate resistance to tension, and other essential properties of the reinforcement proposed.

Such evidence must (*a*) relate to the completed reinforcement rod or bar as produced in readiness for use in the reinforced concrete, and not to constituent members of such reinforcement rod or bar; and (*b*) be suitable and sufficient to enable the Council to decide whether—and, if so, the conditions under which—the reinforcement proposed may be used in that case.

The applicant shall furnish also in each case particulars to the satisfaction of the Council as to the means proposed for (*a*) enabling the district surveyor to ascertain readily that the reinforcement complies with the conditions of the Council's consent; (*b*) supervising the work during progress; and (*c*) ensuring that defective work will not be incorporated in the building to which the consent of the Council relates.

4. The Council will determine in each case the maximum permissible stresses, the limitations for spacing and arrangement to be observed, the minimum thicknesses of concrete cover to be provided, the detail requirements as to anchorage and restraint, and such other requirements as the Council may deem proper for the use of the reinforcement in the manner proposed. Save in exceptional cases, it is probable that the determination of the Council will be upon the following bases.

(*a*) Tensile stress permissible in a reinforcement not exceeding one-half of the yield-point stress (as ascertained to the satisfaction of the Council) of the completed

reinforcement rod or bar as produced in readiness for use in the reinforced concrete. In determining the tensile stress permissible, however, the Council will have regard to the evidence furnished by the applicant that undesirable cracking of the concrete will not occur,

(*b*) Longitudinal compressive stress permissible in a reinforcement not exceeding (i) where the compressive resistance of the concrete is taken into account, the calculated compressive stress in the surrounding concrete multiplied by the modular ratio based upon the modulus of elasticity (as ascertained to the satisfaction of the Council) of the completed reinforcement rod or bar as produced in readiness for use in the reinforced concrete; nor (ii) where the compressive resistance of the concrete is not taken into account, 0·4 of the yield-point stress (as ascertained to the satisfaction of the Council) of the completed reinforcement rod or bar as produced in readiness for use in the reinforced concrete. In determining the longitudinal compressive stress permissible, however, the Council will have regard to the evidence furnished by the applicant that undesirable cracking of the concrete will not occur,

(*c*) Limitations for spacing and arrangement of reinforcement, minimum thicknesses of concrete cover, detail requirements as to anchorage and restraint of reinforcement, and other matters which the Council may deem proper to prescribe for the use of a reinforcement proposed, conforming with the standard provided in the building by-laws, having regard to the special nature and shape of the reinforcement proposed.

APPENDIX IV

LONDON BUILDING ACTS 1930 & 1935

BUILDING BY-LAWS—MEMORANDUM ON COMPUTATION OF STRESSES

The following Memorandum has been issued by the London County Council to afford guidance in the computation of stresses in the construction of buildings and chimney shafts.

GENERAL

1. Computation of stresses should accord with the principles in common acceptance among responsible authorities, should maintain proper relation with the practical purposes for which it is required, and should be regarded as a means for combining economy with adequacy of construction. The preference should be for methods which are simple, direct and sufficiently approximate; methods which involve extraordinary refinement and complexity should be reserved for special cases in which they may be necessary or advantageous.

LOADING

2. Calculations of loading should include all consequential effects (e.g., uplift and sliding) induced by such actions as cantileverage, continuity, fixity, eccentric or unsymmetrical application of loading, horizontal forces and the like.

3. For ordinary construction, the weight of reinforced concrete may be assumed as 144 lb. per cubic foot.

FOUNDATIONS

4. Designs for foundations should include adequate provision for the effects of horizontal and eccentric loading, fixity of columns and the like, which may cause tendencies to uplifting, sliding or overturning, besides affecting the distribution of pressure upon the subsoil.

REINFORCED CONCRETE

5. *Basic assumptions.*—Design of reinforced concrete to resist bending should be based upon the assumptions (i) that both steel and concrete are elastic within the range of the permissible stresses; (ii) that all tensile stresses are taken by the reinforcement; and (iii) that plane sections remain plane. Stresses due to shrinkage or expansion of the concrete need not be calculated.

6. *Moment of Inertia.*—In the absence of conditions rendering such a course unjustifiable, the moment of inertia may be calculated on (i) the entire concrete section, ignoring the reinforcement; *or* (ii) the entire concrete section, including the reinforcement on the basis of the modular ratio; *or* (iii) the compression area of the concrete section, combined with the reinforcement on the basis of the modular ratio. These methods should not be changed or combined in a design. One method having been adopted, that method should be applied throughout.

7. *Bends in bars.*—The internal radius, expressed in bar diameters, of a bend in a reinforcing bar should be not less than the value obtained by dividing the stress in the steel at the commencement of the bend by four times the permissible stress in the concrete in direct compression where the minimum concrete cover is used, and not less than two-thirds of this value where conditions are such that there is no danger of splitting the concrete.

Beams and slabs

8. *Effective span.*—The effective span of a beam or slab should be taken as either (i) the distance between the centres of supports; *or* (ii) the clear distance between supports *plus* the effective depth of the beam or slab.

9. *Lateral stiffness.*—The ratio of length between adequate lateral restraints of a beam to the breadth of its compression flange should not exceed

$$20\left\{3-2\left(\frac{\text{calculated compressive stress}}{\text{permissible compressive stress}}\right)\right\}$$

10. T-*beams and* L-*beams.*—In T-beams the breadth of the flange assumed as taking compression should not exceed the least of the following: (i) one-third of the effective span of the T-beam; (ii) the distance between the centres of the ribs of the T-beams; (iii) the breadth of the rib plus twelve times the thickness of the slab.

In L-beams, the breadth of the flange assumed as taking compression should not exceed the least of the following: (i) one-sixth of the effective span of the L-beam; (ii) the breadth of the rib plus one-half of the clear distance between ribs; (iii) the breadth of the rib plus four times the thickness of the slab.

When a part of a slab is considered as the flange of a T-beam or L-beam, the reinforcement in the slab transverse to the beam should cross the full width of the flange, and should have an aggregate cross-sectional area of at least 0·3 per cent. of the total cross-sectional area of the slab; and where the slab is assumed to be spanning independently in the same direction as the beam, such reinforcement should be near the top surface of the slab.

11. Bending moments in beams and slabs should be calculated for the effective span and all loading thereon. The bending moments to be provided for at a cross-section of a continuous beam or slab should be the maximum positive and negative moments at such cross-section for the following arrangements of superimposed loading: (i) alternate spans loaded and all other spans unloaded; (ii) any two adjacent spans loaded and all other spans unloaded. Nevertheless, provided the maximum positive moments so obtained in any two adjacent spans are increased by an amount not exceeding 15 per cent. of the maximum intermediate support moment, the latter may be reduced by the same numerical amount and the positive moments elsewhere in the span increased accordingly. The computation of bending moments in flat slabs is dealt with later in this memorandum.

12. *Beams and slabs spanning in one direction.*—The bending moments in beams and slabs spanning in one direction may be calculated on one of the following assumptions:

(i) Beams which will have monolithic connection with columns should be designed accordingly, and provision should be made for the maximum bending moments, taking into account the resistance of the columns to bending;

(ii) Where beams or slabs are monolithic, they should be considered as continuous over intermediate supports and capable of free rotation at the supports. Where beams have monolithic connection with external columns (or with other columns which may be subjected to loading similar to that of external columns), they should be designed to resist a negative bending moment equal to the sum of the bending moments in the upper and lower columns, calculated in accordance with the notes later in this memorandum;

(iii) The total bending moments in all cases of uniformly distributed loading over a number of approximately equal spans may be assumed to have the following relations to those induced in a simply supported span similarly loaded:

Near middle of end span	At support next to end support	At middle of interior spans	At other interior supports
$+\frac{8}{10}$	$-\frac{8}{10}$	$+\frac{8}{12}$	$-\frac{8}{12}$

Two spans may be considered as approximately equal when they do not differ by more than 15 per cent. of the longer.

13. *Slabs spanning in two directions at right angles*—

(i) *General.*—To estimate the bending moments in a solid slab spanning in two directions at right angles, the slab may be assumed to act as a perfectly elastic thin plate, Poisson's Ratio being assumed equal to zero.

(ii) *Slabs simply supported on four sides.*—The bending moments at the centre of a rectangular solid slab spanning in two directions at right angles, with loading uniformly distributed, and simply supported on four sides, may be assumed to have the values given by the following equations.

$$M_x = Z_x\left(\frac{wl^2_x}{8}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$M_y = Z_y\left(\frac{wl^2_y}{8}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where M_x and M_y are the bending moments on strips of unit width and spans l_x and l_y respectively ;
w is the total load per unit area ;
l_y is the length of the longer side (see *Fig.* 1) ;
l_x is the length of the shorter side (see *Fig.* 1) ;
and Z_x and Z_y are coefficients shown in Table A.

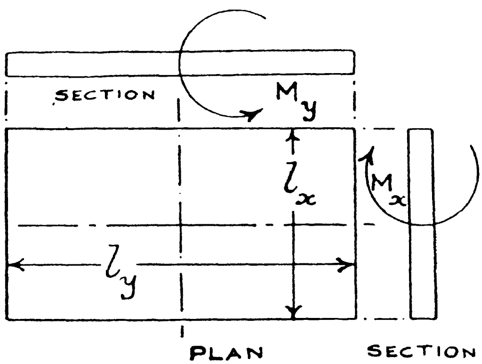


FIG. 1

TABLE A.

l_y/l_x . . .	1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0	2·5	3·0
Z_x . . .	0·500	0·594	0·675	0·741	0·794	0·835	0·904	0·941	0·975	0·988
Z_y . . .	0·500	0·406	0·325	0·259	0·206	0·165	0·096	0·059	0·032	0·022

(iii) *Slabs fixed at or continuous over four sides.*—The negative bending moments at the supports of a rectangular solid slab spanning in two directions at right angles, with loading uniformly distributed, and fixed at or continuous over four sides, may be assumed to have the values given by equations (1) and (2) above, taking the values for Z_x and Z_y shown in Table B ; and the positive bending moments near mid-span may be assumed to have values not less than 80 per cent. of those given by equations (1) and (2) above, taking the values for Z_x and Z_y shown in Table B.

TABLE B.

l_y/l_x . . .	1·0	1·1	1·2	1·3	1·4	1·5	1·75	2·0	2·5	3·0
Z_x . . .	0·295	0·358	0·419	0·477	0·532	0·581	0·681	0·757	0·869	0·940
Z_y . . .	0·295	0·237	0·191	0·154	0·127	0·107	0·071	0·051	0·032	0·022

14. *Resistance to shear*—

(*a*) *General.*—(i) The shear stress "s" at any cross-section in a reinforced concrete beam or slab should be calculated from equation (3).

$$s = \frac{S}{ba} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where S is the total shearing force across the section;
b is the breadth of a rectangular beam or the breadth of the rib of a T-beam;
and a is the arm of the resistance moment.

(ii) Where at any cross-section the shear stress, as calculated from equation (3), exceeds the permissible shear stress for the concrete, the whole shearing force should be provided for by the tensile resistance of the shear reinforcement acting in proper combination with the compression induced in the concrete thereby. Moreover, even with the whole shearing force so provided for, the shear stress as calculated from equation (3) should not exceed four times the permissible shear stress for the concrete alone.

(*b*) *Shear reinforcement.*—(i) Tensile reinforcement which is inclined and carried through a depth of beam equal to the arm of the resistance moment may be considered as shear reinforcement provided it is anchored sufficiently. (ii) Where two or more types of shear reinforcement are used in conjunction, the total shearing resistance of the beam may be assumed to be the sum of the shearing resistances computed for each type separately. (iii) The spacing of stirrups should not exceed a distance equal to the arm of the resistance moment. The resistance to shear "S" should then be calculated from the equation.

$$S = \frac{t_w A_w a}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where t_w is the permissible tensile stress in the shear reinforcement;
A_w is the cross-sectional area of the stirrup;
a is the arm of the resistance moment;
and p is the pitch or spacing of stirrups.

(iv) The resistance to shear at any section of a beam reinforced with inclined bars may be calculated on the assumption that the inclined bars form the tension members of one or more single systems of lattice girders in which the concrete forms the compression members. The shear resistance at any vertical section should then be taken as the sum of the vertical components of the tension and compression forces cut by the section.

15. *Bond and anchorage*—

(i) Exclusive of a hook or other end anchorage, a bar in tension should extend from any section for a distance such that the product of the permissible bond stress, the perimeter of the bar and the length measured from the section is not less than the tension required in the bar. At simply supported ends of beams and slabs at least one-quarter of the main tensile reinforcement should extend to the centre line of the support before the hook or other end anchorage begins. In continuous beams and slabs at least one-quarter of the tensile reinforcement should be carried for a distance not less than one-half the effective depth of the beam or slab beyond points of contraflexure before the hook or other end anchorage begins.

(ii) Subject to the provisions of (i) above, when reinforcement in the form of plain bars is used to resist stresses induced by bending, the bond stress "s_b" due

to a variation in tensile stress, calculated from the following equation (5) should not at any point exceed twice the appropriate permissible bond stress.

$$s_b = \frac{S}{ao} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where S is the total shear across the section ;
a is the arm of the resistance moment ;
and o is the sum of the perimeters of the bars in the tensile reinforcement.

In members of other than uniform depth the effect of the change in depth on the bond stress should also be taken into account.

(iii) A hook at the end of a bar should be of the form indicated in Fig. 2, and its inner diameter should be not less than four times the diameter of the bar ; except that when the hook fits over a main reinforcing or other adequate anchor bar the diameter of the hook may be equal to the diameter of such main reinforcing

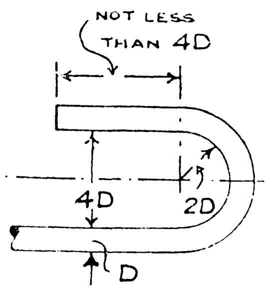


FIG. 2

or anchor bar. The length of the straight part beyond the end of the curve to the end of the hook should be at least four times the diameter of the bar forming the hook. Unless suitable wrapping or other reinforcement is provided, the anchorage value of the hook should not be taken into account if the hook is employed in a place where there is danger of splitting the concrete.

(iv) When a hook is not used, the end anchorage should consist of a length of bar or any combination of suitable attachment and length of bar, having an anchorage value not less than the resistance produced by the permissible bond stress acting over a length of bar equal to fourteen bar diameters. The anchorage value assumed should be such that neither the permissible stress on the concrete in direct compression nor the safe load on the end anchorage itself is exceeded. The permissible stress on the concrete may be increased to three times the value permitted for the concrete in direct compression where the end anchorage is employed in a place where either the cover of the concrete is sufficient, or suitable wrapping or other reinforcement is provided, to prevent local failure of the concrete.

(v) Notwithstanding any of the above notes, in the case of secondary reinforcement such as stirrups and binding, complete bond length and anchorage may be deemed to have been provided when a bend in the bar through an angle of at least 90 degrees passes round a bar of at least its own diameter and the bar is continued beyond the end of the curve for a length of at least eight diameters.

(vi) For flat slabs, see later notes in this memorandum.

Columns

16. *Short columns with lateral ties.*—The axial load " P " permissible on a short column reinforced with longitudinal bars and lateral ties should not exceed that given by the equation

$$P = c(A_c + mA) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where c is the permissible stress for the concrete ;

A_c is the cross-sectional area of concrete excluding any finishing material applied after the casting of the column;
m is the modular ratio (15);
and A is the cross-sectional area of the longitudinal steel.

17. *Short columns with helical reinforcement.*—Where helical reinforcement is used, the axial load "P" permissible on a short column should not exceed that given by equations (6) or (7), whichever is the greater:

$$P = c(A_k + mA) + 2t_bA_b \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where A_k is the cross-sectional area of concrete in the core;
t_b is the permissible stress in the helical reinforcement;
and A_b is the equivalent area of helical reinforcement (volume of helix per unit length of the column).

The sum of the loads contributed by the concrete in the core and by the helix should not exceed $0{\cdot}5\,u\,A_c$, where "u" is the crushing strength of the concrete required from the works test. When, in a column having helical reinforcement, the permissible load is based on the core area, the radius of gyration also should be based on the core area of the column.

18. *Long columns.*—The permissible working load for a long column should not exceed that calculated as above for a short column, multiplied by the appropriate buckling coefficient.

19. *Bending in columns.*—Bending actions applied to a column should be provided for—particularly in an external column and in any other column which will be loaded in like manner.

In a building of large area and of complete skeleton frame construction where the internal columns support a considerable number of spans with symmetrical arrangement both of beams and of loading, calculation of the bending actions on the internal columns (other than those due to eccentricity caused by variations in the superimposed loading) may not be necessary; but even in such cases, care is necessary to ensure that the permissible stresses are not exceeded.

Unless more exact methods are preferred, the bending moments in external (and similarly loaded) columns may be estimated from equations (8), (9), (10) and (11).

Moment at foot of upper column . . $M_e\left(\dfrac{K_u}{K_l + K_u + 0{\cdot}5K_b}\right)$ for a frame of one bay . . (8)

Moment at foot of upper column . . $M_e\left(\dfrac{K_u}{K_l + K_u + K_b}\right)$ for frames of two or more bays (9)

Moment at head of lower column . . $M_e\left(\dfrac{K_l}{K_l + K_u + 0{\cdot}5K_b}\right)$ for a frame of one bay . . (10)

Moment at head of lower column . . $M_e\left(\dfrac{K_l}{K_l + K_u + K_b}\right)$ for frames of two or more bays (11)

where M_e is the bending moment at the end of the beam framing into the column, assuming fixity at the connection;
K_u is the stiffness of the upper column;
K_l is the stiffness of the lower column;
and K_b is the stiffness of the beam.

The equations for the moment at the head of the lower column may be used for columns in a topmost storey by taking K_u as zero.

For the purposes of these equations, the "stiffness" of a member may be taken as the quotient obtained by dividing the moment of inertia of a cross-section by the length of the member, provided the member be of constant cross-section throughout its length.

20. *Combined axial and bending stresses.*—The maximum stresses on longitudinal reinforcement and concrete due to combination of direct load and bending action should not exceed the permissible stresses for bending, multiplied by the appropriate buckling coefficient.

Flat slabs

21. The following definitions and notes should be taken as relating only to the design of floors and flat roofs comprising a series of rectangular slabs of approximately constant thickness arranged in at least three rows in each direction, the ratio borne by the length of a panel to its width not exceeding 4 : 3.

22. *Definitions relating to flat slabs :*

Effective depth (as for any beam or slab) is the distance from the compressed edge of the constructional concrete to the centre of gravity of the tensile reinforcement.

A flat slab is a concrete slab forming part of a floor or flat roof, with reinforcement in two or four directions, and supported directly by columns.

A column strip is a portion of a flat slab panel, of total width usually one-half the panel width, occupying the two areas outside the middle strip.

A middle strip is a portion of a flat slab panel, of width usually one-half the panel width, symmetrical with regard to the centre-line of the panel, and extending throughout the entire length of the panel in the direction for which bending moments are being considered.

A column head is an enlargement of the top of a column supporting a flat slab and designed and constructed to act monolithically with the column and with the flat slab.

A drop is a portion of a flat slab, immediately surrounding the column head, and of greater thickness than the remainder of the flat slab panel.

A direct band is a band of reinforcing bars parallel with an edge of a flat slab panel reinforced in four directions (i.e. edgewise and diagonally).

A diagonal band is a band of reinforcing bars parallel with a diagonal of a flat slab panel reinforced in four directions (i.e. edgewise and diagonally).

Effective area of reinforcement in a diagonal band is the value obtained by multiplying the normal cross-sectional area of the reinforcement by the cosine of the angle at which the band is inclined to the direction for which its effectiveness is required.

23. The lengths and/or widths of any two adjacent panels in a series should not differ by more than 10 per cent. of the greater length or width.

24. The drops should (i) be square or rectangular in plan, and have a length in each direction not less than one-third nor more than one-half of the panel length in that direction ; or (ii) be continuous between columns, and have a width not less than one-third nor more than one-half of the panel width.

25. The width of the column strip should be taken as one-half the width of the panel, excepting that where drops are used it may be taken as the width of the drop.

26. The width of the middle strip should be taken as one-half the width of the panel ; excepting that where drops are used, and the column strip is taken as the width of the drop, the width of the middle strip should be taken as the difference between the panel width and the drop width.

27. For interior panels, fully continuous, the bending moments to be provided for at the sections indicated in *Fig.* 3, i.e. (1) Positive moment sections along the centre-lines of the panel ; (2) Negative moment sections along the edges of the panel on lines joining the centres of the columns and around the perimeter of the column heads ; should be :

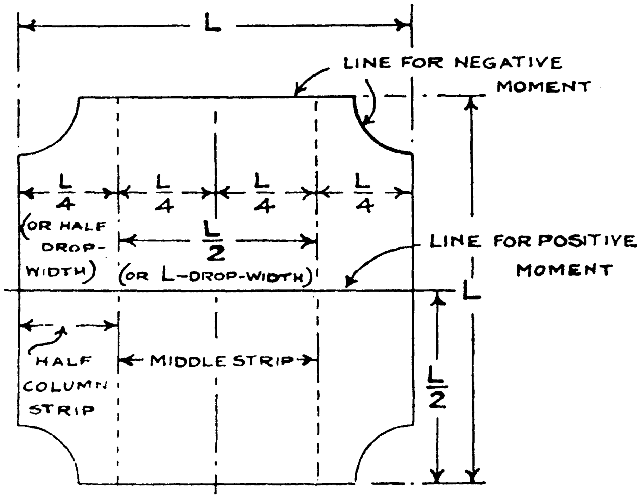
For panels without drops—

	Positive Moment.	*Negative Moment.*
Column strip	$0{\cdot}022wL_2\left(L_1 - \frac{2D}{3}\right)^2$	$0{\cdot}042wL_2\left(L_1 - \frac{2D}{3}\right)^2$
Middle strip	$0{\cdot}018wL_2\left(L_1 - \frac{2D}{3}\right)^2$	$0{\cdot}018wL_2\left(L_1 - \frac{2D}{3}\right)^2$

For panels with drops—

	Positive Moment.	*Negative Moment.*
Column strip	$0{\cdot}022wL_2\left(L_1 - \frac{2D}{3}\right)^2$	$0{\cdot}046wL_2\left(L_1 - \frac{2D}{3}\right)^2$
Middle strip	$0{\cdot}016wL_2\left(L_1 - \frac{2D}{3}\right)^2$	$0{\cdot}016wL_2\left(L_1 - \frac{2D}{3}\right)^2$

In the above and subsequent formulæ relating to flat slabs, L_1 is the length in the direction of span, L_2 is the width at right angles to that direction of the panel measured from centre-line to centre-line of columns, L is the average of L_1 and L_2. D is the diameter of the column head, and w is the total load per unit area. The formulæ give moments on the whole width of the strip.



PLAN — FIG. 3

28. Where the column strip is taken as equal to the width of the drop, and the middle strip is thereby increased in width to a value greater than half the width of the panel, the moments to be taken on the middle strip should be increased in proportion to its increased width. The moments to be taken by the column strip may then be decreased by an amount such that there is no reduction in either the total positive or the total negative moments taken by the column strip and middle strip.

29. In slabs reinforced in two directions only, the reinforcement in each strip should be so disposed that the strip is reinforced over its full width.

30. In slabs reinforced in four directions, the width of the direct bands of reinforcement should be two-fifths of the panel width at right angles to the direction of the reinforcement, and the width of the diagonal bands should be one-half the panel length—or one-half the average panel length in the case of panels which are not square.

31. In four-way systems, the reinforcement should be apportioned as follows :

(i) the reinforcement in the direct band should take the entire positive moment in the column strip ;

(ii) the reinforcement in the diagonal band should take the entire positive moment for the middle strip ;

(iii) the reinforcement in the direct band, plus the reinforcement in the diagonal bands (the effective area of which should be calculated as described above) should take the negative moment in the column strip ;

(iv) additional reinforcement should be provided to take the negative moment in the middle strip.

32. The effective depth of the slab (or of the slab and drop where a drop is used) should be determined from considerations of bending and shear, provided the effective area of tensile reinforcement used for the calculations should not be more than 1 per cent. of the product of the effective depth (which for this purpose may be taken as 1 inch less than the total thickness of the slab or drop) and the width of the strip or drop.

33. Each strip should be at all sections capable of resisting the moments specified without the use of steel in compression, except in side and end panels, and except where openings in panels necessitate rearrangement of reinforcement.

34. *Two-way systems of reinforcement:*

(i) In each strip or band 40 per cent. of the positive reinforcement should be continuous in the lower part of the slab, and extend to within a distance of 0·125L, measured from the line joining the column centres.

(ii) The negative reinforcement should extend in the top of the slab into adjacent panels for an average distance measured from the line joining the column centres not less than 0·25L, and in no case less than 0·2L.

(iii) The full area of negative reinforcement should be provided for a distance measured from the line joining the column centres not less than 0·2L. The full area of positive reinforcement should be provided for a distance measured from the centre-line of the panel not less than 0·25L.

35. *Four-way systems of reinforcement:*

(i) For direct bands, the rules above (for two-way reinforcement) should be applied.

(ii) In each diagonal band, 40 per cent. of the positive reinforcement should be continuous in the lower part of the slab, and should extend to within a distance of 0·2L measured from a line through the centre at right angles to the direction of the band.

(iii) The negative reinforcement should extend in the top of the slab into adjacent panels for an average distance of 0·4L beyond a line through the column centre at right angles to the direction of the band, and in no case less than 0·35L.

(iv) In each diagonal band the full area of negative reinforcement should be provided for a distance not less than 0·3L, measured from a line through the column centre at right angles to the direction of the band. The full area of positive reinforcement should be provided for a distance not less than 0·35L measured from a line through the centre of the panel at right angles to the direction of the band.

(v) The additional reinforcement required to take the negative moment in the middle strip should extend for a distance not less than 0·25L on each side of the line joining the column centres.

36. In all strips, the percentage of reinforcement in the direction of the strips should be not less than 0·3 per cent. of the product of the width of the strip and the effective depth.

37. *Moments in side or end panels not fully continuous.*—For side or end panels in which the slab is not continuous upon one edge or upon two adjacent edges:

(i) The positive moments to be used for sections parallel with the discontinuous edges (reinforcement at right angles to the edges) should be greater by 25 per cent. than those given above in note 27. For this purpose, provided the slab thickness is not less than that of adjacent fully continuous panels, the effective area of steel in tension may exceed the limit in note 32 and compression reinforcement may be used where necessary.

(ii) At the discontinuous edges, the negative moment (reinforcement at right angles to the edges) in the column strip should be taken as not less than 90 per cent., and in the middle strip as not less than 60 per cent. of those given in note 27.

(iii) At all discontinuous edges, the positive and negative reinforcement should extend to within three inches of the edge of the panel, and should be anchored effectively.

(iv) Where end spans are shorter than interior spans (see note 23), the moments given in paragraphs (i) and (ii) of this note may be suitably modified.

38. In a half-column strip adjacent to an edge beam the reinforcement parallel with the beam need not exceed one-quarter of that specified for the column strip in note 27 above.

39. *Openings in panels*—

(i) Except for openings complying with paragraphs (ii), (iii) and (iv) of this note, openings should be completely framed on all sides with beams to carry the loads to the columns; and an opening should not encroach upon a column head or drop.

(ii) Openings of a size such that the greatest dimension in a direction parallel with a central line of the panel does not exceed 0·4L may be formed in the area common to two intersecting middle strips, provided the total positive and negative moments be maintained as specified in note 27, and provided these total positive and negative moments be redistributed between the remaining principal design sections to meet the changed conditions.

(iii) Openings of aggregate length or width not exceeding one-tenth of the width of the column strip may be made in the area common to two column strips provided that the reduced sections are capable of carrying the appropriate moments specified in note 27.

(iv) Openings of aggregate length or width not exceeding one-quarter of the width of the strip may be made in any area common to one column strip and one middle strip, provided that the reduced sections are capable of carrying the appropriate moments specified in note 27.

40. *Shearing stresses*.—The shearing stress in the slab or drop, upon a vertical section at a distance equal to the effective depth from the column head, and the shearing stress upon a vertical section along the perimeter of the drop (where used) should not exceed that permissible.

41. *Columns supporting flat slabs*—

(i) Interior columns should be provided with enlarged heads, the diameter (D) of which should be not less than 0·2L, nor more than 0·25L, except where the column itself is of such diameter.

(ii) The diameter of the column head should be taken on a plane parallel with and $1\frac{1}{2}$ in. below the underside of the slab or drop, and should be the diameter

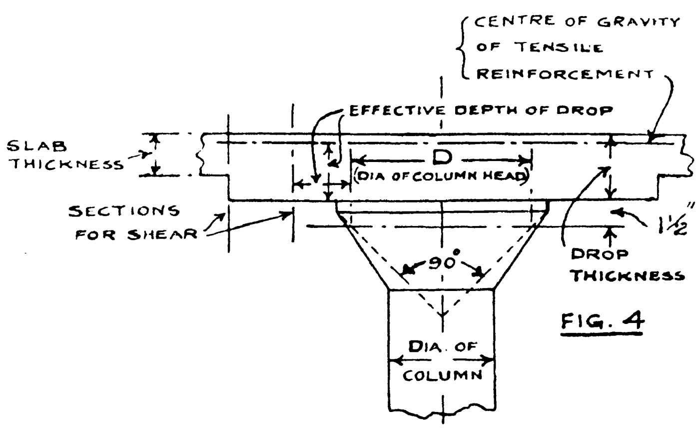


FIG. 4

intercepted by this plane on the largest inverted circular cone contained entirely within the column and its enlarged head below this plane. The vertex angle of the cone should be a right angle, and its axis the centre-line of the column (see Fig. 4).

(iii) All exterior wall columns should be provided with such portion of the enlarged head (specified in paragraph (i) of this note) as will lie within the adjoining walls, or, when rectangular columns are used with beams, the enlarged head may consist of an internal bracket not less than the full width of the inside face of the column.

(iv) The value D for a bracket head in the direction in which the bracket extends may be taken as twice the distance from the centre of the column to a point where the structural portion of the bracket thickness is $1\frac{1}{2}$ in. measured vertically from the underside of the slab or drop to a plane inclined at 45 deg. to the inside face of the column lying entirely within the bracket head, and this value of D averaged with the value of D for an interior column head in the calculations for moment under note 27. The value of D for column strips parallel and adjacent to a non-

continuous edge of a slab where either no marginal beam is used, or where the beam used is not deeper than one-and-a-half times the minimum slab thickness should be taken as equal to the width of the wall column if no bracket is provided in this direction.

(v) The value of D for column strips parallel and adjacent to marginal beams having a depth greater than one-and-a-half times the thickness of the slab at the wall columns should, if no bracket is provided in this direction, be taken as equal to the width of the wall column plus twice the difference between the depth of the beam and the depth of the slab at the column head.

(vi) Moments in internal and in external columns should be provided for and should be taken as equal to 50 and 90 per cent. respectively of the negative moment in the column strip specified in note 27. This moment should be apportioned between the upper and lower columns in proportion to their stiffnesses.

(vii) In the case of external columns carrying portions of the floors and walls as a cantilevered load, the specified column moments may be reduced by the moment due to the dead load on the cantilevered portion.

BRICK CHIMNEY SHAFTS.

42. Calculated loading and stresses for foundations and footings should include the effects of wind pressure and all other applied loading in combination with all constructional loading; and design should provide adequately for the effects of eccentric loading and the like.

Wind pressure is usually taken as 40 lb. per square foot with a reduction factor of 0·6 for a round shaft and 0·8 for any other shape.

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